Inelastic Response Modification of structures Using Semi-Active and Passive Control Devices

S.W. Chen¹, M. Tong² and G.C. Lee³

SUMMARY

In this paper a study on the relative effectiveness of displacement-based control device and velocity-based control device for inelastic structural response modification is presented.

A Multi-Degree-of-Freedom system, whose nonlinear behavior is approximated by bi-linear or tri-linear force-displacement relationships, is used as the basic structural model. The relative effectiveness of the two types of structural control devices is observed from the rate of nonlinear displacement increment (normalized by its corresponding linear value) and the rate of nonlinear velocity increment (normalized by its corresponding linear value).

Numerical results using sinusoidal and earthquake excitations show that the change of the rate of nonlinear displacement increment is greater than that of the rate of nonlinear velocity increment when the structure behaves in the inelastic range. Further, the ratio of these two rates increases with increasing ductility. That the displacement response increases faster than velocity response in the inelastic range implies that the displacement-based devices may have advantage over velocity-based devices for inelastic structural response control. The effects of excitation frequency and natural frequency on the relative effectiveness of these two types of devices are also examined.

With earthquake records as excitation, the effectiveness of the velocity-based device and a hybrid control system (combination of the displacement-based device and velocity-based device) is compared. For the displacement-based device, a semi-active mechanical device (RSPM) is used. For the velocity-based device, linear viscous damper is used.

INTRODUCTION

In recent years, an increasing number of structural control devices have been implemented in buildings to achieve response reduction under earthquake excitations. These devices can be generally categorized into:

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displacement-based type and velocity-based type. For structural response in the linear range, the superiority of velocity-based devices over displacement-based devices can be demonstrated theoretically. However, for inelastic structural response reduction, the answer is not clear.

For single-degree-of-freedom systems, previous study of the same authors showed that the displacement changes faster than velocity in inelastic response range \(^5\), which suggests that the displacement-based device may outperform the velocity-based device in the inelastic response range. To further pursue this issue, this paper investigated the behaviors of the displacement-based device and velocity-based device in the inelastic response range by using a multi-degree-of-freedom structural model.

In this paper, current status on the development of structural control theories and practices will not be reviewed. For a state-of-the-art discussion of the various velocity-based and displacement-based structural control devices, references [1][2][3][4] may be consulted.

**VELOCITY-BASED AND DISPLACEMENT-BASED ENERGY DISSIPATION DEVICES**

Most seismic response control devices can be divided in two groups according to their working principles: velocity-based and displacement-based. Viscous dampers are typical examples of velocity-based devices, as their energy dissipation capacity is directly proportional to the magnitude of velocity. Diagonal or eccentric braces and friction dampers are typical displacement-based devices because their effectiveness is mainly governed by the displacement.

For structural responses in the elastic range, studies have shown that velocity-based devices are more effective for narrowband response control \(^5\). But in the inelastic range, it may be different. There are several governing factors. First, the total structural stiffness degenerates as the response becomes inelastic; second, ductility or inelastic deformation significantly increases the structure’s energy dissipation capacity (the concept of structural fuse); and third, as the structural response enters the inelastic range, the rate of increase of displacement, velocity and acceleration no longer obey the same proportional relationship as they do in the elastic response range. These issues provide the motivation to examine the relative efficiencies of velocity-based and displacement-based devices for inelastic response control.

Numerical analysis of a single-degree-of-freedom system has shown that the displacement-based device may be more effective than velocity-based device in the elastic range because the displacement response increases faster than the velocity response \(^5\). To continue that study, this paper considers the behavior of the displacement-based device and velocity-based device by employing a multi-degree-of-freedom system.

In Ref. [5], a ratio of the displacement incremental rate and the velocity incremental rate with respect to the elastic responses is introduced as the measure. This measure will be used in this paper, and it is defined as:

\[
\eta = \frac{\max(d_{\text{non}}(t))/\max(d_{\text{lin}}(t))}{\max(v_{\text{non}}(t))/\max(v_{\text{lin}}(t))}
\]

where \(d_{\text{non}}\) and \(d_{\text{lin}}\) are respectively the inelastic and elastic displacement responses, and \(v_{\text{non}}\) and \(v_{\text{lin}}\) are the inelastic and elastic velocity responses.
NUMERICAL ANALYSIS

Model description

The numerical study is carried out based on a three-degree-of-freedom system with an assumed damping ratio of 0.03, shown in Figure 1.

Two types of inelastic behaviors (bi-linear model and tri-linear model) are adopted in this paper. As shown in Figure 2, the pre-yielding stiffness of bilinear1, bilinear2 and bilinear3 are one time, 1.5 times and 2.5 times of the stiffness of the basic model respectively. The pre-yielding stiffness of trilinear1, trilinear2 and trilinear3 are assumed to be the same as that of bilinear1, bilinear2 and bilinear3 respectively. All these systems have the same post-yielding stiffness. The values of the stiffness and the natural frequency of each system before and after yielding are listed in Table 1. The cases of bilinear2, bilinear3, trilinear2 and trilinear3 are employed for purpose of parametric analysis.

For this numerical analysis, sinusoidal excitations with frequencies ranging from 0.16 Hz to 10.03 Hz are used as input. For analyzing the influence of ductility, the input has been scaled such that the excitations will be able to produce the desired inelastic responses of the structural model.
Table 1 Stiffness and natural frequencies of the systems (Hz)

<table>
<thead>
<tr>
<th>Floor number</th>
<th>Stiffness (kip/in)</th>
<th></th>
<th></th>
<th></th>
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<tr>
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<td>trilinear1</td>
<td>trilinear2</td>
<td>trilinear3</td>
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<tr>
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<td>before yield</td>
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<td>270</td>
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<tr>
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<td>300</td>
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<td>90</td>
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<td>45</td>
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<table>
<thead>
<tr>
<th>Mode number</th>
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<td>0.7309</td>
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</table>
Results of analysis

Figure 3 presents some of the results of the numerical analysis. In this figure, for bilinear1 and trilinear1 with different excitation frequencies, $\eta$ is plotted as a function of ductility for each floor. From this figure, it is shown that $\eta$ is greater than unity in the entire inelastic range and it increases with the increase of ductility. This means that in the inelastic range the displacement response increases faster than velocity response, so the displacement-based device may be more effective. This phenomenon is consistent with the result of the SDOF study\cite{5}. The value of $\eta$ can be greater than 1.5.

Though the values of $\eta$ for different floors may change with different rate, they change following similar fashion, especially when the system is subjected to excitations of higher frequency.
**PARAMETRIC ANALYSIS**

**Ductility**

The results for cases bilinear1 and trilinear1 are shown in Figure 4 and Figure 5 respectively. For bilinear1, the results of the 3rd floor are given. The results of the 1st floor are presented for case trilinear1. For other floors, the results are similar.

From Figure 3, Figure 4 and Figure 5, the ratio \( \eta \) can be observed to increase with increasing ductility. Generally, \( \eta \) increases fastest when the ductility ratio falls in the range of 1 to 5. The increasing of ratio \( \eta \) is slower in the range of 5 to 10 and even slower when the ductility is larger than 10.

There are a few cases, as shown in these figures, that \( \eta \) decreases when the ductility ratio increases (Figure 4 (a) and figure 5 (a)). These cases only occur when the excitation frequency is in a region close to the natural frequency, where the ratio \( \eta \) is shown to be very sensitive.

Similar observations have been obtained from the analysis of cases bilinear2, bilinear3, trilinear2 and trilinear3.

**Excitation frequency**

To investigate the effect of excitation frequency on ratio \( \eta \), a series of sinusoidal excitations with typical earthquake ground motion frequencies ranging from 0.16 Hz to 10.03 Hz were used.

As shown in Figure 4 and Figure 5, the excitation frequency greatly affects the ratio \( \eta \). It shows that the entire frequency range can be divided into three segments according to the sensitivity to the excitation frequency. These are: lower and resonance region, sensitive frequency region and high frequency region. For these two cases (bilinear1 and trilinear1) the first region is from 0.16 Hz to 3.02 Hz, in which region ratio \( \eta \) changes irregularly as shown in Figure 4 (a) and Figure 5(a). This is the most sensitive region. The second region is from 3.34 Hz to 5.25 Hz, in which region \( \eta \) increases in a much smoother fashion, as shown in Figure 4(b) and Figure 5(b), but some irregularities can also be observed. When the excitation frequency is larger than 5.57 Hz, ratio \( \eta \) increases in a consistent fashion and the change of \( \eta \) is not sensitive to the excitation frequency.

This phenomenon has also been observed for SDOF system \[^5\].

Analysis of bilinear2, bilinear3, trilinear2 and trilinear3 produces similar results but these regions appear to move toward larger frequency with increasing natural frequency.

This phenomenon requires further investigation.
Figure 4 Effect of excitation frequency on ratio $\eta$ for case bilinear1.
Figure 5 Effect of excitation frequency on ratio $\eta$ for case trilinear1
Natural frequency

The effect of natural frequency on the ratio $\eta$ is analyzed by comparing the results of bilinear2 with bilinear3 with those of bilinear1.

The maximum value of the ratio $\eta$ in the whole ductility region for each excitation frequency is identified. Figure 6 shows the change of the maximum value of $\eta$ with the excitation frequency for all three floors. The figure shows that in the high frequency range the curves for different cases are of similar shape, while in lower frequency range they are quite different. The current analysis also shows that the curve for the case bilinear1 is lower than that of bilinear2 and the bilinear2 curve is lower than that of bilinear3. From bilinear1 to bilinear3, the natural frequency increases. Further, the structure displays higher nonlinearity, as shown in Figure 2 and Table 1. Similar results are obtained for the tri-linear yielding models.
EARTHQUAKE EXCITATION

With sinusoidal excitation, the ratio of relative incremental rate of displacement to velocity has been shown to increase with ductility. The behavior of this ratio with earthquake input is examined in this section. The structural model used here is the same three DOF system with trilinear2 and the damping ratio is $\xi=0.03$.

The result under the white noise excitation is illustrated in Figure 7, which shows that the ratio $\eta$ decreases first in the smaller ductility range and then increase faster with the increase of ductility.

Then the same analysis is carried out by using the Mexico earthquake record as input. The result is presented in Figure 8, which shows that ratio $\eta$ increases fast when the ductility ratio is lower than 5. Ratio $\eta$ reaches a maximum value greater than 2. When the ductility ratio is larger than 5, ratio $\eta$ begins to decrease in a slow pace and reaches a stable status after the ductility ratio is greater than 15. It is noted that the value of ratio $\eta$ is still larger than 1.5 when the ductility ratio is approaching 20.
To further verify the effectiveness of displacement-based device, the behavior of a hybrid control system is compared with that of passive damping devices. The hybrid control system consisted of a semi-active device and passive damping devices. The Real-time Structural Parameter Modification (RSPM) technology is a semi-active nonlinear control system, which is a displacement-based technology. The details of this system can be obtained in the references [6] [7] [8].

The analysis used the same three DOF structure with trilinear2 as the force-displacement model. The damping device is a linear viscous damper, for which the damping ratio is 15%. In the hybrid control system, an equivalent 15% of the structural stiffness has been assigned to the RSPM along with a 15% equivalent damping ratio contributed from the hybrid device. The selection of the hybrid control parameters is based on the actual configuration of the devices. RSPM is designed to enhance the performance of the passive damper. To show the effect of the semi-active component in the structural response control, the comparisons are carried out by examining the parameters in a wide range: the elastic response range, the yielding point and the inelastic response range. The following results show that the displacement based semi-active control has a non-uniform control effect. The performance of the system is excellent for non-linear displacement response reduction, especially in the higher ductility range.

The analysis has used both the white noise and Mexico earthquake record as excitations. The results are illustrated in Figure 9 and Figure 10. In these figures, the horizontal axis represents the displacement response of the original structure, which means the uncontrolled system. And the vertical axis represents the displacement reduction percentage when control device is applied, that is

\[ p = \frac{(d_u - d_c)}{d_u} \]

in which, \( p \) is the displacement reduction percentage,

\( d_u \) is the maximum displacement response of uncontrolled structure,

\( d_c \) is the maximum displacement response of controlled structure.
Figure 9 presents the results for the white noise input. The result shows that the hybrid control system is more effective than the pure damping device, which is more apparent in the higher ductility region. The effectiveness for displacement reduction has increased 20%–70%.

With Mexico earthquake record as excitation, the displacement reduction percentage can even be doubled for the hybrid control system, shown in Figure 10. The maximum percentage increases from 39.1% to 65.6%, 39.9% to 67.2%, 41.6% to 68.8% for floor 1, floor 2 and floor 3 respectively. The effectiveness for displacement reduction has increased 50%–100%.

Figure 9 Comparison of damping and hybrid control with input of white noise

Figure 10 Comparison of damping and hybrid control with input of Mexico earthquake record
CONCLUSION

Most structural response modification devices can be grouped into displacement-based device or velocity-based device or hybrid device. In the elastic response range, the velocity-based device is more effective than displacement-based device. But in the inelastic response range, numerical results of SDOF system and MDOF system show that the displacement-based device may have advantage over velocity-based device. This result is observed from a parameter, the ratio of the nonlinear displacement increment rate (normalized by its corresponding linear value) and the rate of nonlinear velocity increment (normalized by its corresponding linear value). This ratio $\eta$ is larger than unity in the inelastic response range and it increases when the ductility increases. This means that the displacement response increases faster than velocity response. Furthermore this study also suggests that excitation frequency and natural frequency are important factors affecting this ratio $\eta$.

In addition, comparison between the hybrid control system with linear viscous damper has validated that the former is more effective.

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