TEMPORAL-FREQUENCY RESPONSE SPECTRUM ANALYSIS OF SEISMIC WAVE

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SUMMARY

In this paper a new concept of Temporal-Frequency Response Spectrum (TFRS) is introduced. TFRS is a three-dimensional spectrum, which is the function of not only period but also time. The TFRS of four seismic waves recorded in Imperial Valley (1940), Northridge (1994), Kobe (1995) and Chi Chi (1999) earthquakes are calculated and compared. The characteristics of TFRS are analyzed and TFRS is also applied to analyze the damage mechanism of structures. And a new definition of damage line of structure is proposed. The results show that the damage line of the structure and TFRS of ground motion are important for seismic design, only the input seismic wave has suitable TFRS for the structure, the seismic analysis and seismic design are reliable.

INTRODUCTION

The characteristics of strong ground motion can be described by its three correlated factors: intensity, spectrum and duration. The three characteristics are key factors influencing damage on structures. Static design method only considers the intensity of strong ground motion. Response spectrum theory considers two characteristics of ground motion: spectral amplitude and frequency. Dynamic design theory analyzes whole response procedure of the structure to the input seismic ground motions. Dynamic design theory considers intensity, spectrum and duration of seismic wave, though its result will be greatly influenced by the seismic wave selected in the analysis procedure.

Since the concepts of response spectrum and normalized response spectrum were presented (Housner[1,2]), response spectra are widely used to describe ground motions for seismic design, but response spectra cannot account for duration. Safak and Frankel[3] suggested incorporating duration in response spectra by introducing the concept of 3D response spectra. 3D spectra account for the secondary peaks, as well as the largest peak, of the displacement response of a single-degree-of-freedom oscillator. The decay of peak amplitudes with increasing number of

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crossings gives the information on duration. 3D response spectra provide a new measure of ground motion intensity, defined as the volume under the 3D velocity response spectra, which shows good correlation with damage in structures.

Past earthquakes show that the duration of strong ground motions is an important factor influencing the damage on structure. 3D response spectra suggested by Safak and Frankel[3] account for duration, but it cannot account for the damage procedure of structures, which is a procedure of cumulative effect caused by input ground motion. We have defined one new concept of Temporal-Frequency Response Spectrum (TFRS), which accounts for three characteristics of ground motions: intensity, spectrum and duration, and it can be used to analyze the damage procedure of structures (Zhang and Luo[4], Zhang[5], Luo and Zhang[6], Li[7]).

DEFINITION OF TEMPORAL-FREQUENCY RESPONSE SPECTRUM (TFRS)

Response spectra are the maximum absolute values of response of a series of single-degree-of-freedom oscillators with same damping and different period over the entire time histories (Housner[1,2]) in which the effect of earthquake duration couldn't be taken into consideration. The displace spectrum, velocity spectrum and acceleration spectrum can be described as (Clough and Penzien[8])

\[
S_d(t, \omega, t) = \frac{1}{\omega} \int_0^t y(\tau) \sin \omega(t - \tau) e^{-\xi \omega(t - \tau)} d\tau
\]  \hspace{1cm} (1)

\[
S_v(t, \omega, t) = \int_0^t \ddot{y}(\tau) \sin \omega(t - \tau) e^{-\xi \omega(t - \tau)} d\tau
\]  \hspace{1cm} (2)

\[
S_a(t, \omega, t) = \omega \int_0^t \dddot{y}(\tau) \sin \omega(t - \tau) e^{-\xi \omega(t - \tau)} d\tau
\]  \hspace{1cm} (3)

The new concept of Temporal-Frequency Response Spectrum (TFRS) is the absolute value of response of a series of single-degree-of-freedom oscillators with same damping and different period over the entire time histories, which can be described as

\[
S_d(t, \omega, t) = \frac{1}{\omega} \int_0^t y(\tau) \sin \omega(t - \tau) e^{-\xi \omega(t - \tau)} d\tau
\]  \hspace{1cm} (4)

\[
S_v(t, \omega, t) = \int_0^t \ddot{y}(\tau) \sin \omega(t - \tau) e^{-\xi \omega(t - \tau)} d\tau
\]  \hspace{1cm} (5)

\[
S_a(t, \omega, t) = \omega \int_0^t \dddot{y}(\tau) \sin \omega(t - \tau) e^{-\xi \omega(t - \tau)} d\tau
\]  \hspace{1cm} (6)

Take the seismic record at El Centro in Imperial Valley earthquake (1940) as an example (Figure 1). The three-dimensional network and contour map of its TFRS are shown in Figure 2 and 3, respectively.
Comparing TFRS with Response spectrum, it follows that the usual response spectrum is the function of period, while TFRS is a three-dimensional spectrum, which is the function of not only period but also time. It is obvious that TFRS has almost the same characteristics as response spectrum, because it is also calculated by using Duhamel integral; and the characteristics of seismic waves in the frequency-domain and time-domain can be revealed in TFRS simultaneously.
The normalized response spectrum is defined as

\[
\beta_A = \frac{(\ddot{x} + \ddot{y})_{\text{max}}}{\dddot{y}_{\text{max}}} \tag{7}
\]

\[
\beta_v = \frac{\dot{x}_{\text{max}}}{\ddot{y}_{\text{max}}} = \frac{S_v}{\dddot{y}_{\text{max}}} \tag{8}
\]

\[
\beta_D = \frac{x_{\text{max}}}{y_{\text{max}}} = \frac{S_D}{\dddot{y}_{\text{max}}} \tag{9}
\]

The normalized response spectrum can be used to make comparison for different seismic waves.

**COMPARISON OF TFRS OF SEVERAL SEISMIC WAVES**

TFRS is used to analyze the temporal-frequency characteristics of several seismic waves. The TFRS of accelerations, which were recorded in Imperial Valley (1940), Northridge (1994), Kobe (1995) and Chi Chi (1999) earthquakes, are calculated. The records and their TFRSs are shown in Figure 1 ~ 12, respectively.

![Figure 4 Seismic wave recorded at Jensen Filtration Plant in Northridge earthquake (1994)](image)

![Figure 5 Three-dimensional network of TFRS of Northridge wave](image)
Figure 6 Contour map of TFRS of Northridge wave

Figure 7 Seismic wave recorded in Kobe earthquake (1995)

Figure 8 Three-dimensional network of TFRS of Kobe wave

Figure 9 Contour map of TFRS of Kobe wave
The results show that the amplitudes of these TFRS are different in their time-domain and frequency-domain distributions. The duration of El Centro wave is very long (see Figure 1). The peak TFRS value is 341.7gal at the 2.14 second mark on the time-axis, after that there are several large values, which are almost half of the peak amplitude (see Figure 2 and 3), most of them appear at the 0.5 second mark on the period-axis and the last one appears at about the 26 second mark on the time-axis. In the period domain, the peak amplitudes appear between 0~1 seconds, especially at the 0.5 second mark on the axis. From Figure 3, we can see that there are several peak amplitudes before the 13 second mark and their periods are distributed in the range of 0.3 ∼ 0.5 seconds, and the second peak value appears at the 13 second mark and its period is 0.7 seconds. Analyzing the network map of the TFRS of El Centro wave, we can guess a structure damage model. Once the structure is affected by the El Centro wave, the structure may be damaged before the first 13 seconds and its natural period will be lengthened. When it meets the second
peak amplitude for a period of 0.7 seconds, resonance may occur. From this we can see that the accumulation of the damage of the structure cannot be neglected.

On time-axis the duration of Northridge record is longer than others and the acceleration values are relative large between the 6 and 25 second mark; the peak value appears at the 15 second mark (see Figure 4). On the period-axis, the spectrum of Northridge seismic wave shows that the frequency components are very rich. The peak value appears at the 2.1 second mark, but the proportion of frequency components distributed from the 0.8 to 2.5 second mark are larger than other components (see Figure 5 and 6). The three-dimensional network of Northridge wave (Figure 5) shows that the peak amplitudes of TFRS concentrate in the time range from 6 to 25 seconds and in the frequency range of 0.8 to 2.5 seconds. It implies that once a structure, which has a shorter natural period, is damaged by the shorter period components of the input wave, its natural period become longer and it easily resonates with the longer period components of the later part of the wave.

On time-axis the energy of Kobe record concentrates within the 7 and 15 second mark; the peak value appears at the 5 second mark (see Figure 7). In a short time, there are two peak values and then the amplitude decreases more quickly, and the duration of the whole wave is relative shorter. On the period-axis, the period of the main portion is smaller than 1 second, after 1 second the amplitude decreases steadily along the period axis (see Figure 8 and 9). The three-dimensional network of the Kobe wave (Figure 8) shows that there is a single peak amplitude and its energy is concentrated in a small area not only in the time domain but also in the frequency domain. Comparing the TFRS of different waves mentioned before, the Kobe seismic wave will produce the largest structural response.

The duration of the wave recorded at P088 station in Chi Chi earthquake is very long and the whole record lasts for 144 seconds. Most of the large values appear between the 40 and 60 second mark on the time-axis, and the peak values appear at the 48.6 second and the 53.6 second mark (see Figure 10). On the period-axis, the energy is concentrated in a narrow range, whose period is shorter than 1 second. The peak value appears at about the 0.55 second mark, after that the amplitude decreases along period axis quickly (see Figure 11 and 12). In the three-dimensional network of Chi Chi wave (Figure 11) we can see that the amplitudes last a long time on the time-axis and decrease quickly on the period-axis. The two peak values are like two sharp mountains. It implies that the energy of Chi Chi earthquake concentrates in a narrow period range. Comparing the TFRS of different waves mentioned before, the spectrum amplitude of Chi Chi seismic wave is the largest one (see Table 1).

<table>
<thead>
<tr>
<th></th>
<th>El Centro</th>
<th>Northridge</th>
<th>Kobe</th>
<th>Chi Chi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak TFRS</td>
<td>262.86</td>
<td>291.21</td>
<td>324.12</td>
<td>476.29</td>
</tr>
<tr>
<td>Appear period(s)</td>
<td>0.55</td>
<td>2.2</td>
<td>0.35</td>
<td>0.5</td>
</tr>
<tr>
<td>Appear time(s)</td>
<td>2.18</td>
<td>14.91</td>
<td>10.96</td>
<td>48.98</td>
</tr>
</tbody>
</table>

After all the four peak accelerations are normalized to 100, the peak values of TFRS are still different from each other. For example, the maximum response of the El Centro wave is 263, Northridge 291, Kobe 324, and Chi Chi even reaches 476. From the statistics it can be reasoned out that structures designed
According to the El Centro record would not always survived the Chi Chi earthquake with the same magnitude.

**APPLICATION OF TFRS IN ANALYSIS OF STRUCTURE DAMAGE MECHENISM**

TFRS analysis method is a method which can shows the trend of the earthquake motion's frequency components’ (include amplitude and frequency) changing with time. Due to its convenience for taking all characteristic factors of seismic ground motion into account, TFRS is a useful tool for structural dynamic analysis.

One six-story concrete frame structure (Figure 13) is chosen to analyze its damage process by using TFRS. The top point of the frame is treated as a single-degree-of-freedom system (SDOF). And its elastic-plastic curve is obtained by using the pushover method (Figure 14). The SDOF has equivalent values to the MDOF's damping and natural period in the elastic range. The natural period of the SDOF is 0.98s, equivalent mass 157085Kg and equivalent damping ratio 0.148 (Li[7]).

Then the SDOF is analyzed by using dynamic analysis. The constitutive curve of the structure is the pushover curve in figure 14. The Northridge wave is chosen as the input wave (see Figure 4). When the peak acceleration of input seismic wave is 300 Gal, at first the structure is in the elastic range, about five second later the structure comes to the nonlinear deformation range, and its natural period is changed with the time (see Figure 15). From the time history of the system’s natural period shown in Figure 15, it can be seen that the effect of input seismic wave will influence the structure’s natural frequency.

To analyze the damage mechanism of the structure, the time history of the system’s natural period and the contour map of TFRS of the input wave are shown in Figure 15. From the figure, we can see that after
above 5 seconds later of the earthquake beginning the structure comes to plastic range, its natural period changes acutely from 1 second to 1.8 second. The contour map shows that at that moment the structure meets the large values of ground motion which vibration period is about 1.8 second. As mentioned above, when the vibration period of ground motion is consistent with the changed structure’s natural period, it is easy to produce resonance and destroy the structure.

Here the time history of the system’s natural period is defined as damage line of the structure. It is obvious that each structure, at least each kind of structure, has its own damage line, so that only the seismic wave, which has suitable temporal-frequency response spectrum for the structure, can be used to make seismic analysis for it. If the input wave is not suitable for the structure, the seismic design will not be reliable. As described in the Figure 16, if the TFRS of the input wave did not have large values at the time of above the 6 second mark on the time-axis and at about the 1.6~1.8 second mark on the period-axis, which is correlated with the damage line of the structure, the input wave would not destroy the structure. But the problem is that how to synthesize the time histories to satisfy an appointed TFRS for a special structure in a special range. The probability-consistent scenario earthquake (Luo[9]) might be used to do it.

**DISCUSSION AND CONCLUSION**

Being different from usual response spectrum, TFRS accounts for three main characteristics of strong ground motion, which are intensity, spectrum and duration. From the study on TFRS, five conclusions can be arrived at:

1. The TFRS has almost the same characteristics as response spectrum, because it is also calculated by using Duhamel integral.
2. In the TFRS, the characteristics of seismic wave in the frequency-domain and time-domain can be
revealed simultaneously, including the changing-with-time of the frequency characteristics.
3. The duration of seismic wave is introduced into TFRS, which can be used to reflect the cumulating effect of seismic ground motions.
4. The TFRS may be used to analyze the damage mechanism of structure.
5. The proposed definition of damage line of the structure is important for seismic design. Only when the input seismic wave has suitable TFRS for the structure, should the seismic analysis and seismic design be reliable.

We need to analyze more seismic records and study how to synthesize the time histories to satisfy an appointed TFRS for a special structure and in a special range.

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REFERENCES