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THE SHEAR STRENGTH OF RC COUPLING BEAMS WITH PLASTIC HINGES USING STRUT-AND-TIE MODEL

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SUMMARY

In this study, a strut-and-tie model for the coupling beam accompanying plastic hinges is presented. Assuming the required plastic deformation at both ends of the coupling beam, the strut-and-tie model is constructed. To design earthquake-resistant shear-dominated RC coupling beams, it is important to consider shear strength deterioration with required ductility. This study proposes the method of estimating shear strength of the reinforced concrete coupling beams considering the strength degradation of the diagonally compressed concrete due to the strain in plastic hinges. The proposed method determines the plastic strain from required plastic deformations based on the elastic deformation of coupling beams and ultimate wall deformations. The estimated horizontal strain of beam is then used in calculating the strength of the diagonal strut of arch action with compatibility conditions. The deterioration of shear strength of the coupling beam depends on the decrease of arch action due to plastic deformations.

INTRODUCTION

Structural walls have long been used in the planning of multistory buildings. A reliable earthquake-resistant system is represented by coupled shear walls, in which two vertical elements are interconnected by short and deep beams. Since 1977, when the American Concrete Institute (ACI 318-77) stipulated specific rules for the design of ductile reinforced concrete structures, the concept of ductility capacity has played an increasing role in the seismic design of structures. The beams of a coupled wall are subjected to large cyclic shear deformations during an earthquake. Hence, one of the most critical problems of these structures concerns the brittle failure of the low slenderness coupling beams. If this failure is avoided, a large fraction of the input seismic energy dissipated by these elements that are distributed throughout these walls. Consequently, building safety will be improved. This favorable behavior of coupled walls has been previously demonstrated in other research [1-3].

The performance-based design philosophy of ductile coupled wall systems allows the coupling beams to form plastic hinges adjacent to beam-wall connections. To carry out this design philosophy, the shear strength of the coupling beams should be greater than flexural yielding force at the ultimate deformation state. After flexural yielding, plastic hinges developed near both ends of the beam, followed by yielding of

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the shear reinforcement and crushing of the diagonal compressive concrete strut in the plastic hinge region, which led to a sudden failure of the beam.

Recent experimental studies [4] on RC members have led to empirical formulations that allow for a degradation in the concrete shear resistance with increasing displacement ductility demand. An increase in the axial strain after flexural yielding widen the diagonal cracks in plastic hinge, thus leading to an increase in the principal tensile strain perpendicular to the crack direction. According to the RC panel tests by Vecchio and Collins [5], the effective strength of concrete is found to decease as principal tensile strain increase. Therefore, for RC beams with a short span-depth ratio, the effective strength in the plastic region deceases after flexural yielding due to sudden increase of the strains in the plastic region.

In this paper, a strut-and-tie model for the coupling beam with plastic hinge is presented. The ultimate displacement is obtained by using the relationship between walls and coupling beam rotations. Assuming the required plastic deformation at both ends of the coupling beam, the deformed strut-and-tie model is constructed.

A strut-and-tie model is a representation of internal force flows in reinforced concrete members by discrete compression struts and tension ties joined together at nodes. As the strength models based on the assumption of ultimate state, strut-and-tie models are today considered by researchers and structural engineers to be a rational tool for the design of D-regions, where the stress is not linearly distributed. Strut-and-tie models require sufficient deformation capacity of components of models. However, while reinforcing steel typically exhibits a rather ductile behavior, the behavior of concrete is far from being plastic. In addition, bond shear stresses transferred between reinforcing bars and the surrounding concrete result in a localization of steel strain near the crack, particularly in post-yield range, reducing the overall ductility of bonded reinforcement. Hence, in order to justify the application of strut-and-tie models to structural concrete it is necessary to ensure a sufficient deformation capacity through appropriate detailing with use of effectiveness factor for concrete [6].

RESEARCH SIGNIFICANCE

To design earthquake-resistant shear-dominated RC coupling beams, it is important to consider shear strength deterioration with required ductility. The research reported in this paper provides a method to estimate the shear strength of RC coupling beams after the plastic deformation has been formed. The proposed method takes into account shear deterioration of concrete due to the degradation of the diagonally compressed concrete in the postyield range.

COUPLED WALL SYSTEMS

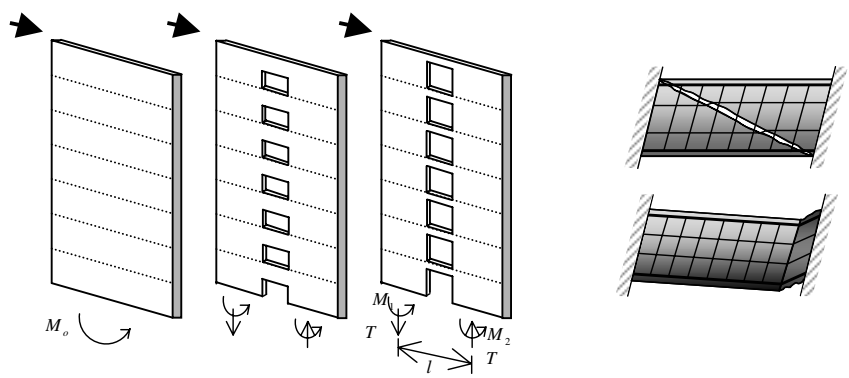
Coupled walls

In many structural walls a regular pattern of openings will be required to accommodate windows or doors. Efficient structural systems, particularly suited for ductile response with very good energy dissipation characteristics, can be conceived when openings are arranged in a regular and rational pattern. Examples are shown in Fig. 1(a) where walls are interconnected or coupled to each other by beams. The walls, which behave predominantly as cantilevers, can then impose sufficient rotations on these connecting beams to make yield. If suitably detailed, the beams are capable of dissipating energy over the entire height of the structures. It is seen that the total overturning moment, M_o, is resisted at the base of the cantilever in the traditionally form by flexural stresses, while in the coupled walls axial forces as well as moments are being resisted. These satisfy the following simple equilibrium statement.

$$M = M_1 + M_2 + lT \tag{1}$$

Coupling beam

The primary purpose of beams between coupled walls during earthquake actions is the transfer of shear from one wall to the other. In considering the behavior of coupling beams it should be appreciated that during an earthquake significantly larger inelastic excursions can occur in such beams than in the walls that are coupled. During one earthquake, larger numbers of shear reversals can be expected in the beams than in the walls. Many coupling beams have been designed as conventional flexural members with stirrups and with some shear resistance allocated to the concrete. Such beams will inevitably fail in diagonal tension, as shown in Fig. 1(b). It is evident that the principal diagonal failure crack will divide a relatively short beam into two triangular parts. Unless the shear force associated with flexural overstrength of the beam at the wall faces can be transmitted by vertical stirrups only, diagonal tension failure will result. Under reversed cyclic loading it is difficult to maintain the high bond stresses along the horizontal flexural reinforcement, necessary to sustain the high rate of changes of moment along the short span. Such horizontal bars, shown in Fig. 1(b), tend to develop tension over the entire span, so that shear is transferred primarily by a single diagonal concrete strut across beam.



(a) (b)

Fig. 1 (a) comparison of flexural resisting mechanisms in structural wall (b) diagonal tension failure and sliding failure

STRUT AND TIE MODEL

Construction of Strut-and-Tie model

Shear forces which are loaded on coupling beam mostly is resisted by truss action, shown in Fig. 2(a), and arch action, shown in Fig 2(b). Truss mechanism transfers a portion of shear force from one support to the other by the stirrups which form a truss together with diagonal concrete strut. Shear force not resisted by the truss mechanism is allocated to a single diagonal strut by arch mechanism. A strut-and-tie model for the RC coupling is constructed by combining the mechanisms as shown in Fig. 2(c).

For the coupling beams with a small aspect ratio, the flexural reinforcement can be expected to be in tension over the entire span of the beam in contrast with simple beam theory. A low stress area in the vicinity of zero bending does not exist. Both top and bottom reinforcement are in tension. Fig. 2 (d) shows that tensile force of the top horizontal chord phases down due to stirrup force.

Equilibrium condition

By Equilibrium condition at Node N_1, the diagonal strut C_1 of the truss mechanism is obtained as:

$$C_1 = \frac{S}{\sin\theta_1} = \frac{A_v f_{yv}}{\sin\theta_1} \tag{2}$$

where S is stirrup force; θ_1 is the inclination angle of the diagonal strut; A_v and f_{yv} are the area and the yield strength of vertical reinforcements in coupling beam, respectively. The horizontal tie force of the middle portion of the bottom chord is determined as follows:

$$T_2 = T_1 - S\cot\theta_1 = A_s f_{yh} - S\cot\theta_1 \tag{3}$$

where T_1 is the tie force of the left portion of the bottom chord; f_{yh} is the yield strength of horizontal reinforcement.

Similarly at node N_2, the diagonal strut force C_2 and the right tie force T_3 are:

$$C_2 = \frac{S}{\sin\theta_2} \tag{4}$$

$$T_3 = T_2 - S\cot\theta_2 \tag{5}$$

where θ_2 is the inclination angle of the diagonal strut.

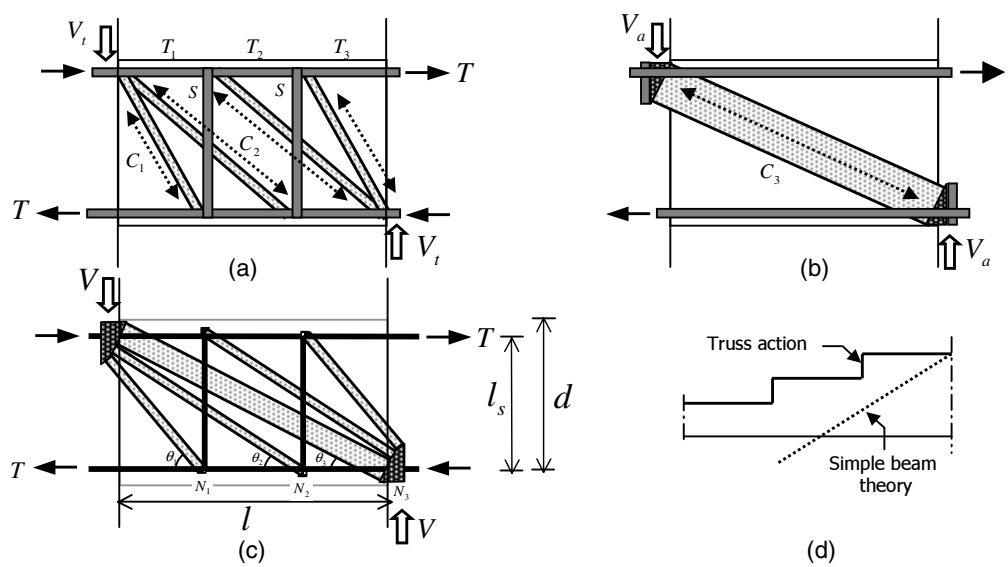


Fig. 2 (a) truss action (b) arch action (c) strut-and-tie model (d) tensile force of the top steel

The diagonal strut force C_3 of arch mechanism is estimated by estimating the vertical length of the nodal zone N_3. The widths of the diagonal strut w_{c1} and w_{c2} are:

$$w_{c1} = \frac{C_1}{b_b f_c^{'}} \tag{6}$$

$$w_{c2} = \frac{C_2}{b_b f_c^{'}} \tag{7}$$

where b_b is the thickness of coupling beams; $f_c^{'}$ is the compressive strength of concrete.

By Equilibrium condition of beam-wall faces, the vertical length of the nodal zone l_{nv} is calculated as:

$$l_{nv} = \frac{T_1 + T_3}{b_b f_c^{'}} \tag{8}$$

The maximum width of diagonal strut w_{c3} is limited by the other diagonal strut, as shown in Fig. 3. Hence, the diagonal strut force C_3 of arch mechanism is:

$$w_{c3} = \frac{\left(l_{nv} - w_{c1}\cos\theta_1 - w_{c2}\cos\theta_2\right)}{\cos\theta_3} \tag{9}$$

$$C_3 = w_{c3} b_b f_c^{'} \tag{10}$$

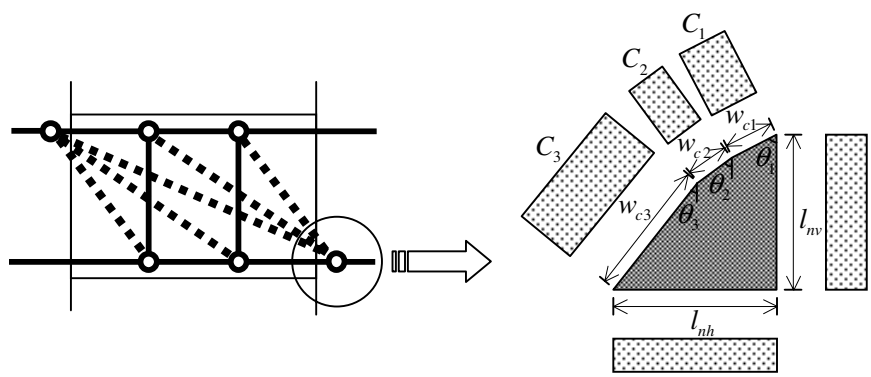


Fig. 3 diagonal strut width of the nodal zone

REQUIRED DEFORMATION

Elastic Deformation

Elastic Deformations in cracked coupling beams consists of: (1) shear distortions by truss mechanism; (2) compressive deformations by arch mechanism; (3) flexural rotations by unequal horizontal tie elongation; (4) elongation of the beam by flexural reinforcement in tension.

Shear rotations by the truss action, as shown in Fig 4 (a), are defined by.

$$\theta_t = \frac{\Delta_t}{l} = \frac{\Delta_{c1} + \Delta_{c2} + \Delta_s}{l} \tag{11}$$

$$\Delta_{c1} = \frac{\frac{l_s}{\sin\theta_1}\frac{V_t}{\sin\theta_1}}{E_c w_{c1} b_b}\frac{1}{\sin\theta_1} = \frac{l_s V_t}{E_c w_{c1} b_b \sin^3\theta_1};\ \Delta_{c2} = \frac{l_s V_t}{E_c w_{c2} b_{beam} \sin^3\theta_2};\ \Delta_s = \frac{V_t l_s}{E_s A_v} \tag{12}$$

where θ_t is shear rotations; Δ_t is the vertical displacements by truss action; Δ_{c1} and Δ_{c2} is the deformation of diagonal strut C_1 and C_2, respectively; E_c is the elastic modulus of cracked concrete.

Shear rotations by the vertical displacement of diagonal strut C_3, as shown in Fig 4 (a) is:

$$\theta_a = \frac{\Delta_a}{l} \tag{13}$$

$$\Delta_a = \frac{l_s V_a}{E_c w_{c3} b_{beam} \sin^3\theta_3} \tag{14}$$

where Δ_a is the vertical displacement of arch mechanism.

As a result of the accumulated strains along the flexural steel the original vertical planes become inclined with respect to the center section, as shown in Fig. 4 (c). Thus the flexural rotation is estimated as:

$$\theta_{fr} = \frac{\Delta_{tl} - \Delta_{tr}}{l_s} \tag{15}$$

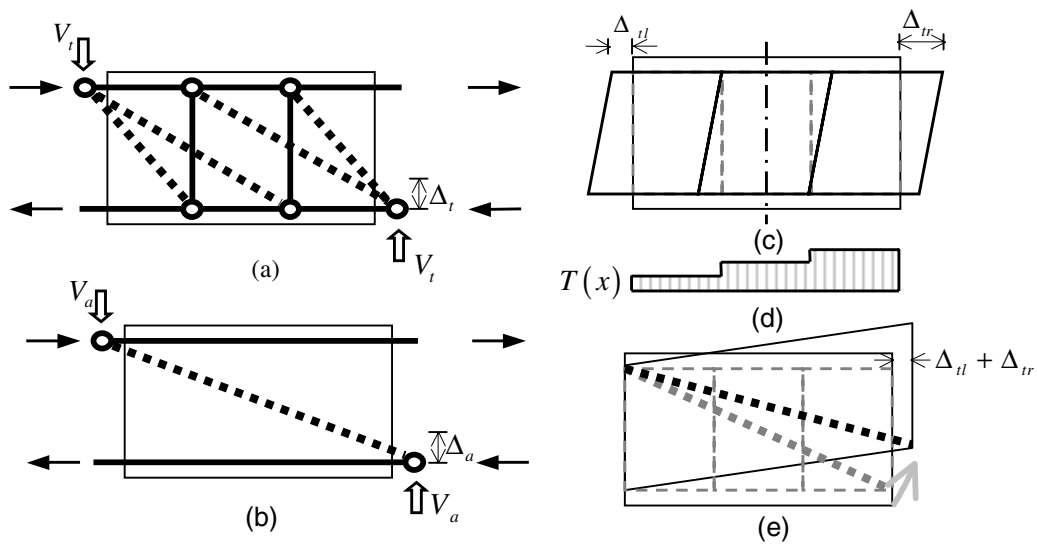


Fig. 4 (a) displacement by truss action (b) displacement by arch action (c) displacement by flexural rotation (d) top chord strain (e) displacement by beam elongation

$$\Delta_{tl} = \frac{1}{E_s A_s}\int_0^{\frac{l}{2}} T(x)dx \tag{16}$$

$$\Delta_{tr} = \frac{1}{E_s A_s} \int_{\frac{l}{2}}^{l} T(x) dx \tag{17}$$

where Δ_{tl} is horizontal displacement of the left potion of the flexural reinforcement; Δ_{tr} is horizontal displacement of the right potion of the flexural reinforcement; E_s is the elastic modulus of reinforcements. $T(x)$ is the tensile force of the flexural reinforcement, as shown in Fig. 4 (d).

The elongation of the flexural reinforcement in the span of the beam is associated with diagonal compressions resulting from truss and arch actions in the concrete. Both of these require a tying action of the flexural reinforcement. Fig. 4 (e) shows the lengthened top reinforcement. The length of the compression diagonal is shown to remain unchanged. Therefore, the resulting rotation is expressed as:

$$\theta_{bel} = \frac{\Delta_{tl} + \Delta_{tr}}{l_s} \tag{18}$$

The superposition of the four previous cases yields the elastic total rotation.

$$\theta_{be} = \theta_t + \theta_a + \theta_{fr} + \theta_{bel} \tag{19}$$

Required plastic deformation

The required deformation of the coupling beams is determined by estimating the deformation capacity of the walls. The required rotation and plastic rotation of the beams are:

$$\frac{\theta_{bd}}{\theta_{wd}} = \frac{D_w + c_1 - c_2}{l} \tag{20}$$

$$\theta_{bp} = \theta_{bd} - \theta_{be} \tag{21}$$

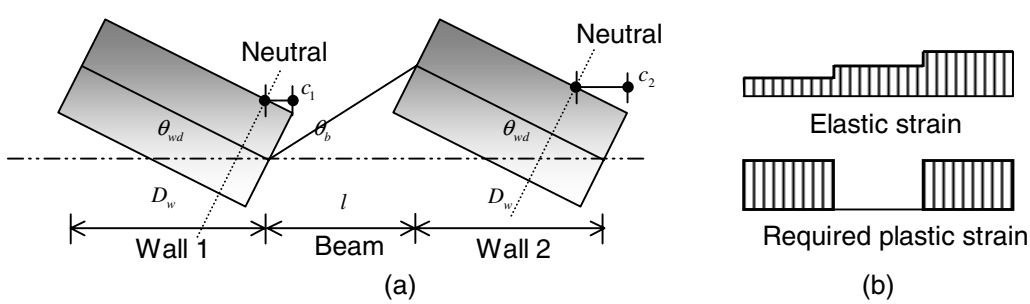


Fig. 5 (a) Relationships between wall and coupling beam rotations
(b) strain of horizontal top chord of coupling beams

where θ_{wd} is the rotation capacity of the walls; D_w is the length of the wall; c_1 and c_2 are the centroid of the walls. θ_{bp} is the plastic rotation of the coupling beams. The elastic rotation of the beams is estimated in previous section.

ESTIMATION OF ULTIMATE SHEAR STRENGTH

In this study, the degradation of the shear strength of the coupling beams is assumed to attribute to the decrease of the shear strength contributed by concrete. An increase of the horizontal strain by plastic deformation of the beam leads to an increase of the principal tensile stress which decreases the compressive strength of the concrete. To investigate the stress-strain characteristics of diagonally cracked concrete, Vecchio and Collins tested reinforced concrete elements in pure shear [5]. Based on these tests, they found that principal compressive stress in the concrete f_2 is a function not only of the principal compressive strain ε_2 but also of the coexisting principal tensile strain ε_1, as shown Fig. 6.

$$f_2 = f_{2\max}\left[2\left(\frac{\varepsilon_2}{\varepsilon_c^{'}}\right)-\left(\frac{\varepsilon_2}{\varepsilon_c^{'}}\right)^2\right] \tag{22}$$

$$\frac{f_{2\max}}{f_c^{'}} = \frac{1}{0.8+170\varepsilon_1} \le 1.0 \tag{23}$$

where $\varepsilon_c^{'}$ is the strain at the peak compressive stress in the concrete .

The horizontal strain of the concrete is:

$$\varepsilon_h = \varepsilon_{he} + \varepsilon_{hp} \tag{24}$$

where ε_{he} and ε_{hp} are the elastic strains and the plastic strain of the concrete, respectively. Considering that beam section near the wall face reaches the flexural capacity in elastic region, the elastic strains of the concrete is assumed as:

$$\varepsilon_{he} = \varepsilon_y \tag{25}$$

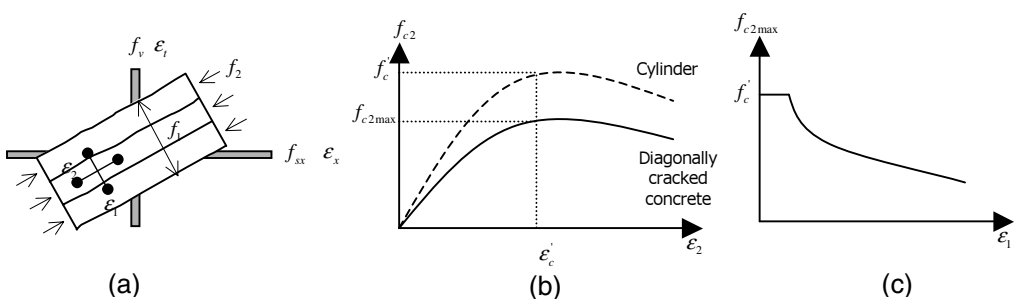


Fig. 6 (a) directions of stress and strain (b) stress-strain relationship in cracked concrete (c) relationship for maximum compressive stress

where ε_y is the yield strain of the reinforcement steel. The plastic strain of the concrete is determined by the plastic rotation of the coupling beam which is estimated in previously section. the horizontal plastic strain is:

$$\varepsilon_{hp} = \frac{\theta_{bp}}{2\cot\theta_1} \tag{26}$$

The vertical strain and the principal compressive strain of concrete are:

$$\varepsilon_v = \varepsilon_y \tag{27}$$

$$\varepsilon_2 = \varepsilon_c^{'} \tag{28}$$

The principal tensile strain ε_1 can be obtained considering a compatibility condition relating the three strains, ε_h, ε_v, and ε_2.

$$\varepsilon_1 = \varepsilon_h + \varepsilon_v - \varepsilon_2 \tag{29}$$

Finally, the diagonal strut force C_3 and shear strength of the coupling beam at the ultimate deformation are:

$$C_{3u} = f_2 b_b w_{c3} \tag{30}$$

$$V_u = C_1 \sin\theta_1 + C_2 \sin\theta_2 + C_{3u} \sin\theta_3 \tag{31}$$

VERIFICATION

The ultimate shear strengths of the coupling beams based on the proposed models are compared with those of experimental programs including results published in literature. The properties of specimen and the comparison of strength are summarized in table 1. All specimens are conventionally reinforced coupling beams. The calculated strength is a good agreement with the tested results. Specimen 312, 315 and P02 with aspect ratio 1.2 are underestimated by approximately 10%. Specimen CB-1A and CB-1B are overestimated by approximately 25%.

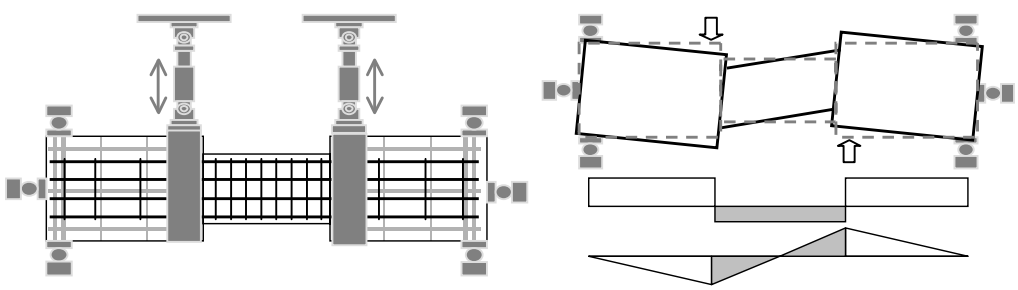


Fig. 7 experimental specimen

Table 1 experimental data and proposed shear strength

author	Specimen	l (mm)	d (mm)	b_b (mm)	$f_c^{'}$ (MPa)	ρ_h (%)	ρ_v (%)	f_{yh} (MPa)	V_u^{test} (KN)	$V_u^{cal.}$ (KN)	$\frac{V_u^{cal}}{V_u^{test}}$
Pauly [1]	312	1016	714	152	35.12	1.58	1.65	306.8	622.2	580.5	0.933
	315	1016	714	152	37.92	1.58	2.43	315.2	755.5	668.6	0.885
Theo. [12]	CB-1A	500	500	130	32.80	0.69	0.93	484.0	212.0	258.2	1.218
	CB-1B	500	300	130	33.00	1.16	0.93	484.0	124.0	158.4	1.277
Lu[13]	P02	600	400	150	44.5	1.05	0.33	567.0	223.9	203.5	0.908

CONCLUSIONS

This research proposes the method of estimating shear strength of the reinforced concrete coupling beams considering the strength degradation of the diagonally compressed concrete due to the strain in plastic hinges. The proposed method estimates the plastic strain from required plastic deformations based on the elastic deformation of the coupling beam and ultimate wall deformations. The estimated horizontal strain of concrete is then used in calculating the strength of the diagonal strut of arch action with compatibility conditions. The deterioration of shear strength of the coupling beams is estimated by the decrease of arch action due to plastic deformations.

This model can be used in the case of the short and deep coupling beam with diagonal tension failure after yielding of flexural reinforcement. The comparisons between experimental results and the proposed method show a good agreement. This paper is not applied to the sliding failure mode. In case of sliding failure, other approaches are needed.

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