ESTIMATION OF INSTANTANEOUS SPECTRUM OF GROUND MOTIONS WITH NEURAL NETWORKS

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SUMMARY

The subject matter of the paper is the estimation of instantaneous spectrum of earthquake ground motions. It has been shown that the nonstationary frequency of earthquake ground motions has significant influence on structural nonlinear response. In order to describe the nonstationary frequency, the instantaneous spectrum can be used for this purpose. Based on time varying ARMA model representation of earthquake ground motions, a method for instantaneous spectrum estimation of earthquake ground motions is set forth by recasting the ARMA model into a single layer recurrent neural network trained by Kalman filter. As an example, the instantaneous spectrum of El Centro (1940, N-S) is estimated and it has been shown, by comparing the estimation results with the Wigner-Ville distribution, that the method is quite suitable for estimating the instantaneous spectrum of earthquake ground motions.

INTRODUCTION

Recent researches have shown the conventional properties of earthquake ground motions, e.g. properties of intensity, frequency and duration, are not so enough as to thoroughly influence the seismic response characteristics of structures, and the analyses of strong earthquake ground motions and the factors that influence the structural seismic responses have indicated that the earthquake processes possess obvious nonstationarity in frequency contents, which usually varies with time and has significant influence on structural seismic response, see Li [1]. Now more and more attentions are paid to the nonstationarity in frequency and that has been a challenge to the notion that the frequency contents of earthquake processes are thought to be stationary in the researches of structural accurate dynamic analysis.

The instantaneous-spectrum can be used to describe the nonstationarity in frequency perfectly, and the estimation of which can not only perform the description of nonstationarity in frequency, but also can represent the energy joint distribution of earthquake ground motions in time and frequency domain. A lot of methods are set forth to estimate instantaneous spectrum, which can be divided into two types, see Auger [2], one is parameterized method which is mostly based on time varying ARMA model and the other is non-parameterized method which consists of two classes solutions, the atomic decomposition (e.g.

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Short-Time Fourier Transform, Wavelet transform and the Gabor Representation) and energy distributions (e.g. Cohen’s class, affine class and reassignment method). The non-parameterized methods are effective on synthesis and simulation of earthquake motions, but they are not suitable for the modeling of instantaneous spectrum, i.e., their properties cannot be formulated as explicit functions and they should be recast beforehand if they are to be applied for practical purpose. While the parameterized methods can estimate the instantaneous spectrum accurately without above disadvantages and the properties of the instantaneous spectrum is merely determined by the estimated parameters. An acceleration sample of large populations with the same statistical characteristics as the actual earthquake ground motions can also be generated using the estimated parameters. In this paper, based on the discrete a method for instantaneous spectrum estimation of earthquake ground motions is set forth by recasting the ARMA model into a single layer recurrent neural network trained by Kalman filter. As an example, the instantaneous spectrum of El Centro (1940, N-S) is estimated and it has been shown, by comparing the estimation result with the Wigner-Ville distribution, that the method is quite suitable for estimating the instantaneous spectrum of earthquake ground motions.

INSTANTANEOUS SPECTRUM ESTIMATION OF EARTHQUAKE GROUND MOTIONS

Discrete time varying ARMA model of ground motions

The earthquake ground motions can be described in following discrete time varying ARMA\((p,q)\) model, see Conte et al. [3],

\[
a_k - \phi_{1,k} a_{k-1} - \cdots - \phi_{p,k} a_{k-p} = \sigma_k e_k - \theta_{1,k} (\sigma_{k-1} e_{k-1}) - \cdots - \theta_{q,k} (\sigma_{k-q} e_{k-q})
\]

where \(\{a_t\}\) is earthquake process, \(\{e_k\}\) is white noise process, \(\{\phi_i, i=1,2,\ldots,p\}\) are the autoregressive parameters, \(\{\theta_i, i=1,2,\ldots,q\}\) are the moving-average parameters, \(\{\sigma_k\}\) is envelope function, the subscript \(k\) means time \(t=k\Delta t\).

Equivalence between time varying ARMA model and neural network

The time varying ARMA\((p,q)\) in Eq. (1) can be represented as a recurrent neural network with single layer and \((p+q)\) nodes as Fig. 1 shown. And the behavior of the neural network can be described by the following nonlinear discrete state space model, see Haykin [4].

\[
W_{k+1} = W_k + \omega_k \quad (2)
\]

\[
a_k = H_k W_{k-1} + e_k \quad (3)
\]

Eq. (2), known as the state (process) equation, specifies that the state of the ideal neural network is characterized as a stationary process corrupted by process (plant) noise \(\omega_k\), and the state vector \(W_k\) is given by the weight parameter values of the network and it consists of \(p\) Auto Regressive and \(q\) Moving Average coefficients according to Eq. (1), i.e.

\[
W_k=[\phi_{1,k}, \ldots, \phi_{p,k}, -\theta_{1,k}, \ldots, -\theta_{q,k}]^T \quad (4)
\]

Eq. (3), known as the observation (measurement) equation, and the desired response \(a_k\) of the network is represented as the nonlinear function of the input \(H_k\), and \(H_k\) is also called history vector.

\[
H_k=[a_{k-1}, \ldots, a_{k-p}, \ e_k, \ldots, e_{k-q}]^T \quad (5)
\]

And besides, the following assumptions are made (Conte 1990):

\[
E[\omega_k]=0, \quad E[\omega_i, \omega_j]=Q_k \delta_{ij} \quad (6)
\]

\[
E[e_k]=0, \quad \text{Var}[e_k]=R_k \quad (7)
\]

\[
\text{Cov}[\omega_k, e_k]=0 \quad (8)
\]

where the covariance matrix \(Q_k\), compared with model parameters \(\sigma_k\) is a small quantity, and commonly is defined to a constant, (e.g. \(Q_k=ul, 10^{-6} \leq u \leq 10^{-2}\)).
Neural network training

A Kalman filter can be used to train the state (i.e., weights) of the state space model. When Kalman filter is applied to state estimation, following linear combination of estimation of $W_k$ at time $k\Delta t$ and the measurement $a_{k+1}$ at time $(k+1)\Delta t$ can be used to estimate the $W_{k+1}$, see Xie [5].

$$\hat{W}_{k+1} = A\hat{W}_k + Ba_{k+1}$$  \hspace{2cm} (9)

where the notation $\hat{W}_{k+1}$ means the estimation of $W_{k+1}$, and the optimal estimation of $W_{k+1}$ means the linear items $A$ and $B$ yield the minimum covariance matrix, $\Delta_{k+1}$

$$\Delta_{k+1} = E\left\{\left[ W_{k+1} - (A\hat{W}_k + Ba_{k+1})\right]\left[ W_{k+1} - (A\hat{W}_k + Ba_{k+1})\right]^T \right\}$$ \hspace{2cm} (10)

i.e., for any other linear items $\tilde{A}$, $\tilde{B}$ and the covariance matrix $D$ they yield, the matrix $D-\Delta_{k+1}$ is nonnegative definite.

According to Eq. (2), Eq. (3) and Eq. (10), the optimal estimation of $W_{k+1}$ can be derived

$$\hat{W}_{k+1} = A\hat{W}_k + Ba_{k+1} = \hat{W}_k + K_{k+1}\left( a_{k+1} - H_{k+1}^T \hat{W}_k \right)$$ \hspace{2cm} (11)

$$A = I - K_{k+1}H_{k+1}^T$$ \hspace{2cm} (12)

$$B = \hat{P}_{k+1}H_{k+1}^T\left( H_{k+1}^T \hat{P}_{k+1}H_{k+1} + R_{k+1} \right)^{-1} \equiv K_{k+1}$$ \hspace{2cm} (13)

$$P_{k+1} = E\left[ (W_{k+1} - \hat{W}_{k+1})(W_{k+1} - \hat{W}_{k+1})^T \right]$$ \hspace{2cm} (14)

$$\hat{P}_{k+1} = P_k + Q_k$$ \hspace{2cm} (15)

Thus the algorithm of Kalman filter can be represented as a set of recursive equations as follows
\[
\hat{P}_{k+1} = P_k + Q_k
\]
\[
K_{k+1} = \hat{P}_{k+1} H_{k+1}^T (H_{k+1} \hat{P}_{k+1} H_{k+1} + R_{k+1})^{-1}
\]
\[
\hat{W}_{k+1} = \hat{W}_k + K_{k+1} (a_{k+1} - H_{k+1}^T \hat{W}_k)
\]
\[
P_{k+1} = \left( I - K_{k+1} H_{k+1}^T \right) \hat{P}_{k+1}
\]
where \( K_{k+1} \) is called Kalman gain, the quantity \( e_{k+1} = a_{k+1} - H_{k+1}^T \hat{W}_k \) in Eq. (18) is called forecast error (innovation) and the quantity \( \hat{e}_{k+1} = a_{k+1} - H_{k+1}^T \hat{W}_k \) is called residual.

**Estimation of instantaneous spectrum**

Theoretically, the instantaneous spectrum of a time varying ARMA model is just determined by its parameters, i.e. the state vector \( W_k \) in Eq. (2). The instantaneous-spectrum of a time varying ARMA\((p,q)\) model is defined as follows:

\[
p(f,k) = 2\sigma^2 \frac{|\Theta(e^{-2\pi f \Delta t})|^2 }{ \Phi(e^{-2\pi f \Delta t})^2} \Delta t = 2\sigma^2 \frac{|1 - \theta_{1,k} e^{-2\pi f \Delta t} - \cdots - \theta_{q,k} e^{-2\pi f \Delta t}|^2 }{ |1 - \phi_{1,k} e^{-2\pi f \Delta t} - \cdots - \phi_{p,k} e^{-2\pi f \Delta t}|^2} \Delta t
\]
where \( f \) belongs to interval \([0, f_{Nyq}]\), \( f_{Nyq} \) is the Nyquist frequency defined as \( f_{Nyq} = 1/2\Delta t \).

**ESTIMATION RESULTS**

Taking the actual earthquake record El Centro (1940, N-S) as an example, the time varying ARMA(2,1) model is applied to estimate the instantaneous spectrum. In order to check the estimation method as described in this paper, the Wigner-Ville distribution of El Centro (1940, N-S component) is used to represent its energy joint distribution in time and frequency domain. For this purpose, a complex valued series associated to the real valued series \( \{a_k\} \), i.e. the earthquake ground motions, see Auger [2].

\[
x_k = a_k + jHT(a_k)
\]
where \( HT(a_k) \) is the Hilbert Transform of \( a_k \) and \( x_k \) is called the analytical signal associated to \( a_k \), and the instantaneous frequency of \( x_k \) is defined as

\[
f(t) = \frac{1}{2\pi} \frac{d \arg x_k}{dt}
\]
The Wigner-Ville distribution of an analytical signal \( x_k \) is given by

\[
W_x(t,f) = \int_{-\infty}^{\infty} x(t + \tau / 2) x^*(t - \tau / 2) e^{-j2\pi f \tau} d\tau
\]
and the instantaneous frequency of \( x_k \) can also be recovered from the Wigner-Ville distribution and its first order moment in frequency,

\[
f_x(t) = \frac{\int_{-\infty}^{\infty} f W_x(t,f) df}{\int_{-\infty}^{\infty} W_x(t,f) df}
\]
The instantaneous frequency of ARMA\((p,q)\) model is determined by the Auto Regressive parameters, let \( \{\lambda_i\}_{i=1,2,\ldots,p} \) be the roots of the polynomial composed by the \( p \) Auto Regressive parameters, and the \( p \) instantaneous frequencies \( f_i \) is given by

\[
f_i = \frac{\log(\lambda_i)}{2\pi \Delta t} \quad i = 1,2,\cdots,p
\]
Usually, instantaneous frequencies $f_i$ of stationary ARMA model are conjugated complex numbers. Fig. 2 shows the instantaneous spectrum of El Centro (1940, N-S), which clearly presents the distribution and changes of energy in time and frequency domains. The energy is mainly concentrated in the first 10 seconds of frequency range 3-4 Hz with several local changes at $t=2.5, 11, 14$ and 30 seconds. Fig. 3 is the Wigner-Ville distribution of El Centro (1940, N-S), from which it be seen that the energy almost distributes overall upper triangular area of the time and frequency surface and there is a distinct interference term and the tendencies both in time and frequency domains are ambiguous.
Comparison between Fig. 2 and Fig. 3 shows that energy distribution in Fig. 2 is localized better both in time and frequency domain than that in Fig. 3 and the energy distribution is overestimated in higher frequency range on account of the interference items.

Fig. 4 shows the instantaneous frequency of El Centro (1940, N-S) derived from time varying ARMA(2,1) model according to Eq. (25), from which we can see it is chiefly contained in the range 3-4.5Hz. In the first 2 seconds it acutely increases from 3.5 to 4.8 Hz, and then keeps to 4 Hz with little fluctuation until 8.5 seconds. There is a sharp local change in the portion from 8.5 to 14 seconds and then it drops slightly from 4.5 to 3Hz. The tendency of instantaneous frequency in Fig. 4 coincides with that in Fig. 2, but Fig. 4 is more accurate and sensitive. Fig. 5 and Fig. 6 is the instantaneous frequency El Centro (1940, N-S) derived from Wigner-Ville distribution and the analytical signal, from them that can be seen they are both irregular and the instantaneous frequency changes sharply with time and it is very difficult to interpret the varying laws from them visually. They need to be reformulated if the varying laws to be used for practical purpose.

Fig. 4 Instantaneous frequency of El Centro (1940, N-S) derived from ARMA model

Fig. 5 Instantaneous frequency of El Centro (1940, N-S) derived from Wigner-Ville distribution

Fig. 6 Instantaneous frequency of El Centro (1940, N-S) derived directly from analytical signal
CONCLUSIONS

In this paper, based on the discrete a method for instantaneous spectrum estimation of earthquake ground motions is set forth by recasting the ARMA model into a single layer recurrent neural network trained by Kalman filter. As an example, the instantaneous spectrum of El Centro (1940, N-S) is estimated and it has been shown, by comparing the estimation results with the Wigner-Ville distribution, that the method is quite suitable for estimating the instantaneous spectrum of earthquake ground motions.

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