3-D FINITE ELEMENT NONLINEAR DYNAMIC ANALYSIS
FOR SOIL-PILE-STRUCTURE INTERACTION

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SUMMARY

A finite element analysis for soil-pile-structure interaction for dynamic loads has been presented. The proposed model for the soil-pile-structure (SPS) system consists of two subsystems: a structure subsystem and a pile-foundation subsystem. Seismic response of the system is found using a successive-coupling incremental solution scheme. Material nonlinearity of soil is introduced using an advanced plasticity based model, HiSS. Proper boundary conditions, simulating radiation effects are used. Both a single pile and a pile group are considered.

Effects of nonlinearity on the pile head response and on the response of the structure are investigated. Both harmonic and transient motions are considered in the analyses. It is observed that nonlinearity significantly affects seismic response of pile foundation as well as that of the structure. Effects of nonlinearity on response are dependent on the frequency of excitation with nonlinearity causing an increase in response at low frequencies of excitation. However, its effects are not significant at high frequencies.

INTRODUCTION

With advent of fast computers, focus of researcher is increasing towards precise analyses. Moreover complex design of nuclear power plants has demanded for rigorous soil-pile-structure interaction analyses for seismic excitations. Significant works [1-5] have been reported for dynamic analyses of soil-pile-structure interaction problems. Heavy damage caused to pile foundations during recent devastating earthquakes (e.g. Bhuj Earthquake of 2001, Chi-Chi Earthquake of 1999, and Kocaeli Earthquake of 1999) has warranted a need to take into account the nonlinear behavior of soil for designing pile supported structures. To adequately account for the soil nonlinearity, dynamic analysis should be performed in the time domain. Nogami [6] introduced material and geometrical nonlinearity in the analysis using discrete systems of mass, spring and dashpots. However, proper representation of damping and inertia effects of continuous soil media is difficult with such discrete systems.

The inclusion of material nonlinearity due to soil plasticity requires the analysis to be performed using finite element approach. Wu [7] presented a quasi-3D method for the analysis of nonlinear pile response.

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Bentley [8] investigated the kinematic response of single piles considering soil plasticity without work hardening. Cai [9] included the material nonlinearity of soil using a finite element technique in the time domain but assumed fixed boundary conditions and neglected damping in the foundation subsystem. Using the hierarchical single surface (HiSS) soil model, Maheshwari [10] examined the effects of material nonlinearity of soil on the free field response as well as on the kinematic response of single piles. Maheshwari [11] extended it for pile groups. In this paper, the effect of the superstructure is analyzed. The analysis is performed for both harmonic and transient excitations and both linear and nonlinear responses are compared.

**MODELING FOR SPS SYSTEM**

Two soil-pile-structure (SPS) systems are considered: one involves a single pile (Fig. 1a) and the other involves a 2*2 pile group (Fig. 1b). The proposed SPS model is a three-dimensional nonlinear finite element model that consists of two subsystems: a structure subsystem and a pile-foundation subsystem (for brevity, foundation subsystem). The two subsystems are connected at the junctions between the pile cap (pile head for a single pile) and the column base(s) of the structure (Fig. 1). The interaction between the two subsystems is transmitted through the motions and dynamic forces of the pile cap and column base(s). The subsystem approach facilitates the computational efficiency.

![Soil-pile-structure systems considered in the analyses: (a) A single pile system, (b) a 2*2 pile group system](image-url)

Full three-dimensional geometric models are used to represent the SPS systems. Taking advantage of symmetry and anti-symmetry, only one fourth of the actual model was built, which dramatically improves the efficiency of computation. Finite element models of the foundations of SPS systems considered are
shown in Fig. 2. The piles have square cross-sections, are fully embedded in the soil and are fixed in the bedrock. For pile groups, the pile spacing ratio, $s/d = 5$. The superstructure considered for the single pile case consists of a massive column with a rectangular cross-section and is directly attached to the pile head (Fig. 1a). For the pile group, a rigid massless cap connects all the pile-heads and the superstructure consists of four massive columns (each is similar to that used in the single pile case). The superstructure is placed such that its center of gravity coincides with the center of the foundation (pile cap), thus maintaining full symmetry.

**Fig. 2. 3D finite element quarter models used for the foundation subsystems:**
(a) A single pile, (b) a 2*2 pile group

The soil and piles are modeled using eight-node brick elements with each node having three translational degrees of freedom. These elements are selected because in the present study response is dominated by shear deformations. Kelvin elements (spring and dashpot) are attached in all three directions along the mesh boundaries (Fig. 3) in order to model the far field conditions and allow for wave propagation. The coefficients of the springs and dashpots are derived separately for the horizontal and vertical directions. The structure is modeled using simple two-node beam elements with six degrees of freedom (3 translations and 3 rotations) at each node. Boundary conditions at the axes of symmetry and anti-symmetry are discussed later.

It is assumed that the soil and pile are perfectly bonded. However, separation between the soil and pile can be considered using no tension elements as shown by Maheshwari [11]. The piles are assumed to behave linearly but their nonlinear behavior can also be modeled using an appropriate constitutive relation. For the nonlinear soil model (HiSS), the initial stress condition in the soil is governed by the confining pressure of the soil and is proportional to the depth (Fig. 3b). The seismic excitation is assumed to act on the fixed base nodes and consist of vertically propagating shear waves.
Fig. 3. Finite element model for the pile group system: (a) Top plan
(b) Front elevation with initial pressure distribution
SUCCESSIVE-COUPLING INCREMENTAL SOLUTION SCHEME

Fig. 4. Schematic of successive-coupling scheme

Soil-Pile-Structure System

\[ t = \Delta t \]

\[ \begin{align*}
F_p &= 0 \\
\ddot{U}_g (\Delta t) &= \ddot{V}_p (\Delta t) \\
\ddot{V}_b (\Delta t) &
\end{align*} \]

Solve for \( \ddot{V}_p (\Delta t) \) \hspace{1cm} \text{Solve for} \( F_p (\Delta t) \)

\[ t = 2\Delta t \]

\[ \begin{align*}
F_p (\Delta t) \\
\ddot{U}_g (2\Delta t) &= \ddot{V}_p (2\Delta t) \\
\ddot{V}_b (2\Delta t) &
\end{align*} \]

Solve for \( \ddot{V}_p (2\Delta t) \) \hspace{1cm} \text{Solve for} \( F_p (2\Delta t) \)

Fig. 4. Schematic of successive-coupling scheme
A successive-coupling incremental solution scheme in the time domain is used to solve the seismic response of the SPS system. In this methodology, the motions from the pile cap and forces from the column bases are transmitted from one subsystem to another while moving to a forward time step (Fig. 4). The time history of seismic excitation is divided into small time steps, each equal to $\Delta t$. This procedure is repeated until the entire response history is determined. It is seen that the continuous response history is well approximated by the discrete step approach using a sufficiently small time step (i.e. $\Delta t = T/80$, where $T$ is the period of the excitation).

FORMULATION AND COMPUTERIZATION

Governing Equation of Motion

The loading history is represented in the time incremental form. The governing equation of motion at time $t+\Delta t$, Bathe [12] is:

$$M^{t+\Delta t} \ddot{U} + C^{t+\Delta t} \dot{U} + K^{t+\Delta t} U = t+\Delta t R$$  \hfill (1a)

where $t+\Delta t R$ is the external load at this time step. The mass matrix $M$ is diagonal because all masses are lumped at the nodal points. $C$ is a global damping matrix and includes the effects of both material damping and radiation damping (dashpots) along the boundary. $K$ is a symmetric stiffness matrix determined assuming full coupling in all three directions of motion and includes the stiffness of springs at the boundary nodes. $U, \dot{U}, \ddot{U}$ are relative nodal displacement, velocity and acceleration, respectively at $t+\Delta t$ due to seismic excitation. Employing the constant average acceleration method of integration (Bathe [12]), Eq. (1a) is solved for displacement $t+\Delta t U$. For the linear case, iterative analysis is not required but is carried out incrementally. When including material nonlinearity due to soil plasticity, matrices $K$ and $C$ do not remain constant but change after each time step. Therefore, an iterative scheme (modified Newton-Raphson) for the solution is used.

The structure subsystem may be subjected to non-uniform foundation motion (support excitation) that is equal to the pile heads motion. The non-uniform foundation motion is obtained from the coupling kinematic-inertial interaction of the soil-pile-structure system (Fig. 4). For the structure subsystem, external forces in Eq. (1a) is given by

$$t+\Delta t R = -M_s R_s t+\Delta t \dot{U}_g$$  \hfill (1b)

where $M_s$ is the mass matrix of the structure subsystem and $R_s$ is a matrix that contains the pseudo-static response influence coefficients. $\dot{U}_g$ is a vector that contains the non-uniform support motion and is equivalent to the pile head motion. For the foundation subsystem, external forces in Eq. (1) is given by:

$$t+\Delta t R = -M_F R_F t+\Delta t \dot{V}_b + t F_p$$  \hfill (1c)

where $M_F$ is the mass matrix of the foundation subsystem and $R_F$ is the pseudo-static response influence coefficients matrix that is updated at each step of time for the nonlinear soil model. $\dot{V}_b$ is the vector of bedrock acceleration due to seismic excitations and is assumed to consist of vertically propagating shear waves. $F_p$ is the pile head force vector and is calculated from the support force vector by solving structure subsystem.
Boundary Conditions

The boundary conditions are shown in Figs. 3a and 3b. The presence of the springs provides stiffness, requiring smaller size of mesh. The constants of the Kelvin elements in the two horizontal directions are calculated using the solution developed by Novak [13]. While for the vertical direction these constants are calculated using the solution developed by Novak [14]. These constants are frequency dependent. For transient excitation, the constants are determined based on the predominant frequency of the excitation. The stiffness and damping of the Kelvin elements are evaluated using the area of the element face (normal to the direction of loading). Further details are described in [11, 12].

All the nodes along the base are fixed in all three directions. The nodes on the axis of symmetry are free to move in the vertical direction and along the direction of the axis of symmetry, and are fixed in the perpendicular horizontal direction (Fig. 3a). The nodes on the axis of anti-symmetry are constrained in the vertical direction and along the direction of this axis and are free to move in the perpendicular horizontal direction (Figs. 3a and 3b). The boundary conditions at the axis of symmetry and anti-symmetry are developed with due consideration of waves and loading patterns and thus they reflect mirror images.

Nonlinear Soil Model

The $\delta_0^*$ version of the HiSS model [15] is used to introduce the effect of soil plasticity and work hardening. The model is based on an incremental stress-strain relationship and assumes associative plasticity. In this model, the dimensionless yield surface $F$ is simplified as:

$$
F = \left( \frac{J_{2D}}{p_a^2} \right) + \alpha_{ps} \left( \frac{J_1}{p_a} \right)^\eta - \gamma \left( \frac{J_1}{p_a} \right)^2 = 0
$$

(2a)

$$
\alpha_{ps} = h_i / \xi_v^{b_2}
$$

(2b)

where $J_1$ is the first invariant of the stress tensor $\sigma_{ij}$; $J_{2D}$ is the second invariant of the deviatoric stress tensor; $p_a$ is the atmospheric pressure; $\gamma$ and $\eta$ are material parameters. The hardening function, $\alpha_{ps}$, is a function of plastic strain trajectory $\xi_v$. Typical yield surfaces for this model are shown in Fig. 5.

![Fig. 5. Shape of yield surfaces in J1-√J2D space](image-url)
DATA USED IN COMPUTATION AND VERIFICATION

Properties of SPS System
The soil is assumed to be clay at Sabine Pass, Texas, Desai [16]. Its properties are: Young’s modulus $E_F = 11.78 \text{ MPa}$, mass density $\rho_F = 1610 \text{ kg/m}^3$, Poisson’s ratio $\nu_F = 0.42$ and material damping ratio $D_F = 5\%$. The material parameters for the HiSS model are: $\gamma = 0.047; \eta = 2.4; h_1 = 0.0034$ and $h_2 = 0.78$. The piles are made of concrete, 10 m long and have square cross-sections (0.5 m x 0.5 m) with the pile slenderness ratio, $l/d = 20$. Young’s modulus, mass density and Poisson’s ratio for the pile are, respectively: $E_F = 25 \text{ GPa}$, $\rho_F = 2400 \text{ kg/m}^3$, $\nu_F = 0.25$. The superstructure considered consist of rectangular column(s) (0.75 m x 0.5 m) and is 6m high with the following properties: Young’s modulus $E_s = 25 \text{ GPa}$, and material damping ratio, $D_s = 5\%$. The total mass of the column is 54 Mg.

Dynamic Loading
The seismic loading is applied as either a harmonic or a transient bedrock motion. The harmonic excitation consists of sinusoidal waves of unit amplitude and varying frequency. The transient motion is the N-S component of the El Centro 1940 Earthquake with a PGA equal to 0.32g (Chopra [17]). The predominant frequency of the excitation is approximately 1.83 Hz. The responses are calculated at the pile cap (or pile head) and at the top of the structure.

Verification of the Model and Algorithm
Since a rigorous approach has been used its verification is imperative. This verification has been performed by comparing the results of the present three-dimensional analysis with those available in literature for elastic and elasto-plastic analyses. This verification has been performed for different modules of the system. Details of these verifications can be found in Maheshwari [18]

EFFECTS OF NONLINEARITY ON SPS RESPONSE

The effects of soil nonlinearity on the seismic response of the soil-pile-structure system are examined. The responses at the pile cap (or pile head) and at the top of the structure are calculated for both the linear (elastic) and nonlinear (HiSS) soil models and the results are compared. The analyses are performed for both harmonic and transient excitations, and for a single pile and a pile group case.

Analyses for Harmonic Excitations
Harmonic excitations with different frequencies are applied to the SPS system and the amplitude of the steady state response is noted in each case. The response at the pile cap (or pile head) and at the top of the structure are plotted as a ratio of the input bedrock motion (i.e. amplification factor) versus the dimensionless frequency, $a_0 = \omega d/V_s$, where $\omega$ is the circular frequency of excitation; $d$ is the dimension of the square pile in cross section and $V_s$ is the shear wave velocity of the soil.

Fig. 6 shows the responses at the pile head and at the top of the structure. For low frequencies ($a_0 < 0.2$ or $f = \omega 2\pi < 3 \text{ Hz}$), the soil nonlinearity increases the pile head response significantly. For higher frequencies, however, the pile head response decreases slightly due to soil nonlinearity. It is also noted from Fig. 6 that the structural response increases slightly due to soil nonlinearity at low frequencies ($a_0 < 0.15$) but decreases significantly for moderate frequencies ($0.15 < a_0 < 0.3$).

The response of the pile group model is shown in Fig. 7. Comparing this with Fig. 6, it can be noted that effect of group interaction decreases the value of peak response for the structure. Guin [19] made a similar observation for an elastic soil model. As far the effect of soil nonlinearity, overall trend of its effect on the pile and structural response is similar to that observed for the single pile model.
Fig. 6. Effect of nonlinearity on the response for a single pile model at different frequencies

Fig. 7. Effect of nonlinearity on the response for the pile group model at different frequencies
The soil nonlinearity increases the pile head response significantly and decreases the structural response. The comparison between Figs. 6 and 7 shows that the effects of soil nonlinearity on the piles and structural responses for the pile group case are less significant when compared to the single pile case. This may be attributed to the fact that the interaction between piles (the group effect) reduces the effects of soil nonlinearity.

**Analyses for Transient Excitations**
The SPS system is subjected to the El Centro excitation and the responses of the pile cap and the structure are calculated. The effect of soil nonlinearity on the SPS system response is investigated. Analyses for a single pile model as well as for pile group model are performed but here only results of single pile model is presented. For further details readers are referred to Maheshwari [18].

Fig. 8a shows a comparison of the pile head response for the linear and nonlinear soil models (only the first 10 sec of the record are shown). It is seen that the response of the pile, in general, increases due to soil nonlinearity. The peak values of linear and nonlinear accelerations are 0.43g and 0.54g, respectively. Fig. 8b shows the effect of soil nonlinearity on the response of the structure. It increases the structural response, and the peak acceleration increases from 1.04g to 1.2g. These observations are consistent with the results obtained using the harmonic excitations. Fig. 6 shows that at the predominant frequency of the transient excitation (f = 1.83Hz or a₀ = 0.11), both the pile head and structural responses increase due to soil nonlinearity.

Fig. 8a. Linear and nonlinear time histories of pile head response

Fig. 8c shows the smoothed Fourier spectra for the responses. It is noted that the soil nonlinearity increases the pile response slightly for most of the frequency content of the excitation. Also, the structural response increases substantially due to soil nonlinearity for the entire frequency content of the excitation, and especially for the frequency range, f < 7 Hz.
Fig. 8b. Linear and nonlinear time histories of response of the structure

Fig. 8c. Linear and nonlinear Fourier spectra
CONCLUSIONS

The effects of soil plasticity on the seismic response of soil-pile-structure systems are investigated using three-dimensional finite element analyses in the time domain. For a harmonic excitation, the soil nonlinearity increases the pile head and structural responses at low frequencies. At high frequencies, both the pile head and the structural responses are slightly affected by the soil nonlinearity. Also for the transient excitation, soil nonlinearity increases both the pile head and the structural responses. Smoothed Fourier spectra show that, in general, nonlinearity increases the responses at low and moderate frequencies but its effect is negligible at high frequencies. The pile group effect decreases the peak values of the response (i.e. reduces the effect of soil nonlinearity).

Based on the range of parameters considered in this study, soil nonlinearity increases the response at low frequencies \( (a_0 < 0.2) \), which represent the range of interest for earthquake loading for SPS system. However, generalization of these results may require further analyses with different soil and pile parameters.

ACKNOWLEDGEMENTS

The research presented here was partially supported by the Mid-America Earthquake Center under National Science Foundation Grant EEC-9701785 and the US Army Corps of Engineers. This support is gratefully acknowledged.

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