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## **COMBINED DECISION-MAKING FOR THE INITIAL SEISMIC CAPACITY OF A BUILDING AND ITS STRENGTHENING TACTICS**

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### **SUMMARY**

In this paper, the synthetical decision-making methodology of seismic design levels and retrofit strategies of structures proposed by the authors in Reference [1](Yan[1]), is applied to calculate and analyze the seismic design level and retrofit strategies of multistory dwelling brick buildings and single story mill buildings in the zone of intensity 8. Some valuable results and conclusions are derived, and beneficial to engineering application.

### **INTRODUCTION**

Due to natural or man-made damage, and the aging effects of materials, the resistance abilities of buildings become more and more inefficient with time. Damage and collapse of buildings built according to a special seismic design level will occur under earthquake excitations, which match with the prescribed design level, during their service period. The situation doesn't accord with forehand expectations, and thus not only waste of the investment is inevitable, but loss induced by earthquakes is relatively more serious. Hence, how we can deal with the union of the seismic design level and the retrofit strategies is the focus of study in this paper.

We have done some research works on the synthetical decision-making methodology of the seismic design level and retrofit strategies of buildings in literature [1](Yan[1]), [2](Liu[2]) and [7]~[13](Yan[7],Min[12]...). In literature [1] we used the optimal control theory and put forward the dynamic decision-making methodology. This paper will briefly introduce and apply this methodology to calculate and analyze a multi-story brick dwellings and a single story mill building in a design zone of intensity 8. Some beneficial results are derived, with the temporal non-stationary feature of seismicity, different indoor property (including the generalized value of lives and economic activity), variability of annual retrofit ability and the corresponding capital needed, being considered.

### **A BRIEF INTRODUCTION OF ANALYSIS METHODOLOGY**

The synthetical decision-making methodology of seismic design levels and retrofit strategies can be described by using the following mathematic model: find  $R(0)$  and  $U(t)$  to make the performance index defined in Equation 1 minimum and meet the state function and maintenance degree restrictions described in Equation 2 and 3.

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$$J = C_0[R(0)] + D(T)C_r[R(T)] + \int_0^T D(t) \left\{ \int_{P_L}^{P_u} F(P, t) L[R(t), P, t] dP + M[R(t), U(t)] \right\} dt \quad (1)$$

$$R(t) = U(t) - A(t)R(t) + B(t) - \int_{P_L}^{P_u} S[R(t), P] F(P, t) dP \quad (2)$$

$$0 \leq U(t) \leq U_u(t) \quad (3)$$

In the above equations,  $R(t)$  is the structural resistance at  $t$ -th year;  $R(0)$  is the seismic design level of structure, that is the structural resistance at first year;  $U(t)$  is the maintenance degree, i.e., the enhance degree of structural resistance at  $t$ -th year through seismic retrofit;  $C_0[R(0)]$  is the structural cost;  $T$  is the service period of construction;  $D(t)$  is the discount coefficient that reflects time value of capital;  $P_u$ , and  $P_L$  are the upper and lower intensity limits of earthquakes possibly suffered in  $T$ , respectively;  $F(P, t)$  is the probability density function with the seismic intensity  $P$  in  $t$ -th year;  $L[R(t), P, t]$  is the economic loss while structure with resistance  $R(t)$  suffers an earthquake of seismic intensity  $P$ ;  $M[R(t), U(t)]$  is the seismic reinforcement cost function;  $A(t)$  and  $B(t)$  are the parameters to reflect the variability of structural resistance in the case of no seismic retrofit and no occurrence of earthquake;  $S[R(t), P]$  shows the degraded level of the structural resistance with seismic intensity  $P$ ;  $U_u(t)$  shows the elevated maximum level of structural resistance through seismic retrofit, and reflects annual retrofit ability.

This is an optimal control problem with variable initial and final states, and its analytical solution is difficult to be obtained. Thus, the assumptions that tally roughly with actual situations of engineering structures are made as the followings:

a) the loss function  $L[R(t), P, t]$  can be expressed as resistance  $R$  quadratic decayed function;

b)  $M[R(t), U(t)] = b_0 U(t) + b_{uR} R(t) U(t)$  (Both  $b_0$  and  $b_{uR}$  are constant.);

c)  $F(P, t) = F_p(P) [a_f \exp(-b_f t) - c_f \exp(-d_f t)]$  (The meaning of  $F_p(P)$  is the annual seism occurrence probability density function.  $a_f$ ,  $b_f$  and  $c_f$  can be obtained according to seismic activity time unhomogeneity distribution. While seismic activity distribution is homogeneous,  $a_f = 1$ ,  $b_f = c_f = d_f = 0$ .);

d) the structural resistance decreases exponentially with time  $t$  if it was not strengthened or retrofitted;

e) the declining degree of structural resistance under earthquake of intensity  $P$  is not related to structural resistance, i.e.,  $S[R(t), P] = S(P)$ ;

f)  $U_u(t) = U_u = \text{constant}$ .

Under the above assumptions, the following analytical solution can be obtained

$$R(0) = C_1 \quad (4)$$

$$U(t) = \begin{cases} 0 & t \notin (t_{j1}, t_{j2}) \\ U_u & t \in (t_{j1}, t_{j2}) \end{cases} \quad (5)$$

$(j=1, 2, \dots, N)$

where  $N$  is the number of retrofit during the service period of the structure;  $(t_{j1}, t_{j2})$  is the time interval of the  $j$ -th retrofit;  $t_{j1}$ ,  $t_{j2}$  ( $j=1, 2, \dots, N$ ) and  $C_1$  can be obtained through calculating a non-linear algebraic equation.

**Table 1. Multistory Dwelling Brick Buildings' Predictable Average Damage Index**

Intensity Resistance	Intensity 6	Intensity 7	Intensity 8	Intensity 9	Intensity 10
<b>Intensity 6</b>	0.020	0.243	0.433	0.593	0.848
<b>Intensity 7</b>	0.010	0.200	0.390	0.580	0.760
<b>Intensity 8</b>	0.000	0.100	0.260	0.410	0.570

**Table 2. Relationship between Damage Index and Seismic Lost**

Degree of damage	Damage index	Retrofit cost	Average retrofit cost	Property loss	Average property loss
Basically undamaged	0.00	0~1	0.5	0.00	0.0
Lightly damaged	0.20	0~1	5.0	0~1	0.5
Medium damaged	0.40	10~30	20.0	0~10	5.0
Heavy damaged	0.60	30~100	65.0	10~30	20.0
Fractional damaged	0.80	50~100	75.0	30~50	40.0
Entirely damaged	1.00	100	100	50~100	75.0

**Table 3. Predicting of Multistory Dwelling Brick Buildings' Average Seismic Damage Loss in Beijing-tianjin-tangshan Area**

Seismic intensity		Intensity 6	Intensity 7	Intensity 8	Intensity 9	Intensity 10	
Resistance	Intensity 6	Estate loss	0.95	8.225	27.425	63.425	81.00
		Property loss	0.05	1.40	8.00	19.25	47.00
Intensity 7	Estate loss	0.725	5.00	17.25	42.25	70.25	
	Property loss	0.025	0.525	4.325	12.50	80.50	
Intensity 8	Estate loss	0.5	2.75	9.5	25.0625	58.25	
	Property loss	0.009	0.25	1.825	7.25	17.75	

**Table 4. Relationship Between Seismic Intensity and Resistance Declining Quantity of Multistory Dwelling Brick Buildings**

Intensity	Intensity 6	Intensity 7	Intensity 8	Intensity 9	Intensity 10
Resistance					
Intensity 6	0.06	0.729	1.299	1.779	2.544
Intensity 7	0.04	0.80	1.56	2.24	3.04
Intensity 8	0.00	0.50	1.30	2.05	2.85
Average declining quantity	0.00333	0.6763	1.3863	2.023	2.8113

### SEISMIC LEVELS AND RETRIOFT STRATEGIES OF MULTISTORY DWELLING BRICK BUILDINGS

The average seismic damage index of multistory dwelling brick buildings is showed in Table1 from Yang[3], and the relationship between varied seismic damage index and seismic loss is also notified in Table2 from Li[4]. The loss of multistory dwelling brick buildings (repairing expenses) and the relevant property loss (see Table.3 and Table.4) are obtained according to inter-interposition in Table 1 and Table 2 when they suffer earthquakes of different intensities. In this paper, seismic resistance of buildings is denoted by the seismic intensities of earthquakes that they can defend in general. The relationship  $I(R, P)$  among seismic damage index  $I$  and structural resistance  $R$ , seismic intensity  $P$  also can be obtained from Table1. It is obvious that earthquakes of making  $I(R, P) = 0$  do not work on resistance  $R$  (i.e.  $S(R, P)=0$ ) and those of making  $I(R, P) = 1$  will make resistance entirely lost. The earthquakes of making  $I(R, P)=1$  ought to make structural resistance decline to  $R - P_L$  on the assumption that seismic intensity of making  $I(R, P) = 0$  is  $P=P_L$ . Therefore the following approximate expression can be given:

$$S(R, P) = (R - P_L) I(R, P) \quad (6)$$

If an arithmetical mean of resistance  $R$  are made according to Equation 6, the average declined degree of the structural resistance  $S(P)$  under earthquakes of intensities from 6 to 10 can be gained.

Assume that a multistory dwelling brick building would be built in the region of intensity 8, and the seismic activity in this region be in an active period during the period from the 20-th year to the 40-th year in future, and the average yearly probability density function of seismic intensity  $P$  be expressed as

$$F(P, t) = 3.476 \times 10^6 (e^{-0.03305t} - e^{-0.03475t}) e^{-2.03P} \quad (7)$$

$$0 \leq t \leq 50, 5 \leq P \leq 11$$

According to Equation 7, the mean yearly probability exceeding seismic intensity 8 in the region is  $2.26 \times 10^{-3}$  and the exceedance probability in fifty years is 11.3%. The functional relationship between the multistory dwelling brick building's cost and its seism design level  $R(0) = R_0$  is supposed as the following

$$C_0(R_0) = 1.18 - 0.09R_0 + 0.01R_0^2 \quad (8)$$

$$(4.5 \leq R_0 \leq 11.0)$$

Equation 8 indicates that the construction cost is 1 unit if the seismic resistance of the building is intensity 6 and the cost enhances 4% or 10%, respectively if the seismic resistance of the building is increased to intensity 7 or 8 from intensity 6. The cost of strengthen is given by the following

$$M(R, u) = [0.012 + 0.008R(t)] U(t) \quad (9)$$

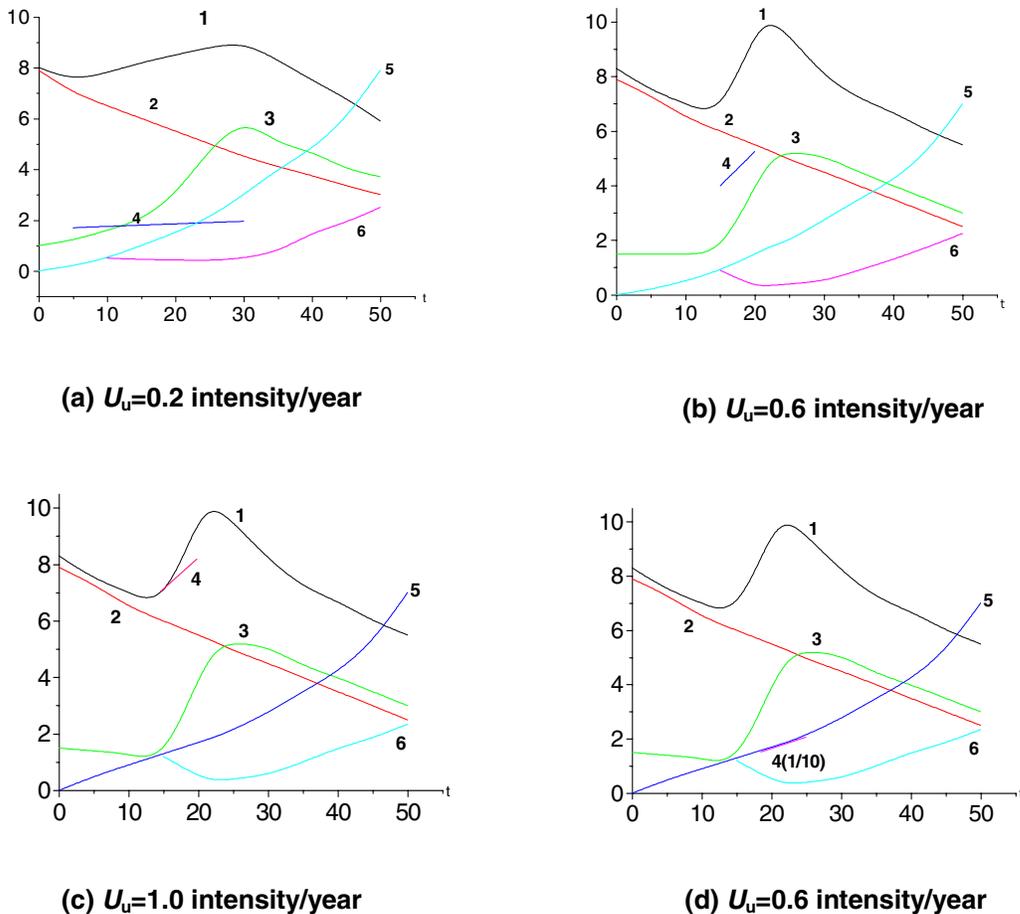
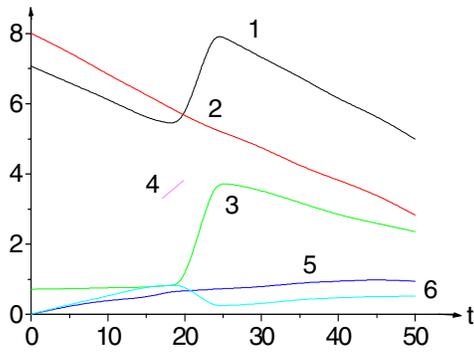
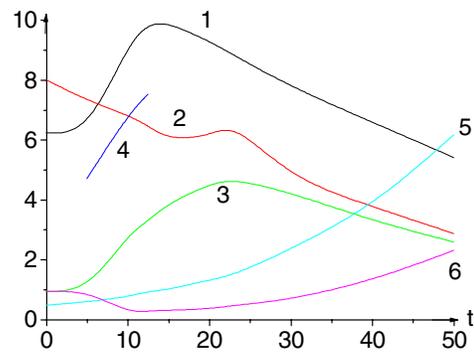


Figure 1. Optimal Resistance and Benefit of Distributing in the Case of Different Yearly Seismic Retrofit Ability ( $b_e=1, r=0$ )



**Figure 2. Optimal Resistance of Distributing ( $b_e=1, r=0.04, U_u=1.4$ )**



**Figure 3. Optimal Resistance of Distributing in the Case of Homogeneous Distribution of Seismic Activity**

It can be known from Equation 8 and 9 that the necessary cost for making structural resistance enhance a certain level through seismically retrofitting is more than the cost for enhancing the same level at constructing. For example, the necessary reinforcement cost for making the structural resistance of intensity 6 raise to intensity 7 is 1.5 times as much as the constructing cost for enhancing to the same level.

The aforementioned functions and data are substituted into a nonlinear algebraic equation, then the unknown parameters  $C_1, t_{1,j}, t_{1,j}$  ( $j=1$  to  $N$ ) are determined. With the parameters being substituted into Equation 4 and 5 the seismic design levels and retrofit strategies of multistory dwelling brick buildings are obtained. Moreover, the optimal resistance distribution, the corresponding yearly loss distribution, the reinforcement cost and minimum value of the performance index are calculated from Equation 1 and 2.

Figure 1 illustrates the optimal resistance distribution  $R(t)$  and the corresponding yearly loss distribution  $Loss_1$  (%) in the case of the deferent yearly retrofit capability (i.e. value of  $U_u$  is different). In the figure, the indoor property of building (including vital value of populace, etc.) is supposed as the same as the constructing cost of the building with seismic resistance of intensity 6 (i.e. indoor property coefficient  $b_e=1.0$ ), and increases by 4% per year; and no time value of capital is considered. In addition,  $R_n(t)$  in the figure is the resistance distribution of the building constructed with the resistance of intensity 8 and without seismic retrofit during its service period;  $Loss_0$  (%) is the corresponding yearly loss distribution; Cost (%) is the yearly seismic reinforcement cost distribution; and Ratio =  $Loss_0 / Loss_1$ , explains the benefit distribution.

**Table 5. Optimal Retrofit Time Interval and Benefit ( $r=0.0, U_u=1.6$ )**

$b_e$	$t_1$	$t_2$	$r_1$	$r_2$
0.1	14.02	15.75	1.903	1.582
0.3	14.85	16.79	2.238	1.913
0.8	16.62	18.83	3.080	2.793
1.0	17.25	19.50	3.387	3.163

The optimal resistance distribution with the consideration of time cost of capital (discount coefficient  $D(t) = e^{-rt}$ , discount rate  $r=0.04$ ) is showed in Figure 2, and the optimal resistance distribution without consideration of time cost of capital is showed in Figure 3, in which seismic activity distribution is homogeneous with time.

**Table 6. Optimal Retrofit Time Interval and Benefit in the Case of Homogeneous Seismic Activity Distribution**

$b_e$	$t_1$	$t_2$	$r_1$	$r_2$
0.1	7.90	19.54	1.849	0.2
0.3	7.77	21.83	2.192	0.2
0.8	4.53	10.85	2.590	0.9
1.0	4.91	11.43	2.784	0.9

Table 5 shows the optimal retrofit time intervals ( $t_1$ ,  $t_2$ ), the ratios of the whole seismic loss without reinforcing the building with a resistance of intensity 8 at constructing to the whole loss with the above optimal resistance distribution ( $r_1$ ), and the ratios of the margin of above-mentioned pair to the whole cost of reinforcement ( $r_2$ ) in the case of different indoor property (i.e. the coefficient of indoor property  $b_e$  is different. For example  $b_e=0.4$  indicates that indoor property is 40% of the construction cost of the building with resistance of intensity 6). Table 6 presents the values of the above-mentioned parameters in the case of homogeneous seismic activity distribution.

The following conclusions can be obtained from the above tables and figures:

- the optimal seismic design level, the residual resistance of the building in the ending of its service period, and the economic benefit resulted from optimizing the seismic design level and the retrofit strategies of the building are insignificantly related to the variability of yearly seismic retrofit ability;
- the economic benefit increases with the increase of indoor property and the discount rate  $r$  has greater influence on each economic index;
- different seismic activity distributions in time have great effects to the optimal seismic design level, the optimal retrofit occasion, the optimal retrofit degree, the benefit, and the seismic loss; and if seismic activity distribution with time is homogeneous, the seismic loss may be underestimated;
- the maximum resistance of the building may not be its optimal seismic design level (i.e., the resistance at constructing of the building), which is related to the time distribution of seismic activity;
- the economic benefit produced by optimizing the seismic design level and the retrofit strategies of the building also is obvious even if the indoor property is too tiny to be considered.

### DETERMINATION OF SEISMIC DESIGN LEVELS AND RETROFIT STRATEGIES OF SINGLE STORY MILL BUILDING

The predicted seismic damage rates of single story mill building can be seen in Reference 5, and the building loss and the indoor property loss from different seismic damage levels are given in Reference 6. The decrease degree of the seismic resistance of a single story mill building because of earthquakes can be ascertained by using the aforementioned methodology in this paper.

Suppose that a single story mill building would be constructed in the region of intensity 8, and the seismic activity in this region be in an active period during the period from the 20-th year to the 40-th year in future, and its intensity distribution accord with the extremum III distribution, i.e.:

$$F(P, t) = 0.254(e^{-0.03305t} - e^{-0.03475t})(11 - P)^{3.7} \exp\left[-\left(\frac{11 - P}{4.45}\right)^{4.7}\right] \quad (10)$$

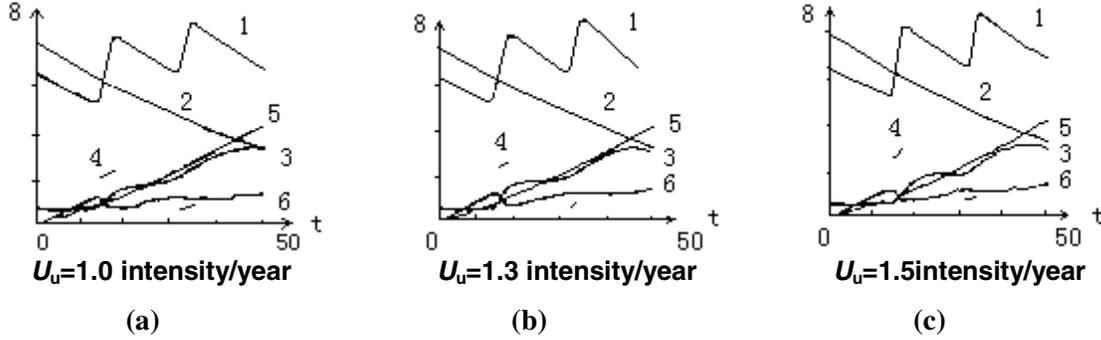
$$(0 \leq t \leq 50, P \leq 11.0)$$

Equation 10 indicates that the mean yearly probability exceeding seismic intensity 8 and the exceedance probability in 50 years in the region are  $2.6 \times 10^{-3}$  and 13%, respectively; the mean yearly probability exceeding seismic intensity 6.5 (the intensity of frequently occurred earthquakes) and the exceedance probability in fifty years are 1.26% and 63%, respectively; and the mean yearly probability exceeding seismic intensity 9 (the intensity of seldom occurred earthquakes) and the exceedance probability in fifty years are  $4.16 \times 10^{-4}$  and 2.08%, respectively.

The equation that defines the vital value of population in the mill building is (Yan[7]) :

$$V = N \cdot \frac{Z}{N} \cdot Y \cdot B \quad (11)$$

In which,  $V$  is the vital value of populace;  $N$  is the quantity of population;  $Z$  is the average length of service of population; and  $B=0.5$  is the average living coefficient of population. This equation shows that one's vital value in his whole life is half of the produced value by him in 30 years. The relationship between the cost and the seismic design level of the mill building is showed in Equation 8, and the retrofit cost is showed in Equation 9.



**Figure 4. Optimal Resistance Distribution in the Case of Different Yearly Seismic Retrofit Ability ( $b_e=1$ ,  $r=0$ )**

**Table 7. Optimal Retrofit Interval and Economic Benefit  $t$  of the Single Story Mill Building**

Seismic Activity	Discount rate $r$	The ratio of Decreased loss and retrofit cost	Optimal time interval ( $u_i=1.30$ )	
			$(t_1, t_2)$	$(t_3, t_4)$
Homogeneity	0.08	6.08	(15.86,18.20)	(35.25,34.31)
Unhomogeneity	0.08	5.69	(14.58,17.21)	(31.62,33.70)
Homogeneity	0.00	33.53	(21.15,23.45)	(37.74,39.10)
Unhomogeneity	0.00	39.11	(22.26,24.43)	(37.97,39.29)

Suppose that the ratio of the fixed equipment investment in the mill building to its constructing cost with resistance of intensity 6 is 1:2; the yearly production value of this factory is 13 percent of its whole investment and increase 10 percent rate, i.e.,  $Z=0.39e^{0.1t}$  (the constructing cost of the mill building with resistance of intensity 6 is one unit). The whole investment sum of principal and profit can be refunded in 10 years in case the yearly interest rate of debit and credit of 8%. Because the indoor equipment's value in the mill building maybe increase due to the equipment updating and replacing by new generation equipment, it is conservative to consider the value of the indoor equipment a constant. For the work in three shifts in a day is put into practice in the mill building, the vital value of population is a third part of the value sum of whole employee, i.e.,  $V=1.95e^{0.1t}$ . Therefore, the property value of the mill building can be computed according to the following equation:

$$C_r(t) = 2.34e^{0.1t} + 2.0 \quad (12)$$

Figure 4 gives the optimal resistance distribution of the mill building (the discount rate or the debit and credit interest rate is 0.08, i.e.,  $r=0.08$ ) in case of different yearly retrofit abilities. Table 7 shows the optimal retrofit interval and the corresponding benefit with and without the consideration of discount in the case of homogeneous and unhomogeneous seismic activity distributions.

The following opinions can be given from Figure 4 and Table 7:

a) The yearly retrofit ability has minor influence on the optimal seismic design level of the mill building, its residual resistance in the ending of its service period, and the economic benefit resulted from optimizing synthetically the seismic design level and the retrofit strategies;

- b) The discount rate has a important effect on the index of economic benefit;
- c) The optimal seismic design levels, the optimal retrofit degree and the corresponding occasion, and the retrofit benefit vary significantly with seismic activity distribution.;
- d) for indoor property including equipments, life value and the value created by production are far more than the cost of mill building, the economic effect of optimizing the seismic design levels and the retrofit strategies is thoroughly obvious; the optimal retrofit benefit with consideration of the discount is 6 times over that of the retrofit cost, and the benefit without consideration of the discount is 40 times over that of the retrofit cost.

## CONCLUSIONS

In this paper, the synthetical decision-making methodology for seismic design levels and retrofit strategies of a building that has been proposed by the authors in Reference 1 is applied to compute and analyze multistory dwelling brick buildings and single story mill buildings in the zone of intensity 8. There exist several assumptions in the process deriving the relationship between the construction cost of a structure and its seismic design level, the relationship among the retrofit cost, the retrofit degree and the structural resistance when it is retrofitted, and so on. Some of the assumptions may not completely tally with the actual situations of engineering structures, but the significant conclusions can be obtained qualitatively.

The optimal seismic design level, the retrofit degree and the corresponding occasion of a structure vary with seismic activity distribution. To consider seismic activity distribution to be homogeneous can likely make the earthquake damage and loss of the structure be underestimated, and the retrofit occasion of the structures be chosen more difficultly. The yearly retrofit ability has minor effects on seismic design levels and retrofit strategy of the structure. And the retrofit time interval may be shorter if the yearly retrofit ability is greater, otherwise it may be longer, which is of importance for engineering application. The union of seismic design level and retrofit strategy can bring more economic benefit, especially for structures with great indoor property.

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