TORSIONALLY COUPLED RESPONSE CONTROL OF STRUCTURES USING CIRCULAR TUNED LIQUID COLUMN DAMPERS

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SUMMARY

In this paper, the control performance of Circular Tuned Liquid Column Dampers (CTLCD) to torsional response of structure excited by ground motions is investigated. Based on the equation of motion for the CTLCD-structure system, the optimal control parameters of CTLCD are given through some derivations supposing the ground motion is stochastic process. The influence of systematic parameters on the equivalent damping ratio of the structures is analyzed with purely torsional vibration and translational-torsional coupled vibration, respectively. The results show that Circular Tuned Liquid Column Dampers (CTLCD) is an effective torsional response control device.

INTRODUCTION

Modern tall buildings have become relatively light and flexible with the plentiful application of high-strength materials in civil engineering, which will result that the structures may collapse or exceed the comfort limitation at the action of dynamic loads, such as seismic and wind. The structural vibration control in tall buildings can be a successful method of mitigating the effects of these dynamic responses. A number of control devices have been developed and some of them have been applied to the real structures in recent years [1].

Tuned Liquid Column Dampers (TLCD) is an effective control device developed from Tuned Liquid Dampers (TLD), which increases structural damping through the vibration of liquid in the U-shaped container to suppress the structural dynamical response. The potential advantages of liquid vibration absorbers include: low manufacturing and installation costs; the ability of the absorbers to be incorporated during the design stage of a structure, or to be retrofitted to serve as a remedial role; relatively low maintenance requirements; and the availability of the liquid to be used for emergency purposes, or for the everyday function of the structure if fresh water is used [2, 3]. The control method using TLCD for civil structures was firstly presented by Sakai et al [4]. The orifice-opening ratio of TLCD was optimized through experiments and the control method for high-rise strictures was analyzed by Qu et al [5]. A variation of TLCD which has a different cross-sectional area to vertical column and horizontal column, called Liquid Column Vibration Absorber (LCVA), has also been investigated in recent years [2, 3, 6, 7, 8]. Yan et al. presented the adjustable frequency tuned liquid column damper by adding springs to the

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TLCD system, which modified the frequency of TLCD and expended its application ranges [9].

The earthquake is essentially multi-dimensional and so is the structural response excited by earthquake, which will result in the torsionally coupled vibration that cannot be neglected. So, the torsional response for structure is very important [10]. Previously, Li et al. presented the method of reducing torsionally coupled response by installing TLCDs in structural orthogonal directions [11]. Circular Tuned Liquid Column Dampers (CTLCD) is a type of control device sensitive to torsional response. The results of free vibration and forced vibration experiments showed that it is effective to control structural torsional response [12, 13]. However, how to determine the parameters of CTLCD to effectively reduce torsionally coupled vibration is still necessary to be further investigated. In this paper, the optimal parameters of CTLCD are presented based on the stochastic vibration theory. Also, the torsional vibration and torsionally coupled vibration are controlled using CTLCD in this paper.

**EQUATION OF MOTION FOR CONTROL SYSTEM**

The configuration of CTLCD is shown in Fig.1. Through Lagrange principle, the equation of motion for CTLCD excited by seismic can be derived as

\[
\rho A(2H + 2\pi R)\dddot{h} + \frac{1}{2}\rho A \xi \dot{h}^2 \ddot{h} + 2\rho A g h = -2\rho A \pi R^2 (\ddot{u}_\theta + \dddot{u}_\phi) \tag{1}
\]

where \( h \) is the relative displacement of liquid in CTLCD; \( \rho \) means the density of liquid; \( H \) denotes the height of liquid in the vertical column of container when the liquid is quiescent; \( A \) expresses the cross-sectional area of CTLCD; \( g \) is the gravity acceleration; \( R \) represents the radius of horizontal circular column; \( \xi \) is the head loss coefficient; \( \ddot{u}_\theta \) denotes the torsional acceleration of structure; \( \dddot{u}_\phi \) is the torsional acceleration of ground motion.

Because the damping in the above equation is nonlinear, equivalently linearize it and the equation can be re-written as

\[
m_T \dddot{h} + c_{\text{eq}} \dot{h} + k_T h = -\alpha m_T R (\ddot{u}_\theta + \dddot{u}_\phi) \tag{2}
\]

where \( m_T = \rho A L_{cc} \) is the mass of liquid in CTLCD; \( L_{cc} = 2H + 2\pi R \) denotes the total length of liquid in the column; \( c_{\text{eq}} = 2m_T \omega_T \xi_T \) is the equivalent damping of CTLCD; \( \omega_T = \sqrt{2g / L_{cc}} \) is the natural circular frequency of CTLCD; \( \xi_T = \frac{\xi}{2\sqrt{\rho g L_{cc}}} \) is equivalent linear damping ratio[14]; \( \sigma_h \) means the standard deviation of the liquid velocity; \( k_T = 2\rho A g \) is the “stiffness” of liquid in vibration; \( \alpha = 2\pi R / L_{cc} \) is the configuration coefficient of CTLCD.

![Fig. 1 Configuration of Circular TLCD](image)
For a single-degree-of-freedom (SDOF) structure, the equation of torsional motion installed CTLCD can be written as

\[ J_\theta \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta = -J_\theta \ddot{\theta}_c + F_\theta \quad (3) \]

where \( J_\theta = m_f r^2 \) is structural moment of inertia to vertical axis; \( m_s \) and \( r \) represent the lumped mass and gyrus radius of structure; \( c_\theta = 2J_\theta \omega_\theta \zeta_\theta \) denotes the damping of structure; \( \omega_\theta \) and \( \zeta_\theta \) mean the torsional natural circular frequency and damping ratio of structure; \( k_\theta = J_\theta \omega_\theta^2 \) express the stiffness of structure; \( F_\theta \) is the control force of CTLCD to structures, given by

\[ F_\theta = -m_f R (\ddot{R}\dot{\theta} + R \dddot{\theta}_\varphi + \dddot{\theta} \dot{\varphi}) \quad (4) \]

Combining equation (1) to (4) yields

\[
\begin{bmatrix}
1 + \lambda & \alpha \lambda / R & 0 & 2\zeta_\theta \omega_\theta \\
\alpha \lambda / R & \lambda / R^2 & 0 & 2\zeta_\theta \omega_\tau / R^2 \\
0 & 0 & \lambda \omega_\tau^2 / R^2 & 0 \\
0 & 0 & 0 & \lambda \omega_\tau^2 / R^2
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_c \\
\dot{\theta}_c \\
\ddot{\theta}_\theta \\
\dot{\theta}_\theta
\end{bmatrix}
= \begin{bmatrix}
1 + \lambda \\
\{ \alpha \lambda / R \} \\
\{ \alpha \lambda / R \} \\
\{ \alpha \lambda / R \}
\end{bmatrix}
\dddot{\theta}_c
\]

(5)

where \( \lambda = \frac{m_f R^2}{J_\theta} \) denotes inertia moment ratio. Let \( \dddot{\theta}_c(t) = e^{i\omega t} \), then

\[
\begin{bmatrix}
\dot{\theta}_c \\
\ddot{\theta}_c \\
\dot{\theta}_\theta \\
\ddot{\theta}_\theta
\end{bmatrix}
= \begin{bmatrix} H_\theta(\omega) \\ H_\theta(\omega) \end{bmatrix} e^{i\omega t}
\]

(6)

where \( H_\theta(\omega) \) and \( H_\theta(\omega) \) are transfer functions in the frequency domain. Substituting equation (6) into equation (5) leads to

\[
\begin{bmatrix}
-\{ (1 + \lambda) \omega^2 + 2\zeta_\theta \omega \omega + \omega_\theta^2 \} & -\alpha \lambda \omega^2 / R \\
-\alpha \lambda \omega^2 / R & -\lambda \omega^2 / R^2 + 2\zeta_\theta \omega \omega / R^2 + \lambda \omega_\tau^2 / R^2
\end{bmatrix}
\begin{bmatrix}
H_\theta \\
H_\theta
\end{bmatrix}
= \begin{bmatrix}
1 + \lambda \\
\{ \alpha \lambda / R \}
\end{bmatrix}
\dddot{\theta}_c
\]

(7)

From the above equation, the transfer function of structural torsional response can be expressed by

\[
H_\theta(\omega) = \frac{\hat{\lambda}(1 + \lambda) - \alpha^2 \lambda^2}{\{ (1 + \lambda) \omega^2 + 2\zeta_\theta \omega \omega + \omega_\theta^2 \} - \lambda \omega^2 + 2\zeta_\theta \omega \omega + \lambda \omega_\tau^2 - \lambda \omega_\tau^2 \omega^2} - \alpha^2 \lambda^2 \omega^2
\]

(8)

Then, the torsional response variance of structure installed CTLCD can be obtained as

\[
\sigma_{\theta c}^2 = \int_{-\infty}^{\infty} |H_\theta|^2 S_{\dddot{\theta}_c}(\omega) d\omega
\]

(9)

If the ground motion is assumed to be a Gauss white noise random process with an intensity of \( S_0 \) and define the frequency ratio \( \gamma = \omega_\tau / \omega_\theta \), the value of \( \sigma_{\theta c}^2 \) can be calculated by

\[
\sigma_{\theta c}^2 = \frac{\pi S_0}{2\omega_\theta'} \frac{2(1 + \lambda)^2 \zeta_\theta \gamma^2 + 2A_1 \zeta_\theta (1 + \lambda)^2 \gamma^4 + 2B_1(1 + \lambda)(2B_1 + \alpha^2 \lambda) \gamma^2 + 2C_1 \zeta_\theta \gamma + 2D_1 \zeta_\theta (1 + \lambda - \alpha^2 \lambda)^2}{2(1 + \lambda)^2 \zeta_\theta \gamma^2 + 2A_1 \zeta_\theta \gamma^4 + 4B_1 \zeta_\theta \gamma^2 + 2D_1 \zeta_\theta \gamma + 2\zeta_\theta \gamma^2}
\]

(10)

where

\[
\begin{align*}
A_1 &= \alpha^2 \lambda + 4\zeta_\theta^2 (1 + \lambda) \\
B_1 &= (2\zeta_\theta^2 - 1)(1 + \lambda) + 2\zeta_\theta^2 + \alpha^2 \lambda \\
C_1 &= 4\zeta_\theta^2 (1 + \lambda)^2 + \alpha^4 \lambda^2 \\
D_1 &= 4\zeta_\theta^2 + \alpha^2 \lambda
\end{align*}
\]

**OPTIMAL PARAMETERS OF CITULAR TUNED LIQUID COLUMN DAMPERS**
The optimal parameters of CTLCD should make the displacement variance $\sigma_{u_p}^2$ minimum, so the optimal parameters can be obtained according to the following condition

$$\frac{\partial \sigma_{u_p}^2}{\partial \sigma_{u_p}} = 0 \quad \frac{\partial \sigma_{u_p}^2}{\partial \gamma} = 0$$  \hspace{1cm} (11)$$

Neglecting the structural damping ratio $\zeta_o$ and solving above equation, the optimal damping ratio $\zeta_T^{opt}$ and frequency ratio $\gamma^{opt}$ for CTLCD can be formulized as

$$\zeta_T^{opt} = \frac{1}{2} \sqrt{\frac{\lambda \alpha^2 (1 + \lambda - \frac{5}{4} \lambda \alpha^2)}{(1+\lambda)(1+\lambda-\frac{3}{2} \lambda \alpha^2)}} \quad \gamma^{opt} = \sqrt{\frac{1+\lambda-\frac{3}{2} \lambda \alpha^2}{1+\lambda}}$$  \hspace{1cm} (12)$$

Fig. 2 shows the optimal damping $\zeta_T^{opt}$ and optimal frequency $\gamma^{opt}$ of CTLCD as a function of $\lambda$ ranging between 0 to 5% for $\alpha=0.2, 0.4, 0.6$ and 0.8. It can be seen that as the value of $\lambda$ increases the optimal damping ratio $\zeta_T^{opt}$ increases and the optimal frequency ratio $\gamma^{opt}$ decreases. For a given value of $\lambda$, the optimal damping ratio $\zeta_T^{opt}$ increases and the optimal frequency ratio decreases with the rise of $\alpha$. It can also be seen that the value of $\gamma^{opt}$ is always near 1 for different $\alpha$ and $\lambda$ value from the relationship of $\gamma^{opt}$ with $\lambda$ in Fig.2. If let $\gamma = 1$ and solve $\frac{\partial \sigma_{u_p}^2}{\partial \zeta_T} = 0$, the optimal damping ratio of CTLCD $\zeta_T^{opt}$ is obtained as

$$\zeta_T^{opt} = \frac{1}{2} \sqrt{\frac{\lambda^2 (\alpha^2 + 1 + \lambda) + \alpha^2 \lambda (1 + \lambda)^2}{(1+\lambda)^3}}$$  \hspace{1cm} (13)$$

The optimal parameters of CTLCD cannot be expressed with formulas when considering the structural damping for the complexity of equation (10), we can only get numerical results for different values of structural damping, as shown in Table 1. Table 1 shows that for different structural damping, the optimal damping ratio of CTLCD increases and the optimal frequency ratio decreases with the rise of $\lambda$ value, which is the same as Fig. 2. Table 1 also suggests the value of structural damping has little effect on the optimal parameters of CTLCD, especially the optimal damping ratio $\zeta_T^{opt}$. 
Table 1 The optimal parameters of CLTCD ($\alpha = 0.8$)

<table>
<thead>
<tr>
<th>$\zeta_\theta$</th>
<th>$\gamma_{opt}$</th>
<th>$\zeta_{opt}$</th>
<th>$\zeta_T^{opt}$</th>
<th>$\zeta_{opt}$</th>
<th>$\zeta_T^{opt}$</th>
<th>$\zeta_{opt}$</th>
<th>$\zeta_T^{opt}$</th>
<th>$\zeta_{opt}$</th>
<th>$\zeta_T^{opt}$</th>
</tr>
</thead>
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<tr>
<td>0%</td>
<td>0.005</td>
<td>0.0282</td>
<td>0.005</td>
<td>0.0282</td>
<td>0.005</td>
<td>0.0282</td>
<td>0.005</td>
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</tr>
<tr>
<td>1%</td>
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<td>0.01</td>
</tr>
<tr>
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<td>0.015</td>
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</tr>
<tr>
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<td>0.0561</td>
<td>0.02</td>
<td>0.0561</td>
<td>0.02</td>
<td>0.0561</td>
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</tr>
<tr>
<td>4%</td>
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<tr>
<td>5%</td>
<td>0.03</td>
<td>0.076</td>
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<td>0.076</td>
<td>0.03</td>
<td>0.076</td>
<td>0.03</td>
</tr>
</tbody>
</table>

ANALYSIS OF STRUCTURAL TORSIONAL RESPONSE CONTROL USING CTLCD

The objective of damper installed in the structures is to increase the damping of the structure system and reduce the response of structure. To analyze the effect of different system parameters on the torsional response of structure, the damping of structure with CTLCD is expressed by equivalent damping ratio $\zeta_e$ [14]

$$\zeta_e = \frac{\pi S_\theta}{2\omega_\theta^2}$$

(14)

The relationships of $\zeta_e$ with different parameters of control system are shown in Fig. 3 to Fig. 7.

Fig. 3 shows the equivalent damping ratio of structure as a function of the damping ratio of CTLCD for $\lambda=0.005, 0.01, 0.015, 0.02$. It is seen from the figure that the equivalent damping ratio $\zeta_e$ increases rapidly with the increase of $\zeta_T$ initially, whereas it decreases if the damping ratio of CTLCD $\zeta_T$ is greater than a certain value. Fig 2 also suggests that the value of $\zeta_e$ increases with the rise of inertia moment ratio $\lambda$ for a given value of $\zeta_T$.

Fig. 4 shows the equivalent damping ratio of structure $\zeta_e$ as a function of the damping ratio of CTLCD for $\gamma=0.6, 0.8, 0.9, 1.0$. It is seen from the figure that the value of $\zeta_e$ increase with the rise of frequency ratio $\gamma$.

Fig. 5 shows the equivalent damping ratio of structure $\zeta_e$ as a function of the damping ratio of CTLCD for $\zeta_\theta=0.005, 0.01, 0.03$ and 0.05. It is seen from the figure that as the rise of the damping ratio of
structure \( \zeta_\theta \), the equivalent damping ratio \( \zeta_e \) increases.

Fig. 6 shows the equivalent damping ratio of structure \( \zeta_e \) as a function of \( \lambda \) for \( \alpha = 0.5, 0.6, 0.7, 0.8 \) and 0.9. It can be seen from the figure that the damping ratio of structure \( \zeta_e \) increases with \( \lambda \) initially. Whereas, the curve will be gentle after the value of \( \lambda \) is greater than a certain value. It also can be concluded from the figure that the damping ratio of structure \( \zeta_e \) increase with the rise of configuration coefficient \( \alpha \).

![Fig. 5 The structural equivalent damping ratio with the damping ratio of CTLCD](image1)

![Fig. 6 The structural equivalent damping ratio with the inertia moment ratio](image2)

Fig. 7 shows the equivalent damping ratio of structure \( \zeta_e \) as a function of frequency ratio between CTLCD and structure. It is seen from the picture that the value of \( \zeta_e \) will be maximum at the condition of \( \gamma = 1 \). So, the value of \( \gamma \) can be set to approximate 1 in the engineering application to get the best control performance.

![Fig. 7 The structural equivalent damping ratio with the frequency ratio](image3)

**ANALYSIS OF STRUCTURAL TORSIONALLY COUPLED RESPONSE CONTROL USING CTLCD**

The torsional response of structure is usually coupled with translational response in engineering, so it is necessary to consider torsionally coupled response for vibration control of eccentric structure. In this paper, a single-story structure only eccentric in \( x \) direction is taken as an example, which means that the displacement in \( y \) direction is coupled with torsional response of structure. The equation of torsionally coupled motion for the eccentric structure installed CTLCD can be written as
The above equation can be simplified as
\[
\mathbf{M} \ddot{\mathbf{u}} + \mathbf{C}_s \dot{\mathbf{u}} + \mathbf{K}_s \mathbf{u} = -\mathbf{M} \ddot{\mathbf{u}}_g + \mathbf{F}
\]  
(16)
where \( e_s \) is eccentric distance; \( u_y \), \( \ddot{u}_y \) and \( K_y \) are the displacement, ground acceleration and stiffness of structure in \( y \) direction, respectively; The control force \( \mathbf{F} \) is calculated by
\[
\begin{align*}
F_y &= -m_y (\ddot{u}_y + \ddot{u}_y) \\
F_\theta &= -m_\tau R (R \ddot{u}_\theta + R \ddot{u}_\theta + \alpha \dot{\theta})
\end{align*}
\]
(17)
It is assumed that the damping matrix in Equation (16) is directly proportional to the stiffness matrix, that is
\[
\mathbf{C}_s = \alpha \mathbf{K}_s
\]
(18)
where the proportionality constant \( \alpha \) has units of second. The proportionality constant \( \alpha \) was chosen such that the uncoupled lateral mode of vibration has damping equal to 2% of critical damping. This was to account for the nominal elastic energy dissipation that occurs in any real structure. [15] The critical damping coefficient \( c_c \) for a single degree-of-freedom (SDOF) system is given by
\[
c_c = 2m_y \omega_y
\]
(19)
where \( \omega_y = \sqrt{\frac{K_y}{m_y}} \) is natural frequency of the uncoupled lateral mode. From the equation (18) and (19), the constant \( \alpha \) is determined by
\[
\alpha = \frac{0.02 \times 2m_y \sqrt{\frac{K_y}{m_y}}}{K_y}
\]
(20)
Combining the Equation (3) and (15), the equation of motion for torsionally coupled system can be written as
\[
\begin{bmatrix}
1 + \mu & 0 & \frac{\mu R^2}{r^2} & \frac{\mu R}{\omega_\tau} & 0 \\
0 & 1 + \frac{\mu R^2}{r^2} & \alpha \mu R & \frac{\mu R}{\omega_\tau} & 0 \\
0 & 0 & \frac{\mu R}{\omega_\tau} & \frac{\mu R}{\omega_\tau} & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_y \\
\dot{h} \\
\dot{h}
\end{bmatrix}
+ \begin{bmatrix}
\frac{a \omega_y^2}{r^2} & \frac{a \omega_\tau^2}{r^2} & 0 \\
\frac{a \omega_\tau^2}{r^2} & \frac{a \omega_\tau^2}{r^2} & 0 \\
0 & 0 & 2\mu \alpha \omega_\tau
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_y \\
\dot{u}_\theta \\
\dot{u}_\theta
\end{bmatrix}
= \begin{bmatrix}
1 + \mu & 0 & \frac{\mu R^2}{r^2} & \frac{\mu R}{\omega_\tau} & 0 \\
0 & 1 + \frac{\mu R^2}{r^2} & \alpha \mu R & \frac{\mu R}{\omega_\tau} & 0 \\
0 & 0 & \frac{\mu R}{\omega_\tau} & \frac{\mu R}{\omega_\tau} & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_g \\
\dot{u}_g \\
\dot{u}_g
\end{bmatrix}
\]
(21)
where \( \mu = \frac{m_T}{m_s} \) is ratio between the mass of CTLCD and the mass of structure; \( \omega_\theta = \sqrt{\frac{k_\theta}{mr^2}} \) denotes the natural frequency of the uncoupled torsional mode. The following assumptions are made in this paper: \( \ddot{u}_g \) and \( \ddot{u}_\theta \) are two unrelated Gauss white noise random processes with intensities of \( S_1 \) and \( S_2 \),
respectively; \( H_{yy}, H_{y\theta}, H_{\theta y} \) and \( H_{\theta\theta} \) are transfer functions from \( \dddot{y} \) to \( y \), \( \dddot{y} \) to \( \theta \), \( \dddot{y} \) to \( u_\theta \) and \( \dddot{y} \) to \( u_\theta \), respectively. Then, the displacement variance of structure can be obtained by

\[
\sigma_{u_{yy}}^2 = S_1 \int_{-\infty}^{\infty} |H_{yy}|^2 d\omega \\
\sigma_{u_{y\theta}}^2 = S_2 \int_{-\infty}^{\infty} |H_{y\theta}|^2 d\omega \\
\sigma_{u_{\theta\theta}}^2 = S_1 \int_{-\infty}^{\infty} |H_{\theta\theta}|^2 d\omega \\
\sigma_{u_{\theta\theta}}^2 = S_2 \int_{-\infty}^{\infty} |H_{\theta\theta}|^2 d\omega
\]

(22)

where \( \sigma_{u_{yy}} \) and \( \sigma_{u_{y\theta}} \) are displacement variances in \( y \) direction caused by the ground motion in \( y \) direction and \( \theta \) direction, respectively; \( \sigma_{u_{\theta\theta}} \) and \( \sigma_{u_{\theta\theta}} \) are displacement variances in \( \theta \) direction caused by the ground motion in \( y \) direction and \( \theta \) direction, respectively. So, the equivalent damping ratios of structure are given as [14]

\[
\zeta_{e_{yy}} = \frac{\pi S_1}{2\omega_1^3 \sigma_{u_{yy}}} ; \quad \zeta_{e_{y\theta}} = \frac{\pi S_1}{2\omega_1^3 \sigma_{u_{y\theta}}} ; \quad \zeta_{e_{\theta y}} = \frac{\pi S_1}{2\omega_1^3 \sigma_{u_{\theta y}}} ; \quad \zeta_{e_{\theta\theta}} = \frac{\pi S_1}{2\omega_1^3 \sigma_{u_{\theta\theta}}}
\]

(23)

where \( \zeta_{e_{yy}} \) and \( \zeta_{e_{y\theta}} \) are equivalent ratios in \( y \) direction caused by the ground motion in \( y \) direction and \( \theta \) direction, respectively; \( \zeta_{e_{\theta y}} \) and \( \zeta_{e_{\theta\theta}} \) are equivalent ratios in \( \theta \) direction caused by the ground motion in \( y \) direction and \( \theta \) direction, respectively. Then, the total equivalent damping ratio \( \zeta_{e_{y}} \) in \( y \) direction and \( \zeta_{e_{\theta}} \) in \( \theta \) direction can be defined as

\[
\zeta_{e_{y}} = \zeta_{e_{yy}} + \zeta_{e_{y\theta}} \\
\zeta_{e_{\theta}} = \zeta_{e_{\theta y}} + \zeta_{e_{\theta\theta}}
\]

(24)

Define \( \Omega = \frac{\omega_\theta}{\omega_1} \) as the frequency ratio between the uncoupled torsional mode and uncoupled translational mode and \( \omega_1 \) as the first frequency of torsionally coupled structure. The relationships of equivalent damping ratio \( \zeta_{e_{y}} \) and \( \zeta_{e_{\theta}} \) with parameters of control system are shown in Fig. 8 to Fig. 11. Fig. 8 shows equivalent damping ratio \( \zeta_{e_{y}} \) and \( \zeta_{e_{\theta}} \) as functions of frequency ratio \( \omega_\tau / \omega_1 \) for mass ratio \( \mu = 0.005, 0.01, 0.015 \) and 0.02. It is seen from the figure that the values of \( \zeta_{e_{y}} \) and \( \zeta_{e_{\theta}} \) are maximum when the value of frequency ratio \( \omega_\tau / \omega_1 \) is approximate 1. The Fig. 2 also suggests that damping ratio \( \zeta_{e_{y}} \) and \( \zeta_{e_{\theta}} \) increase with the rise of mass ratio \( \mu \).

Fig. 8 Equivalent damping ratio of structure with frequency ratio \( \omega_\tau / \omega_1 \)
Fig. 9 shows equivalent damping ratio $\zeta_{cy}$ and $\zeta_{c\theta}$ as functions of mass ratio $\mu$ for configuration coefficient $\alpha = 0.5, 0.6, 0.7$ and $0.8$. It is seen from the figure that the values of $\zeta_{cy}$ and $\zeta_{c\theta}$ increase initially and approach constants finally with the rise of mass ratio $\mu$. It can also be concluded that the values of $\zeta_{cy}$ and $\zeta_{c\theta}$ increase with the rise of configuration coefficient $\alpha$.

![Fig. 9 Equivalent damping ratio of structure with mass ratio $\mu$](image)

Fig. 10 shows equivalent damping ratio $\zeta_{cy}$ and $\zeta_{c\theta}$ as functions of damping ratio $\zeta_T$ for $\alpha = 0.5, 0.6, 0.7$ and $0.8$. It is seen from the figure that the values of $\zeta_{cy}$ and $\zeta_{c\theta}$ rapidly increase initially with the rise of $\zeta_T$; after a certain value of $\zeta_T$, $\zeta_{cy}$ will decrease to a constant and $\zeta_{c\theta}$ decrease first, then increase gradually.

![Fig. 10 Equivalent damping ratio of structure with damping ratio $\zeta_T$](image)

Fig. 11 shows equivalent damping ratio $\zeta_{cy}$ and $\zeta_{c\theta}$ as functions of frequency ratio $\Omega$ for $e_s/l_r = 0.4, 0.6, 0.8$ and $1.0$. It is seen from the figure that $\zeta_{cy}$ and $\zeta_{c\theta}$ are approximate zero for the structure with $\Omega$ near to $e_s/l_r$; for the structure $\Omega < e_s/l_r$, $\zeta_{cy}$ and $\zeta_{c\theta}$ decrease with the rise of frequency ratio $\Omega$ and increase with the rise of $e_s/l_r$; for the structure with $\Omega > e_s/l_r$, $\zeta_{cy}$ and $\zeta_{c\theta}$ increase with the rise of frequency ratio $\Omega$ and decrease with the rise of $e_s/l_r$.

![Fig. 11 Equivalent damping ratio of structure with frequency ratio $\Omega$](image)
CONCLUSION

In this study, the control performance of CTLCD on suppressing the torsional and torsionally coupled vibration of structure is analyzed. A set of generalized equations of motion for control system are established and some useful design formulas for optimal properties of CTLCD are derived. Based on the analysis in this paper, the following conclusion can be obtained:

1. The optimal damping ratio of CTLCD $\zeta_T^{opt}$ increases with the rise of $\lambda$ and $\alpha$, but the optimal frequency ratio $\gamma^{opt}$ decrease with the rise of $\lambda$ and $\alpha$.

2. For torsional vibration, in a certain range the equivalent damping ratio of structure $\zeta_e$ increases with the rise of the damping ratio of CTLCD $\zeta_T$ and inertia moment ratio $\lambda$, otherwise $\zeta_e$ will decrease; the greater the value of $\zeta_s$ and $\alpha$, the greater $\zeta_e$; The value of $\zeta_e$ is maximum when the frequency ratio $\gamma$ is equal to 1.

3. For torsionally coupled vibration, the equivalent damping ratio $\zeta_{ey}$ and $\zeta_{e\theta}$ are maximum when the frequency of CTCD $\omega_T$ is tuned to the first frequency of torsionally coupled structure $\omega_1$; $\zeta_{ey}$ and $\zeta_{e\theta}$ increase with the rise of mass ratio $\mu$, configuration coefficient $\alpha$ and damping ratio $\zeta_T$; $\zeta_{ey}$ and $\zeta_{e\theta}$ are about zero for the structure with frequency ratio $\Omega$ near to $e_s / r$; for the structure with $\Omega < e_s / r$, $\zeta_{ey}$ and $\zeta_{e\theta}$ decrease with the rise of $\Omega$ and increase with the rise of $e_s / r$; for the structure with $\Omega > e_s / r$, $\zeta_{ey}$ and $\zeta_{e\theta}$ increase with the rise of $\Omega$ and decrease with the rise of $e_s / r$.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the support from the Outstanding Youth Science Foundation of the National Natural Science Foundation of China and reviewers’ comments.

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