



## **TUNED LIQUID DAMPERS FOR CONTROL OF EARTHQUAKE RESPONSE**

**Pradipta BANERJI<sup>1</sup>**

### **SUMMARY**

Earlier studies have shown that a tuned liquid damper (TLD) is effective in controlling the response of a structure to small amplitude and narrow-banded motions. However, since it is a tuned damper it has been implicitly assumed that a TLD is effective only for such excitations. A preliminary paper co-authored by the present author showed, through a few numerical simulations, that if the TLD is properly designed, it also has the ability to be effective in controlling response of structures to broad-banded excitations, such as ground motions generated by earthquakes. A selection of numerical simulations and experiments, done here over the past couple of years, are presented in this paper to conclusively show that a TLD is effective in controlling the response of a structure to broad-banded, long duration earthquake ground motions. It is shown that a TLD water particle motion formulation based on a shallow-wave theory proposed by earlier researchers is reasonable for predicting the response of a structure with a TLD attached to it and subjected to large amplitude earthquake type motions at its base. It is, however, interesting to note that experiments show that the above theory consistently under-predicts the reduction in structural response for a wide variety of structures and ground motions. This is possibly due to energy being dissipated by breaking waves, which is seen to occur during the excitation phase in the experiments and is only approximately modeled in the numerical simulations. The TLD parameters such as the ratio of water depth to tank length (called the depth ratio) and the ratio of water mass to the structure mass (called the mass ratio) are shown to control the effectiveness of a TLD. The response of a typical single degree-of-freedom (SDOF) structure is typically reduced by about 30% if a TLD has a depth ratio of 0.15 and mass ratio of 4%.

### **INTRODUCTION**

Tuned liquid dampers have been proposed by Modi [1] and Fujino [2] and used successfully over the past few years (see Tamura [3]) for controlling response of structures subjected to wind excitations. Since a TLD is essentially a tank of water (typically rectangular or circular in cross-section) with a controlled dimension and water-depth, it is an economical control device that requires very little maintenance, and is effective in controlling response to bi-directional excitations too, as the water in the tank is not constrained to move only in one direction. Although most of the early work was on harmonic excitations, some of the work, e.g. Koh [4], has concentrated on arbitrary excitations. However, the design of the TLD was dependent on the fact that the TLD base was subjected to only relatively small amplitude motions.

---

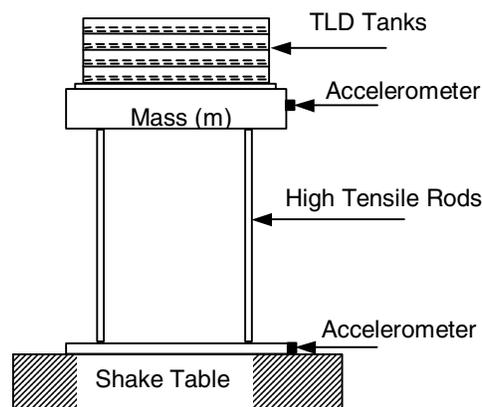
<sup>1</sup> Professor of Civil Engineering, IIT Bombay, Mumbai, INDIA. Email: pbanerji@iitb.ac.in

The response of a tuned liquid damper, even for small amplitude motions, is highly non-linear due to liquid sloshing. Considerable work has been done in developing mathematical models to describe the behavior of a TLD when subjected to an excitation at its base, e.g. Modi [1], Fujino [2], Sun [5], and Yu [6]. All of them approximate the behavior of the water under the effect of the TLD base excitation, as the actual water motion is complex and not amenable to exact mathematical modeling. In this study the model suggested by Sun [5] has been used for numerical simulation of the TLD-structure system with the base of the structure excited by an earthquake-type motion. The formulation models the non-linear behavior of the TLD, but assumes the free surface of the water to be continuous at all times, i.e. waves do not break. However, by considering correction factors, determined empirically, for some of the terms in the equation of fluid motion, it tries to approximately account for the energy dissipation through wave breaking.

In this study a series of experiments have been done to determine the behavior of a SDOF structure with a TLD attached to it – no attempt has been made to study the water motion in the TLD – when subjected to a variety of large amplitude base excitations, as can be expected in strong-motion earthquakes. The results of these experiments are used to study the applicability of the theory proposed by Sun [5] for predicting the response of a structure subjected to large amplitude, broad-banded base excitation. Furthermore, the applicability of the design procedure proposed by Banerji [7] for a TLD to be effective for broad-banded earthquake excitations is also studied.

### EXPERIMENTAL SETUP

Figure 1 shows a schematic representation of the experimental setup and TLD-structure model used for the study. The model comprises of a TLD mounted on top of a simple structural frame model, which is further mounted on a shake table. The base plate of the structural model is directly welded to the table to avoid any relative displacement between the structural base and the table. The TLD tank base is rigidly connected to its base frame, which is fixed on the top of the structure by welding. The TLD consists of four tanks that are stacked one above the other and are connected by two steel rods to act as a single unit. The liquid-to-structure mass ratio is controlled by selectively filling water in the tanks. Accelerometers are placed at the top and at the base of the structural model to measure structural and base acceleration, respectively.



**Figure 1. Schematic representation of the experimental set-up and TLD-structure model**

The structural model is made up of mild steel plates, which represent the rigid roof, supported on four high tensile steel rods of 7-mm diameter, which represent the columns. As welding a high tensile rod makes it brittle, which eventually causes it to break even at small displacements, a barrel-and-wedge system is used to connect the both the roof and base steel plates rigidly to the high tensile rods. This *innovative* technique not only offered the desired flexible structure but also flexibility in changing the frequency of this single-degree-of-freedom model by changing the position of mild steel plates along high tensile steel rods. Thus, the same structural model with varying plates being attached to it and these being positioned at different heights along the rods gives all the six different structure-TLD systems considered in the experimental study. The dynamic properties of the structure and the attached TLD details for all six cases are given in Table 1 below. The TLD parameters in these experiments have been chosen based on the experience gained from an earlier theoretical study by Banerji [7], in which the optimal parameters for a TLD to control earthquake-type base excitations were developed.

**Table 1. Structure properties and TLD parameters**

Case No.	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	
Mass [Kg]	91	92.82	130.2	101.2	61.67	54.97	
Structure Frequency $f_s$ [Hz]	1.10	1.10	1.42	1.11	1.73	1.10	
Structure Damping (%)	1.6	1.6	1.3	1.2	1.2	1.0	
Tank Size	Length [mm]	153	228	175	280	119	160
	Width [mm]	228	141	280	175	280	520
Depth Ratio ( $\Delta$ )	0.083	0.127	0.143	0.151	0.157	0.078	
Mass Ratio ( $\mu$ [%])	0.49, 0.97, 1.95	1.00, 2.01, 4.01	0.94, 1.88, 3.77	2.05, 4.09	1.01, 2.02, 4.04	1.89, 3.78	

## NUMERICAL MODEL FORMULATION

### Structure Idealization

The single-degree-of-freedom (SDOF) structure with a TLD attached to it shown in Fig. 1 is subjected to a ground motion, characterized by the ground acceleration time history, denoted by  $a_g$ . The equation of motion for this SDOF structure is

$$m_s \ddot{v}_s + c_s \dot{v}_s + k_s v_s = -m_s a_g + F \quad (1)$$

where  $m_s$ ,  $k_s$  and  $c_s$  represent the mass, stiffness and damping of the structure, respectively. Moreover,  $v_s$  is the displacement of the structure relative to the ground, and  $F$  denotes the shear force developed at the base of the TLD due to water sloshing. Equation (1) when normalized with respect to the structural mass is given as

$$\ddot{v}_s + 2\xi_s \omega_s \dot{v}_s + \omega_s^2 v_s = -a_g + \frac{F}{m_s} \quad (2)$$

where  $\omega_s = 2\pi f_s$  and  $\xi_s$  are the structure's natural circular frequency and damping ratio, respectively, and  $f_s$  is the natural frequency. The TLD base shear force  $F$ , shown on the right hand side of equation (2), is determined by solving the equations of motion of water in the TLD.

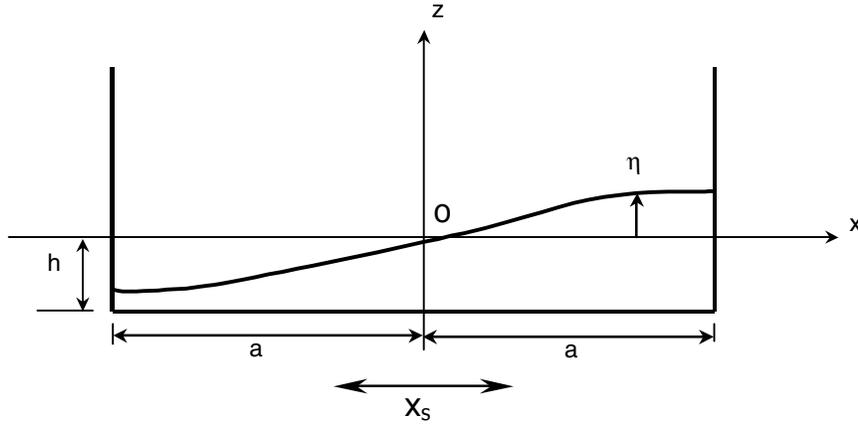
### Formulation of TLD Equations

The rigid rectangular TLD tank, which is shown in Fig. 2, has a length  $2a$  and width  $b$  (not shown in the figure), and an undisturbed water depth of  $h$ . It is subjected to a lateral base excitation,  $x_s$ , which is identical to the excitation of the structure's top. The equations of motion of the water inside the tank can be defined in terms of the free surface motion, as the water depth is assumed to be shallow (Fujino [2]). Since strong earthquake ground motion generally results in large amplitude TLD excitation, the equations of motion should include the effects of wave breaking. The formulation used here has been suggested by Sun [5], and the governing equations of motion of the water are

$$\frac{\partial \eta}{\partial t} + h\sigma \frac{\partial(\phi u)}{\partial x} = 0 \quad (3)$$

$$\frac{\partial u}{\partial t} + (1 - T_H^2)u \frac{\partial u}{\partial x} + C_{fr}^2 g \frac{\partial \eta}{\partial x} + gh\sigma\phi \frac{\partial^2 \eta}{\partial x^2} \frac{\partial \eta}{\partial x} = -C_{da}\lambda u - \ddot{x}_s \quad (4)$$

where the independent variables are  $\eta(x,t)$  and  $u(x,\eta,t)$ . They denote the free surface elevation above the undisturbed water level and the horizontal free surface water particle velocity, respectively. Both these variables are a function of the horizontal distance,  $x$ , from  $o$  (see Fig. 2) and time  $t$ . The horizontal acceleration of the TLD base, which is identical to the total acceleration of the structure's top, is  $\ddot{x}_s$ , and the acceleration due to gravity is  $g$ .



**Figure 2. Schematic sketch of TLD for horizontal motion**

Equation (3) represents the integrated form of the continuity equation for the water, and equation (4) is derived from the two-dimensional Navier-Stokes equation. The parameters  $\sigma$ ,  $\phi$ , and  $T_H$  in equations (3) and (4) are given by the expressions

$$\sigma = \tanh kh / kh \quad \phi = \tanh k(h + \eta) / \tanh kh \quad T_H = \tanh k(h + \eta) \quad (5)$$

where  $k$  is the wave number. The  $\lambda$  in equation (4) is a damping parameter that accounts for the effects of the boundary layer along the tank bottom, side walls, and the water's free surface contamination that can be given semi-analytically as

$$\lambda = \frac{1}{(\eta+h)} \frac{1}{\sqrt{2}} \sqrt{\omega_l \nu} \left[ 1 + \left( \frac{2h}{b} \right) + s \right] \quad (6)$$

in which  $\omega_l$  is the fundamental linear sloshing frequency of the water in the tank,  $\nu$  denotes the kinematic viscosity of water, and  $s$  denotes a surface contamination factor which can be taken as unity. The fundamental linear sloshing frequency of the TLD is given by

$$\omega_l = \sqrt{\frac{\pi g}{2a} \tanh(\pi \Delta)} \quad (7)$$

where  $\Delta$  is the ratio of undisturbed water depth  $h$  to the tank length  $2a$ , called the water depth ratio.

The coefficients  $C_{fr}$  and  $C_{da}$  in equation (4) are incorporated to modify the water wave phase velocity and damping, respectively, when waves are unstable ( $\eta > h$ ) and break. These coefficients take on a unit value when waves do not break. Conversely, when waves break,  $C_{fr}$  is found empirically (Sun [5]) to essentially have a constant value of 1.05, whereas  $C_{da}$  has a value that is dependent on the amplitude,  $(x_s)_{\max}$ , of motion of the structure's top when it does not have a TLD attached to it. This  $C_{da}$  value is given as

$$C_{da} = 0.57 \sqrt{\frac{h^2 \omega_l}{a \nu} (x_s)_{\max}} \quad (8)$$

where, as before,  $h$  and  $a$  are the water depth and half tank length, respectively, and  $\omega_l$  is the sloshing frequency given by equation (7).

By solving equations (3) and (4) simultaneously for the free surface elevation  $\eta$ , and neglecting higher order terms and shear stresses along the bottom of the tank, a reasonable estimate of the shear force,  $F$ , at the base of the TLD is given by the following expression:

$$F = \frac{\rho g b}{2} \left[ (\eta_n + h)^2 - (\eta_0 + h)^2 \right] \quad (9)$$

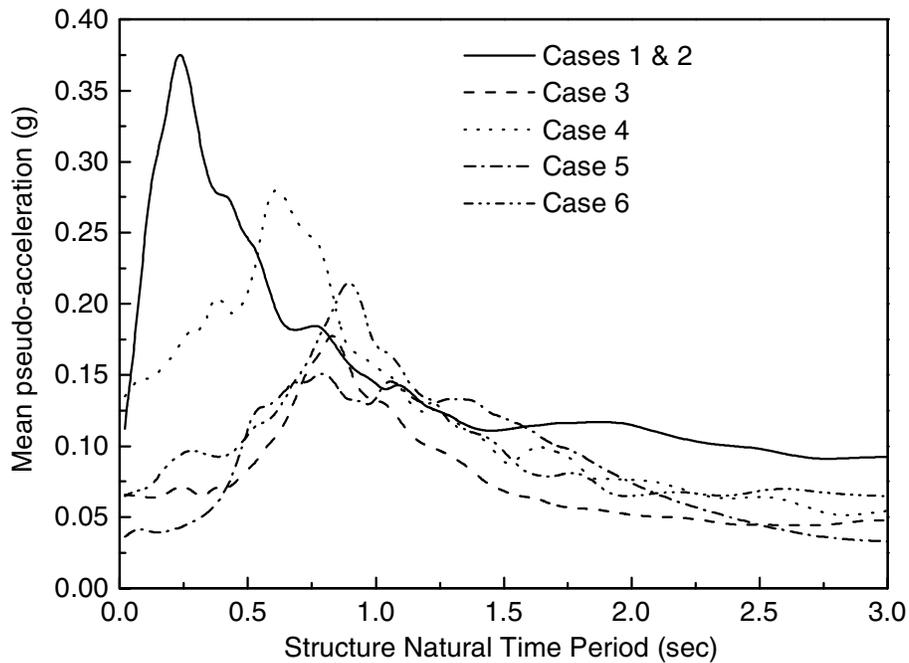
where  $\rho$  is the mass density of water,  $b$  is the tank width, and  $\eta_n$  and  $\eta_0$  are the free surface elevations at the right and left walls, respectively, of the tank.

### Analysis Procedure

Equations (2) through (4) have to be solved simultaneously to find the response of a SDOF structure with a TLD attached. Although the structure's behavior is linear, the water motion is non-linear. Therefore, an iterative numerical procedure is needed to compute the structure's response. Equations (3) and (4) are discretized, with respect to  $x$ , into difference equations and then they are solved using the Runge-Kutta-Gill procedure. Equation (2) is solved using a central difference scheme in which the time step depends on defining the sloshing phenomenon properly but is also small enough to ensure numerical stability.

## RESULTS AND DISCUSSION

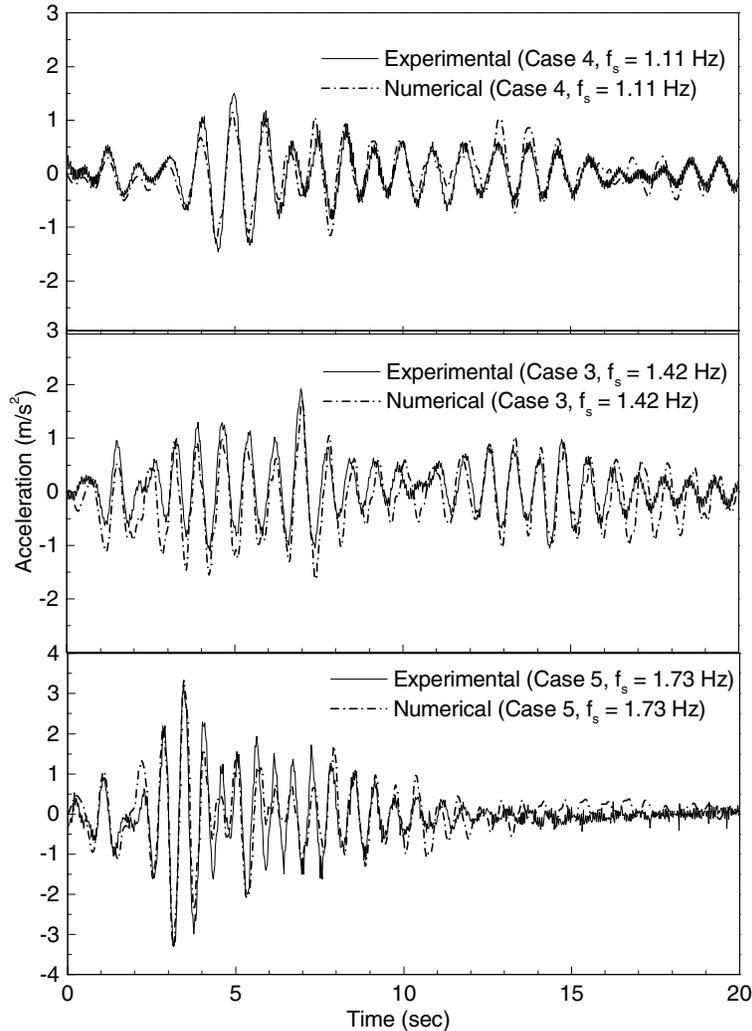
The experiments have been done for a variety of artificially generated ground motions: narrow-banded motions, broad-banded motions with the excitation frequencies in a band around the structure frequency, broad-banded motions with the excitation frequencies away from the structure frequency, small amplitude and large amplitude motions. Here the results for only a representative sample of ground motions are shown (more results and details are available in Banerji [8]). For each type of motion, the structure is subjected to an ensemble of 15 artificially generated acceleration time histories. The mean pseudo-acceleration response spectrum for the ground motions considered in this study for each structure case given in Table 1 is shown in Fig. 3.



**Figure 3. Mean pseudo-acceleration spectra for 2% damping for the artificially generated base motions used in the shake table experiments for all the structure cases**

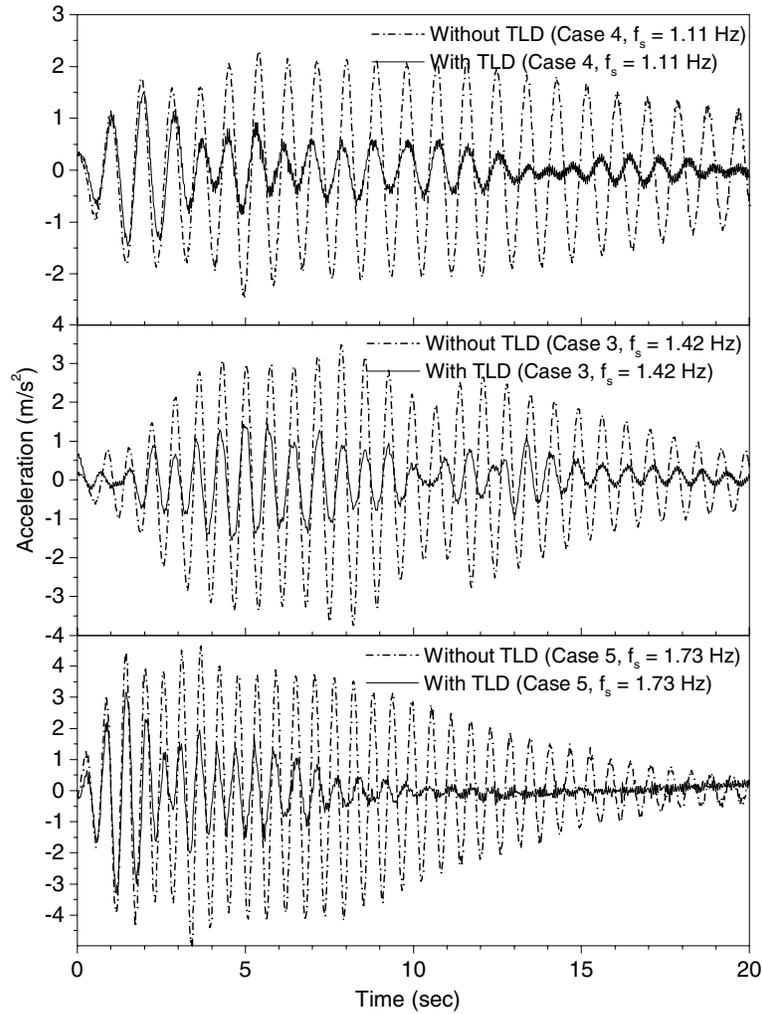
The acceleration response time history for the SDOF structure, when the TLD-structure system is subjected to a particular base excitation, is measured from the experiment and also determined using the numerical formulation given earlier in this paper. Typical plots of these comparative time histories for the three different structure cases with optimally designed TLD (depth ratio = 0.15 and mass ratio = 4%), as given in Table 1, are given in Fig. 4. It is instructive to note that the response time histories obtained from numerical simulation for the measured base excitation is comparable to the measured response time histories, at least in the strong motion part of the time history for all the structure and ground motion cases. This was uniformly true for all large amplitude broad-banded base excitations, although less so for harmonic base excitations. It can, therefore, be reasonably concluded that the theoretical formulation of Sun [5] is applicable for determining the structural response in a structure-TLD system for large amplitude broad-banded base excitations, although less so for large amplitude harmonic excitations. This is probably due to the fact that during the experiments a visual record showed that the free surface of the water in the

TLD tanks was reasonably clearly visible, with some amount of wave breaking due to steep waves, for the broad-banded motions, which is broadly within the assumptions of the theoretical formulation. The free surface, however, was never properly visible for large amplitude harmonic motions, with significant churning of the water, thus violating the basic assumptions of the theoretical formulation.



**Figure 4. Comparison of typical numerical and measured structure acceleration response histories for three different cases with TLD mass ratio  $\mu = 4\%$**

The effectiveness of a properly designed TLD in reducing structural response for the three structural cases with optimally designed TLD is clearly illustrated in Fig. 5, where a comparison of typical measured acceleration response time histories for the SDOF structures with and without the TLD is shown. The experiments show that the maximum reduction in structural response is for large amplitude harmonic motions with excitation frequency close to the structure frequency. However, the reduction in structural response for broad-banded base motions is also significant.



**Figure 5. Typical measured structural acceleration time histories for a few different cases to illustrate the effectiveness of a TLD with mass ratio  $\mu = 4\%$**

The earlier numerical study, Banerji [7], showed that it is important to define the two TLD parameters, the mass and depth ratios, properly to ensure the effectiveness of a TLD for large amplitude motions. Various mass ratios from 0.5% to 4% (higher values are not effective as they add to the inertial load on the structure due to base excitation) and depth ratios from 0.08 to 0.16 (the higher value is close to the limit of shallow water levels as defined by Fujino [2]) have been considered in the experiments (see Table 1).

The effect of the variation in mass ratios on reduction in structural response for the broad-banded motions considered in this study is illustrated in Table 2. It is seen that a mass ratio of 1% or less has little effect on the response of a structure. The reduction in structural response increases with increasing mass ratio. The reduction in response for a mass ratio of 4% varies from a low of 24.4% for the case 2 structure to a high of 40.2% for the case 4 structure, which is not inconsiderable. Furthermore, it can be seen that the

reductions predicted by the numerical simulations are comparable to those obtained from the experiments, although they are consistently lower. This has been seen consistently over all the experiments that have been reported in Banerji [8].

**Table 2. Effect of mass ratio on percentage reductions in mean peak structure acceleration (PSA)**

Mass ratio	$\mu = 1\%$		$\mu = 2\%$		$\mu = 4\%$	
	Exp.	Num.	Exp.	Num.	Exp.	Num.
Case 1	8.8	7.9	15.6	13.2	-	-
Case 2	9.1	5.8	16.1	14.1	24.4	23.4
Case 3	17.0	14.1	28.9	27.0	38.2	35.0
Case 4	-	-	29.1	27.1	40.2	37.0
Case 5	13.0	11.5	19.6	18.9	28.5	26.8
Case 6	-	-	21.4	19.8	34.0	32.2

The effect of the depth ratio on the reduction in structural response is illustrated in Table 3. All other structure and TLD parameters for the case 6 and case 4 structures are identical (within a small range as can be seen in Table 1, which is all that can be achieved in experiments). One point that is immediately obvious, is the similarity in the values obtained from experiments and numerical simulations, as has been discussed earlier. The other point is the increase in effectiveness of the TLD due to increase in depth ratio. This is true for both 2% and 4% mass ratios. One point that was further obvious from numerical simulations (Banerji [8]), was that increasing the depth ratio led to a corresponding increase in the bandwidth of effectiveness of a TLD, i.e. the TLD with a larger depth ratio was effective over a larger range of structural frequencies for a given broad-banded earthquake-type base excitation. It is, therefore, concluded that a depth ratio of about 0.15 and a mass ratio of 4% is optimum and makes a TLD most effective for controlling the structural response to broad-banded base excitation.

**Table 3. Effect of depth ratio on percentage reductions in mean peak structure acceleration (PSA)**

Mass ratio	Experimental		Numerical	
	$\Delta=0.078$ (Case 6)	$\Delta=0.151$ (Case 4)	$\Delta=0.078$ (Case 6)	$\Delta=0.151$ (Case 4)
2 %	21.4	29.1	19.8	27.1
4 %	34.0	40.2	32.2	37.0

A point that could be raised against all the above results is that all of them, both experimental and numerical, are for artificially generated ground motions, and need not represent the true behavior of a TLD-structure system when subjected to actual ground motions that are seen in real earthquakes. The constraints of the experimental setup at IIT Bombay as well as the need to study the behavior for different types of base motions, as mentioned earlier, required the use of artificially generated ground motions in the experiments. However, one important conclusion that can be drawn from the above results, is the closeness of the structural response estimation using numerical simulation to that determined from experiments for a wide variety of broad-banded base excitations. Therefore, it is possible to study the behavior of a TLD-structure system for actual ground motions using the numerical simulation procedure. An ensemble of 30 recorded ground motions, available from the CSMIP database for California earthquakes, is considered for this analysis. As has been shown in Banerji [7], considering only one

ground motion leads to anomalies in understanding the behavior of the TLD-structure system. The characteristics and identities of the ground motions are given in Table 4 below. The ground motions represent a range of earthquake intensities too.

**Table 4. List of recorded earthquake ground motions considered for analysis**

Earthquake	Station name	Comp.	PGA(g)
Loma Prieta 1989	Foster City Redwood Shores	0 <sup>0</sup>	0.258
	Gilroy#2-HWY 101/ Bolsa Rd. Motel	0 <sup>0</sup>	0.351
	Hayward-Bart Station	220 <sup>0</sup>	0.156
	Sago South- Hollister Cienega Rd.	261 <sup>0</sup>	0.072
	Hollister-South Street and Pine Drive	0 <sup>0</sup>	0.369
	Richmond-City Hall Parking Lot	190 <sup>0</sup>	0.125
	San Francisco Bay – Dumbarton Bridge	267 <sup>0</sup>	0.129
	Woodside – Fire Station	0 <sup>0</sup>	0.099
	Capitola – Fire Station	0 <sup>0</sup>	0.472
	San Francisco International Airport	0 <sup>0</sup>	0.235
	Yerba Buena Island	0 <sup>0</sup>	0.029
	Coyote Lake Dam – Downstream	195 <sup>0</sup>	0.158
	Gilroy # 6 – San Ysidro	90 <sup>0</sup>	0.170
	Olema – Point Reyes Ranger Station	90 <sup>0</sup>	0.102
	Agnew – Agnews State Hospital	0 <sup>0</sup>	0.166
	San Francisco – Diamond Heights	0 <sup>0</sup>	0.098
Northridge 1994	New Hall LA Country Fire Station	360 <sup>0</sup>	0.308
	Camarillo	180 <sup>0</sup>	0.125
	Alhambra – Fremont School	360 <sup>0</sup>	0.08
	Los Angeles – Baldwin Hills	90 <sup>0</sup>	0.239
	Los Angeles – Hollywood Storage grounds	90 <sup>0</sup>	0.231
	Los Angeles – Obergon Park	90 <sup>0</sup>	0.355
	Mt. Wilson – Caltech Seismic Station	90 <sup>0</sup>	0.133
	Pacoima – Kagel Canyon	90 <sup>0</sup>	0.30
	Point Mugu – Naval Air Station	90 <sup>0</sup>	0.143
	Rolling Hills Estates – Rancho Vista Sch.	90 <sup>0</sup>	0.116
	Sanpedro – Palos Verdes	90 <sup>0</sup>	0.095
Whitter, 1987	Vasquez Rocks Park	360 <sup>0</sup>	0.151
	Lake Hughes # 9	90 <sup>0</sup>	0.225
	Inglewood – Union Oil Yard	220 <sup>0</sup>	0.156

The response of a variety of structures, with different natural frequencies and 2% damping, for each of the above ground motions is determined using the numerical procedure outlined earlier in this paper. The mean peak acceleration response of each structure to the ensemble of ground motions and the reduction in the response by an optimally designed TLD (4% mass ratio, 0.15 depth ratio) is presented in Table 5. It is obvious from these results that the optimally designed TLD is equally effective in controlling the earthquake response of a structure for actual ground motions. It should be noted that the response reduction values presented in Table 5 have been obtained from numerical simulation, and noting the conclusion that actual reduction as observed in experiments is typically more, the reduction in response

across the band of structure natural frequencies considered here is at least between 20% for structures with natural frequencies well away from the band of ground frequencies to about 35% for structures with natural frequencies in the band of ground frequencies. This is comparable to the results obtained for artificially generated ground motions and presented earlier in this paper.

**Table 5. Percentage reduction in mean peak structure acceleration (PSA) by a TLD with  $\mu=4\%$ , when structures with 2% damping is subjected to the ensemble of the 30 recorded ground motions**

$f_s$ (Hz)	0.5	0.67	1.0	1.33	1.5	2.0
$a_0$ (m/s <sup>2</sup> )	2.03	3.24	6.34	8.77	8.47	9.68
%age red.	20.2	20.7	22.1	32.4	17.7	21.8

## CONCLUSIONS

The major emphasis in this paper is on using the results of extensive experimental and numerical simulations to illustrate that a TLD, which is one of the most economical control devices currently available, can be designed to effectively control the response of a structure subjected to large amplitude broad-banded base excitations, such as those experienced during an earthquake. Some of the major conclusions are:

1. A numerical simulation procedure based on the TLD formulation proposed by Sun [5] can reasonably predict the structural response in a TLD-structure system that is subjected to a large amplitude broad-banded base excitation, although it slightly underestimates the reduction in structural response by a TLD, probably due to an underestimation of energy dissipation by wave breaking during the strong shaking phase of the base excitation.
2. The TLD-to-structure mass ratio and the depth ratio (ratio of water depth to tank length in direction of shaking) are TLD parameters that have a significant effect of the ability of a TLD to control structural response to large amplitude base excitations. A 4% mass ratio and a 0.15 depth ratio enable a TLD to be most effective for broad-banded ground motion. This optimal TLD can reduce the response of a SDOF structure typically by about 30%, which is sufficient from a design point of view.

Although most of the results are presented for artificially generated ground motions, a set of results show that the conclusions drawn from this paper are equally valid for actual earthquake ground motions. The author strongly feels that further studies after actual implementation of TLDs as earthquake vibration control devices on actual structures would lead to greater confidence in this economical device.

## ACKNOWLEDGEMENT

This study is part of a project that was partially funded by the Ministry of Human Resources Development, Government of India under its R&D scheme. This funding is gratefully acknowledged. The author also acknowledges the contribution of his various research students who have contributed to the body of knowledge on which this paper is based.

## REFERENCES

1. Modi VJ, Welt F, Irani MB. "On the suppressing of vibrations using nutation dampers." *Journal of Wind Engineering and Industrial Aerodynamics* 1990; 33: 273-82.
2. Fujino Y, Sun LM, Pacheco BM, Chaiser P. "Tuned Liquid Dampers (TLD) for suppressing horizontal motion of structures." *Journal of Engineering Mechanics, ASCE* 1992; 118(10): 2017-30.
3. Tamura Y, Fujii K, Ohtsuki T, Wakahara R. "Effectiveness of Tuned Liquid Dampers under wind excitations." *Engineering Structures* 1995; 17(9): 609-21.
4. Koh CG, Mahatma S, Wang CM. "Theoretical and experimental studies on rectangular tuned liquid dampers under arbitrary excitations." *Earthquake Engineering Structural Dynamics* 1994; 23: 17-31.
5. Sun LM, Fujino Y, Pacheco BM, Chaiser P. "Modeling of Tuned Liquid Damper (TLD)." *Journal of Wind Engineering and Industrial Aerodynamics* 1992; 41-44: 1883-97.
6. Yu J, Wakahara T, Reed DA. "A non-linear numerical model of the tuned liquid damper." *Earthquake Engineering Structural Dynamics* 1999; 28: 671-86.
7. Banerji P, Murudi M, Shah AH, Popplewell N. "Tuned liquid dampers for controlling earthquake response of structures." *Earthquake Engineering Structural Dynamics* 2000; 29: 587-602.
8. Banerji P., Murudi M, Sarada S, Chavan S. "Passive Structural Control using Tuned Liquid Dampers." Department of Civil Engineering Research Report, Mumbai: IIT Bombay, 2003.