RESPONSE OF CONCRETE LIQUID CONTAINING STRUCTURES IN DIFFERENT SEISMIC ZONES

J. Z. CHEN¹ AND M. R. KIANOUSH²

SUMMARY

Liquid storage tanks as part of environmental engineering facilities are primarily used for water and sewage treatment plants and other industrial wastes. Normally, they are constructed of reinforced concrete in the form of rectangular or circular configurations. The behaviour of liquid storage tanks during earthquakes is more important than the economic values of the tanks and their contents. A good understanding of the seismic behaviour of these structures is necessary in order to meet safety objectives while containing construction and maintenance costs.

In this paper, a new procedure to determine hydrodynamic pressures for rectangular tanks is discussed. The method considers the effect of wall flexibility on impulsive pressures. The behaviour of three types of open top tanks is studied under seismic ground motions. These tanks are classified as shallow, medium and tall tanks. Three suites of time history representing low, moderate and high earthquake zones are used for dynamic time history analysis. It is concluded that a lumped mass approach can not realistically represent the true behaviour of concrete liquid storage tanks.

INTRODUCTION

Concrete liquid containing structures as part of environmental engineering structures are considered as essential facilities during earthquakes. While the leakage of tanks containing hazardous materials is essential to be controlled in water tanks, the contents are important for firefighting operations as well as for meeting the public demands. To date, intensive research has been conducted on the dynamic response of liquid storage tanks. However, most of them are related to steel tanks. Little attention has been drawn to reinforced concrete tanks. Reinforced concrete tanks are widely used in environmental engineering applications in the form of rectangular or circular configuration. It is necessary to have a good understanding of the seismic behaviour of these structures to meet safety objectives while containing construction and maintenance costs.

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Loading conditions of liquid storage tanks subjected to earthquakes are very complex. Beside the inertial force due to the weight of the tank walls, the hydrodynamic pressures are also applied on the tank walls. As the nonlinear hydrodynamic pressure loads are strongly dependent on the input of ground motion, in this study, rectangular reinforced concrete tanks are subjected to seismic ground motions of different intensities. The results of this study will provide useful information on the response of concrete tanks subjected to seismic ground motions.

The effect of hydrodynamic pressure to tank walls has been investigated by many researchers. Housner [1] proposed an approximate method including the effect of hydrodynamic pressure for a two fold-symmetric-fluid container subjected to horizontal acceleration. The fluid response was represented by impulsive and convective components. The fluid was assumed to be incompressible and the container was assumed to have rigid walls. Veletsos [2], and Yang and Veletsos [3] considered the effect of the wall flexibility on the magnitude and distribution of the hydrodynamic pressures and associated tank forces. They assumed that the tank-fluid system behaved like a single degree of freedom system and the base shear and moment were evaluated for several prescribed modes of vibration. It was found that for tanks with realistic flexibility, the impulsive forces are considerably higher than those in rigid wall. For rectangular tanks, Haroun [4] gave a very detailed analysis method in the typical system of loadings. The hydrodynamic pressures were calculated by classical potential flow approach. The formula of hydrodynamic pressures only considered the rigid wall condition. Park et al. [5] and Kim et al. [6] carried out analytical studies on the dynamic behaviour of rectangular tanks. Dynamic response of fluid storage tanks subjected to earthquakes was obtained using time history analysis.

**HYDRODYNAMIC PRESSURE**

Fig. 1(a) shows a 3-D rectangular tank. It is assumed that the liquid storage tank is fixed to the rigid foundation and a Cartesian coordinate system (x, y, z) is used with the origin located at the center of the tank base. Furthermore, it is assumed that the width to the length ratio of the tank is so large that the unit width of tank can represent the tank and the corresponding 2-D model as shown in Fig. 1(b).

The fluid filled in the rectangular tank is of height, \( H_l \) above the base. The fluid is considered to be ideal, which is incompressible, invicid, and with a mass density \( \rho_l \). The response of the body of fluid to an earthquake can be treated as gravity waves on its free surface, which is irrotational in most instances.

The governing equation of motion in matrix form can be expressed by:

\[
[M] \{\ddot{u}_r\} + [C] \{\dot{u}_r\} + [K] \{u_r\} = -[M] \{\ddot{u}_g\} + \{P\} \tag{1}
\]

Where:
- \( \{u_r\}, \{\dot{u}_r\}, \{\ddot{u}_r\} \): Displacement, velocity and acceleration of rectangular wall relative to the ground motion
- \( \{\ddot{u}_g\} \): Horizontal ground acceleration in x direction
- \( \{P\} \): Hydrodynamic pressures on the wall surface
- \( [K] \): Stiffness matrix of rectangular tank wall
- \( [M_w]\) : Mass matrix of rectangular tank wall
- \( [C] \): Damping matrix of rectangular tank wall
The solution of velocity potential, which satisfies the boundary conditions, can be solved by the method of separation of variables introduced by Currie [7]. The Hydrodynamic pressure distribution on the flexible wall related to the velocity potential can be expressed by:

\[ p = \sum_{i=1}^{\infty} \frac{2 \cdot \rho_i \cdot \tanh(\lambda_i \cdot L_x)}{\lambda_i \cdot H_l} \cos(\lambda_i \cdot y) \int_0^H \cos(\lambda_i \cdot y) \cdot \ddot{u}(t) dy \]  

(2)

Where \( \lambda_i = (2i-1)\pi/2H_l \). The detailed derivation of the above equation is discussed by Chen [8]. As the series in Eq.2 convergence very fast, only the first term of the series may be used in practical application.

For the rigid tank \( \ddot{u}(t) = \ddot{u}_g(t) \) which means that the acceleration along the height of the wall is same as the acceleration of ground motion, then Eq.2 becomes:

\[ p = \sum_{i=1}^{\infty} \frac{2 \cdot (-1)^i \cdot \rho_i}{\lambda_i^2 \cdot H_l} \tanh(\lambda_i \cdot L_x) \cdot \cos(\lambda_i \cdot y) \cdot \ddot{u}_g(t) \]  

(3)

This result is the same as the hydrodynamic pressure equation for the rigid wall derived by Haroun [4].

**THE SEQUENTIAL METHOD**

In order to consider effects of flexibility of the tank wall on hydrodynamic pressures in dynamic analysis, a sequential method is used in this investigation. The sequential method is a technique in which the two fields of fluid and structure are coupled by applying results from the first analysis as loads or boundary conditions for the second analysis. Basically the dynamic response of liquid storage tank must be solved...
by “strong” coupled method, which is that the data must be transferred or shared between at each step of the solution to maintain accuracy of dynamic response analysis. From Eq.1, the hydrodynamic pressure can be treated as the external forces applied on the rectangular tank wall, and the boundary conditions of rectangular tank wall determines the hydrodynamic pressure in Eq.2. Actually these two equations must be solved simultaneously because the interaction between the rectangular tank wall and the hydrodynamic pressure occurs at the same time. Since it is difficult to solve the dynamic response of wall and hydrodynamic pressure directly from Eq.1, the sequential method can be applied to approximate it.

The sequential method is carried out by the following procedure. First the dynamic response of the flexible tank wall subjected to an earthquake is analyzed at time step t. Then the hydrodynamic pressure is determined, which also includes the effect of flexibility of the tank wall. Finally the hydrodynamic pressure is applied on the tank wall at the next time step t+Δt. The procedure is then repeated at each time step until the analysis is complete. Fig. 2 shows in a flowchart format the procedure for analysis and Fig. 3 shows how the data is transferred between rectangular tank wall and fluid.

Further details of the analytical approach are discussed by Chen and Kianoush [8].

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**Fig. 2 Procedure for Sequential Analysis**
Selection of tank parameters

Three different tanks are studied based on a two-dimensional model as described earlier in this paper. The one-meter strip of tank walls is discretized into a number of 2-D plane rectangular elements in the horizontal and vertical directions. The height of tank wall, $H_w$, the height of liquid filled inside the tank, $H_l$, and the corresponding thickness of tank wall, $t_w$, for the three different tanks are listed below:

(a) Tank 1 (shallow tank): $H_w = 3.0$ m, $H_l = 2.7$ m, $t_w = 0.3$ m
(b) Tank 2 (medium tank): $H_w = 6.0$ m, $H_l = 5.5$ m, $t_w = 0.6$ m
(c) Tank 3 (tall tank): $H_w = 9.0$ m, $H_l = 8.1$ m, $t_w = 0.9$ m

Other dimensions and properties of the tanks are as follows:

$L_x = 15$ m, $\rho_w = 2300$ kg/m$^3$, $\rho_l = 1000$ kg/m$^3$, $E = 26440$ MPa, $\nu = 0.17$

For the liquid storage tanks, three different tank models are considered for dynamic time-history analysis as shown in Table 1. In Model 1, the impulsive mass of liquid is determined using the procedure described by Housner [1]. Both the impulsive mass and the inertial mass of wall are lumped at an equivalent height $h$, determined by:

$$h = \frac{M_i \cdot h_i + M_w \cdot h_w}{h_i + h_w}$$  \hspace{1cm} (4)

Where:

$h_i$ = height from the base of the wall to the center of gravity of the impulsive lateral force

$h_w$ = height from the base of the wall to the center of gravity of the tank wall

$M_i$ = equivalent mass of impulsive component of stored liquid

$M_w$ = equivalent mass of wall

The period of vibration of the tank wall in Model 1 is determined using the classical approach for a cantilever wall. This represents a typical model for tank wall used in most of the current codes and standards for concrete liquid containing structures.
### Table 1. Tank Models for Dynamic Analysis

<table>
<thead>
<tr>
<th>Model</th>
<th>Conditions of Analysis</th>
<th>Schematic of Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Housner)</td>
<td><strong>Fluid</strong>&lt;br&gt;Impulsive Component: Lumped&lt;br&gt;Boundary Condition: Rigid&lt;br&gt;<strong>Wall</strong>&lt;br&gt;Inertial Mass: Lumped&lt;br&gt;Wall Type: Flexible&lt;br&gt;<strong>Method</strong>&lt;br&gt;Mode Superposition Method</td>
<td><img src="image1" alt="Schematic 1" /></td>
</tr>
<tr>
<td>2 (Proposed)</td>
<td><strong>Fluid</strong>&lt;br&gt;Impulsive Component: Distributed&lt;br&gt;Boundary Condition: Rigid&lt;br&gt;<strong>Wall</strong>&lt;br&gt;Inertial Mass: Distributed&lt;br&gt;Wall Type: Flexible&lt;br&gt;<strong>Method</strong>&lt;br&gt;Mode Superposition Method</td>
<td><img src="image2" alt="Schematic 2" /></td>
</tr>
<tr>
<td>3 (Proposed)</td>
<td><strong>Fluid</strong>&lt;br&gt;Impulsive Component: Distributed&lt;br&gt;Boundary Condition: Flexible&lt;br&gt;<strong>Wall</strong>&lt;br&gt;Inertial Mass: Distributed&lt;br&gt;Wall Type: Flexible&lt;br&gt;<strong>Method</strong>&lt;br&gt;Direct Step-by-Step Integration Method and Sequential Method</td>
<td><img src="image3" alt="Schematic 3" /></td>
</tr>
</tbody>
</table>

In Model 2, both the impulsive mass of the liquid and the mass of the wall are distributed over the height of the wall. In this case, the impulsive mass is determined assuming a rigid wall condition. In Model 3, the hydrodynamic impulsive pressure is determined considering the wall flexibility. This model is expected to provide the most accurate results among the three different models. In Models 1 and 2, the mode superposition method is used for dynamic analysis while Model 3 is analyzed using the direct step-by-step method including the sequential procedure as described earlier.

Damping ratio for all models is assumed to be 5% of critical. The Rayleigh damping \[ c = \alpha [M] + \beta [K] \] is used in the direct step-by-step integration method. The values \( \alpha \) and \( \beta \) are determined using the procedure as described by Bathe et al. [9].

Table 2 shows the dynamic properties of Models 1 and 2. The lumped mass for Model 1 includes the impulsive mass \( M_i \) and the inertial mass of wall \( M_w \). The height of lumped impulsive mass from the tank base and the equivalent height of the total lumped mass for the three tanks are also demonstrated. The
natural periods of vibration for the two models are also shown. The fundamental periods for Model 2 are about 50% greater than that for Model 1.

Table 2. Tank Properties

<table>
<thead>
<tr>
<th>Tank Model</th>
<th>Tank Properties</th>
<th>Tank 1</th>
<th>Tank 2</th>
<th>Tank 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hw = 3m Hl = 2.7m</td>
<td>M, +Mw (10^3 kg)</td>
<td>6.28</td>
<td>25.74</td>
<td>56.39</td>
</tr>
<tr>
<td></td>
<td>hi (m)</td>
<td>1.01</td>
<td>2.06</td>
<td>3.04</td>
</tr>
<tr>
<td></td>
<td>h (m)</td>
<td>1.2</td>
<td>2.4</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>T, (sec)</td>
<td>0.05</td>
<td>0.11</td>
<td>0.15</td>
</tr>
<tr>
<td>Model 2</td>
<td>T, (sec)</td>
<td>0.07</td>
<td>0.15</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>T, (sec)</td>
<td>0.014</td>
<td>0.028</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Selection of earthquake records for dynamic analysis

The time history of ground motions adopted in this study are based on the SAC project [10]. Three suites of time history in locations of Boston, Seattle and Los Angeles corresponding to seismic zone 2, 3 and 4 respectively for the 2% probability of exceedance in 50 years are used for dynamic analysis. Detailed information of these earthquake records is listed in Table 3. An integration time step of 0.005 sec is used to guarantee the accuracy for sequential analysis. The proportion in peak ground acceleration (PGA) with respect to the different seismic zones is also considered.

Table 3. Detailed Information of Three Suites of Time History Ground Motions

<table>
<thead>
<tr>
<th>SAC Name</th>
<th>Record</th>
<th>Earthquake Magnitude</th>
<th>Distance (km)</th>
<th>Scale Factor</th>
<th>Duration (sec)</th>
<th>PGA (cm/sec^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BO32</td>
<td>Nahanni, 1985 Station 3</td>
<td>6.9</td>
<td>18</td>
<td>2.63</td>
<td>19.015</td>
<td>381.09</td>
</tr>
<tr>
<td>SE23</td>
<td>1992 Erzincan</td>
<td>6.7</td>
<td>2</td>
<td>1.27</td>
<td>20.775</td>
<td>593.60</td>
</tr>
<tr>
<td>LA25</td>
<td>1994 Northridge</td>
<td>6.7</td>
<td>7.5</td>
<td>1.29</td>
<td>14.945</td>
<td>851.62</td>
</tr>
</tbody>
</table>

Results of analysis and discussions

A summary of the analytical results for the three models as described above is listed in Table 4. This is in terms of the maximum base shear, base moment and top displacement during the time history analysis.

The direct step-by-step integration method together with the proposed sequential analysis is applied to Model 3. In this case, the effect of flexibility of tank wall on hydrodynamic pressure is considered. In this model, it is observed that the base shear may be larger or smaller than that of Model 2 calculated using the rigid wall boundary condition subjected to the same ground motion. Further study shows that in Model 3, the hydrodynamic pressure is still amplified as a result of the flexibility of tank wall at each time step.
under the specific ground acceleration. But the maximum base shear during the entire time history may be less than that calculated in Model 2. The reason is that as both ground acceleration and hydrodynamic pressure load are applied as external forces on the wall, the combination of these two loads may reduce the total effect during the time history analysis. Consequently, the maximum base shear may be reduced. This means that the hydrodynamic pressure load is highly dependent on the input of ground acceleration record.

Table 4. Summary of Analytical Results

<table>
<thead>
<tr>
<th>Zone</th>
<th>Response</th>
<th>Tank 1 (H_w = 3m)</th>
<th>Tank 2 (H_w = 6m)</th>
<th>Tank 3 (H_w = 9m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Model 1</td>
<td>Model 2</td>
<td>Model 3</td>
</tr>
<tr>
<td>2</td>
<td>Top Displacement (mm)</td>
<td>2.2</td>
<td>2.1</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>Base Shear (kN)</td>
<td>70.5</td>
<td>27.8</td>
<td>23.4</td>
</tr>
<tr>
<td></td>
<td>Base Moment (kNm)</td>
<td>86.3</td>
<td>55.4</td>
<td>49.7</td>
</tr>
<tr>
<td></td>
<td>Equivalent Height (m)</td>
<td>1.2</td>
<td>2.0</td>
<td>2.1</td>
</tr>
<tr>
<td>3</td>
<td>Top Displacement (mm)</td>
<td>1.2</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Base Shear (kN)</td>
<td>37.7</td>
<td>21.4</td>
<td>35.8</td>
</tr>
<tr>
<td></td>
<td>Base Moment (kNm)</td>
<td>46.1</td>
<td>42.6</td>
<td>58.6</td>
</tr>
<tr>
<td></td>
<td>Equivalent Height (m)</td>
<td>1.2</td>
<td>2.0</td>
<td>1.6</td>
</tr>
<tr>
<td>4</td>
<td>Top Displacement (mm)</td>
<td>2.3</td>
<td>2.9</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Base Shear (kN)</td>
<td>74.1</td>
<td>39.0</td>
<td>51.3</td>
</tr>
<tr>
<td></td>
<td>Base Moment (kNm)</td>
<td>90.7</td>
<td>77.6</td>
<td>95.4</td>
</tr>
<tr>
<td></td>
<td>Equivalent Height (m)</td>
<td>1.2</td>
<td>2.0</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Figures 4, 5 and 6 show the variation of results between the three models for the three seismic zones. As shown in Figure 4, for Model 1, the results show that the base shear is much higher than those observed from the other two models. This is an indication that the procedure adopted in the codes and standards are too conservative in estimating the base shear.

The variation of base moment for different tank heights for the three models is compared as shown in Figure 5. A similar trend to that as for base shear is noticed. As shown in Figure 6, the top displacement for Model 2 is higher than the other two models except for the tall tank in zone 2 in which the displacements for Model 1 exceed that of the other two models.

Figures 7, 8 and 9 show the variation of base shear, base moment and displacement for the three models for different seismic zones. For Model 1, there is a noticeable difference in response between the three zones. However, for Models 2 and 3, the response of zone 3 becomes very close to that of zone 4.

In order to improve the understanding of the basic dynamic properties of each model, the ratio of base moment to base shear or equivalent height is studied. For Model 1, they are 1.2m for H_w = 3m, 2.4m for H_w = 6m and 3.4m for H_w = 9m. For the case of more accurate Model 2, they are 2.0m for H_w = 3m, 3.8m for H_w = 6m and 5.6m for H_w = 9m. The discrepancy between the two models is an indication that Eq.4 which is based on a single degree of freedom system (SDOFS) and specified in the current design...
standards and codes cannot reflect the dynamic response of structures appropriately. In Model 3, the values of equivalent height are between the two limits of those of Models 1 and 2, but vary in the three tanks for the different suites of time history. They are approximately 1.8 m for $H_w = 3m$, 3.1 m for $H_w = 6m$ and 4.5 m for $H_w = 9m$.

Another argument between the three models is related to the added mass in Models 1 and 2 and applied external force in Model 3, where the hydrodynamic load is applied on the tank wall. As Model 1 is a SDOFS, the fundamental natural period is the basic parameter for the dynamic response of the tank walls. The hydrodynamic pressure load is approximate by the lumped mass which becomes part of the properties of dynamic system. This situation is also true for Model 2. But as Model 2 is a multi degree of freedom system, additional dynamic modes contribute to the dynamic response of the tank. In these two models, the hydrodynamic pressures represented by the added mass have no relationship with the input ground motions. However, in Model 3, the ground motion can affect the hydrodynamic pressures applied on the tank walls. In this case, the magnitude of hydrodynamic pressures may be different for different earthquake records. As such, the hydrodynamic pressures are not related to the basic dynamic properties of the tank system. The hydrodynamic pressures are affected by the input of ground motion as well as other parameters such as stiffness and mass of the tank wall. For this reason, the hydrodynamic pressures are treated as external loads in dynamic analysis. The major discrepancy in the results between Models 2 and 3 are due to this effect.

CONCLUSIONS

In this paper, the dynamic response of liquid storage tanks in three different seismic zones is investigated. Three different tank models are subjected to three suites of time history ground motions located in different seismic zones. Based on the proposed model, using the sequential analysis, the hydrodynamic pressures are no longer required to be approximated by added mass but they can be treated as external forces. The advantage of the proposed model is that it can consider the effect of the flexibility of the tank wall on the magnitude of hydrodynamic loads.

Based on the study presented in this paper, it is concluded that the dynamic response of liquid storage tanks determined based on the current design approach in terms of base shear is too conservative. This is due to the inaccuracy in the determination of equivalent height for the impulsive mass of the liquid. It is shown that a single lumped mass can not accurately represent the behavior of liquid storage tanks. It is also concluded that the hydrodynamic load is highly dependent on the input of ground motion. It is recommended that the hydrodynamic load be treated as external force in the dynamic analysis of liquid storage tanks.
Fig. 4 Effect of Model Type on Base Shear

Fig. 5 Effect of Model Type on Base Moment
Fig. 6 Effect of Model Type on Displacement

(a) Zone 2

(b) Zone 3

(c) Zone 4

Fig. 7 Seismic Zone Effect on Base Shear

(a) Model 1

(b) Model 2

(c) Model 3

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**Legend**
- Model 1
- Model 2
- Model 3

- Zone 2
- Zone 3
- Zone 4
Fig. 8 Seismic Zone Effect on Base Moment
Fig. 9 Seismic Zone Effect on Displacement
REFERENCES


http://nisee.berkeley.edu/data/strong_motion/sacsteel/draftreport.html