UPGRADING OF RC STRUCTURES FOR A TARGET RESPONSE SHAPE

Georgia E. THERMOU1, Stavroula J. PANTAZOPOULOU2, Amr S. ELNASHAI3

SUMMARY

The objective of this paper is to investigate criteria for the formulation of upgrading strategies for seismic rehabilitation of substandard RC buildings. The stated goal of upgrading is to modify the response so as to simultaneously reduce demand in the critical regions as well as to enhance the force and deformation supply. An important tool in controlling the extent of damage through the proposed strategy is modification of vibration mode shapes to achieve a target distribution of interstorey drift. The proposed procedures are implemented within the framework of displacement-based design and assessment, using Yield Point Spectra so as to evaluate the possible alternative redesign scenarios. The performance of the proposed procedures is assessed through an extensive parametric study conducted on characteristic example RC frame structures. The significance of design decisions made in selecting the target response of the rehabilitated system is evaluated through comparisons of the estimated demands.

INTRODUCTION

Earthquakes worldwide have repeatedly demonstrated the poor level of seismic performance of old, substandard reinforced concrete construction. Although there is an ongoing convergence of requirements of major seismic design codes at a baseline level of seismic protection, older construction in has been designed to a wide range of less stringent guidelines. These characterize the respective period of construction, as codes have been evolving over several decades to reach their modern format, as well as construction practices and materials are markedly different today from the past. Deficiencies of old construction can be classed in two broad categories – those concerning systemic problems of the structure, such as eccentricities and soft-storey formations, and local inadequacies associated with poor reinforcement detailing. Response modification through seismic upgrading is mostly focused on the first class of deficiencies, which are addressed through global interventions aiming to reduce the magnitude and localization of demands. It is implicitly assumed that the necessary measures to

1PhD Candidate, Civil Eng. Dept., Demokritus University of Thrace, GR, Email: gthermou@civil.duth.gr
2Professor & Assoc. Chair, Civil Eng. Dept., Demokritus University of Thrace, GR, Email: pantaz@civil.duth.gr
3Willett Professor of Engineering, Acting-Director, Mid-America Earthquake Center, Civil and Envir. Eng. Dept., UIUC, USA, Email: aelnash@uiuc.edu
enhance deformation capacity of individual members or zones will accompany the global upgrading scheme, as such technologies and the necessary detailing rules are now becoming available (e.g. Pantazopoulou and Tastani [1]).

The decision on retrofitting to globally modify the structural behavior is neither straightforward nor obvious, for apart from the implications of a certain solution scheme on seismic demand and supply there are often economic or architectural restrictions to be considered. Development of a complete strategy guiding the retrofit solution through established objectives or criteria is an ongoing effort of the earthquake engineering research community. The aim of the retrofit strategy as a working framework is to balance supply and demand. *Supply* refers to the capacity of the structural system, which has to be assessed in detail before selecting the intervention scheme. *Demand* is expressed by either the code design spectrum or the site-specific record as a function of period and vibration characteristics of the upgraded system. By modifying strength, stiffness and/or ductility of the system alternative retrofit options are obtained, as shown in Figure 1 (Elnashai and Pinho [2]). Ductility enhancement applies to systems with poor detailing (sparse shear reinforcement, insufficient lap splicing), stiffness and strength enhancement to systems with inherently low deformation capacity (so as to reduce displacement demand), whereas strength and ductility enhancement to systems with low capacity or where seismic demand is high.

![Image](image.png)

**Figure 1: Alternative retrofit strategies**

The retrofit design framework considered in this paper is displacement-based. The Yield Point Spectrum (YPS) representation (Aschheim and Black [3]) is implemented in order to obtain the admissible design region for each of the alternative retrofit scenarios and to estimate the seismic demands of the retrofitted system. The key role that the vibration shape has in the retrofit procedure is investigated and assessed. Therefore, in the proposed strategy damage control and redistribution of deformation demand is engineered through modification of the response so as to approach a pre-selected target shape of vibration.
The selection of the target shape depends greatly on the structural system type and on the diagnosed deficiencies of the initial condition of the structure. In the last section of the paper, the efficacy of stiffness modification heightwise in order to achieve a desired approximation of a target deflection shape is explored through parametric investigations on a group of shear-type frames. Objective of this effort is to obtain automatic rules by which to gauge the alteration in demands effected by changes in the structural system through upgrading. In this regard, the influence of the stiffness distribution on the selected shape is expressed by pertinent weighting factors.

### Lateral Drift Control

#### The role of the deflection shape on the distribution and localization of damage

According to the generalized coordinate approach any structure of arbitrary form may be treated as an equivalent SDOF system under the assumption that its displacements are restricted to a single shape (Clough and Penzien [4]). The displacements can be expressed in terms of the generalized coordinate, \( Y(t) \), and the assumed shape vector, \( \Phi(x) \), and given by the product \( u(x,t) = \Phi(x) \cdot Y(t) \). The generalized coordinate expresses the displacement at the top of the building, whereas the shape vector can be an arbitrary shape or the fundamental mode shape (normalized to have unit value at the top storey).

The equation of motion of a MDOF system is given by:

\[
M \ddot{u} + C \dot{u} + K u = -M \ddot{\mathbf{1}} \cdot \ddot{u}_g
\]  
(1)

Substituting the displacement by \( u = Y \cdot \Phi \) and pre-multiplying by \( \Phi^T \) the above equation is converted to:

\[
\Phi^T \cdot M \cdot \Phi \cdot \ddot{Y} + \Phi^T \cdot C \cdot \Phi \cdot \dot{Y} + \Phi^T \cdot K \cdot \Phi \cdot Y = - \Phi^T \cdot M \cdot 1 \cdot \ddot{u}_g
\]  
(2)

which upon further transformation yields the equation of motion of the ESDOF oscillator:

\[
M^* \ddot{Y} + C^* \dot{Y} + K^* \cdot Y = -L^* \ddot{\mathbf{1}}_g
\]  
(3)

The terms \( M^*, C^*, K^* \) represent generalized mass, damping and stiffness, given by,

\[
M^* = \sum m_i \cdot \Phi^2_i
\]  
(4)

\[
K^* = \sum K_i \cdot \Delta \Phi_i^2
\]  
(5)

and the period of the generalized system:

\[
T^* = 2\pi \sqrt{\frac{M^*}{K^*}}
\]  
(6)

From the above, the influence of the vibration shape on the characteristics of the ESDOF system is evident. The contribution of the stiffness of each floor to the generalized stiffness is denoted by the term \( \Delta \Phi_i^2 \). Altering the distribution of stiffness along the height of the building modifies the response and hence the deflected shape; but the magnitude of the change effected in the characteristics of the ESDOF and on the associated Spectral Demands depend on the \( \Delta \Phi_i \) values. Hence, to actually achieve modification of the response so as to control damage through an optimized distribution of drift necessarily requires that \( \Delta \Phi \) rather than \( \Phi \) need be changed to meet a desired pattern. Note that in the general case the displacement profile of a structure under lateral load falls between the flexural-type and the shear-type bounds as illustrated in Figures 2a, b. A flexural-type profile resembles the deflected shape of a cantilever and is characteristic of structures that have a pure wall type of behavior (systems with distributed mass and elasticity). Interstorey drift is higher at the upper stories of such structures. The shear-type profile is representative of structures where mass and stiffness are concentrated in discrete locations (lumped-mass systems). Interstorey drift is higher in the ground floor and decreases in upper stories. This type of behavior is particularly prevalent in older residential multistorey structures that frequently had strong and stiff beams; in S. Europe these also often came with an open first floor (known as pilotis, used for parking, shops, and other communal facilities Figure 2c). In this case beyond yielding deformation will tend to localize in the “weak link” of the structure and the displaced lateral load profile at failure will have the form (Kotsoglou et al. [5]):
\[ u = u_y + u_p = \Phi_y Y + (1,1,\ldots,1,0,\ldots,0)Y_u - Y \]

where \((1,1,\ldots,1,0,\ldots,0)^T\) corresponds to the i-th level. Hence, at system failure the vibration shape is,

\[ \Phi_u = \frac{1}{\mu} \Phi_y + \left( \frac{\mu - 1}{\mu} \right) (1,1,\ldots,1,0,\ldots,0)^T \]

In equation (8), \(\Phi_y\) is the fundamental mode shape (calculated from elastic response assumptions), and \(\mu\) is the apparent ductility of the system (i.e. ratio of ultimate and yield displacements at the top storey, \(\frac{\gamma_u}{\gamma_y}\)). The i-th level is the floor or level where deformations have localized thereby effectively behaving as a soft-storey.

Equation (8) indicates that the inelastic component of the interstorey displacement at the critical floor level, \(\Delta \Phi_{\text{cr},p}\), equals \((\mu - 1)/\mu\), i.e. 0.50, 0.67, 0.75 for \(\mu = 2, 3\) and 4 respectively. This means that 50\%, 67\% and 75\% respectively of the total inelastic displacement experienced by the system develops in the critical floor where localization takes place. Evidently to obtain a successful retrofit solution, storey stiffness should be reconsidered to alter the localization pattern.

At this juncture it is in the interest of time and effort to conduct an a-priori assessment of the relative influence on the global scale that would be achieved through an intervention in the stiffness of the j-th floor. Global stiffness of the ESDOF is the work-equivalent term, \(K^*\) (Eqn. 5). To modify the response as per the target deflection shape, the stiffness distribution along the height of the building needs to be multiplied by a vector, \(w\), which is unique for the structure and the selected target shape. Hence, the stiffness of each storey is given by the product of the total stiffness (direct sum of the floor stiffnesses) and the weighting factor \(K_{\text{storey},i} = K_{\text{tot}} w_i\), where \(K_{\text{tot}} = K^*/(\sum w_i \Delta \Phi_{i}^2)\). In case that the contributing influence of the stiffness of each storey to the stiffness of the oscillator need be quantified then \(K^*_{\text{storey},i} = K_{\text{tot}} w_i \Delta \Phi_{i}^2\).

Figure 2: Lateral displacement profiles (a) flexural-type, (b) shear-type, (c) soft-storey, (d) drift components

**Drift as a damage index**

Because lateral drift is the most representative index in quantifying damage at the structural level it is related directly to the performance objectives of the rehabilitation procedure. A range of values for the drift limits at target performance levels have been proposed by SEAOC, Vision 2000 [6] and FEMA 273 [7] in the order of 2\% and 4\% for life safety and collapse prevention states of concrete frames.

The change in the vibration shape may be dictated by the retrofit strategy selected for seismic upgrading of the structure. As the vibration shape changes and shifts from the shear to the flexural profile, the influence of the tangential component of drift increases. At this point it is important to stress the difference between interstorey drift and tangential drift (Figure 2d). In shear buildings the tangential drift is negligible and all the storey displacement is attributable to interstorey drift. On the other hand in flexural-type buildings the storey displacement comprises two components, namely the interstorey drift and the tangential interstorey drift. The later is responsible mainly for structural and non-structural damage. Naturally, this information need be reflected in the retrofit process – thus, the target shape used in upgrading aims to a better distribution of tangential interstorey drift, so as to achieve a uniform distribution of damage. To realize
this objective, selective stiffness increases are required; this is reflected in the respective weight factors that quantify the work-equivalent effects of storey-stiffness increase on the level of seismic protection supplied by the upgraded structure. To illustrate this point a parametric investigation on the relationship between weight factors and resulting modified response is presented in the last section of this paper.

SELECTION OF THE RETROFIT DESIGN SCENARIO

Retrofit with use of the YPS

The Yield Point Spectrum (YPS) format was proposed by Aschheim and Black [3] as an alternative to the Capacity Spectrum method. Yield Point Spectra are inelastic constant ductility ADRS spectra ($\mu = \Delta_u/\Delta_y$) with the abscissa corresponding to the yield displacement, $\Delta_y$, the ordinate to the yield strength coefficient, $C_y$ and the radial lines to the period, $T$. Yield Point Spectra can be generated from either a code-based format or a site specific record. This is a graphical procedure; to construct the YPS from the elastic spectra, the hysteretic response is required, relating the force reduction factors, $R$, with the ductility as a function of period ($R-\mu-T$ relationships). The accuracy of the computation of the constant ductility spectra can be improved by selecting pertinent $R-\mu-T$ relationships to specifically address the behavior of retrofitted structures.

In Figure 3, the YPS for the El Centro ground motion (1940, Imperial Valley) is depicted for ductility levels $\mu=1$, 2, 4 and 8, viscous damping 5% of critical and an elastoplastic bilinear force-deformation pushover curve of an ESDOF system representing the initial (prior to upgrading) condition of a MDOF structural system. Given the yield displacement and the period of the ESDOF system the yield strength coefficient and the peak displacement may be estimated directly from the YPS. (For example, for a system with secant-to-yield period equal to $T=1.0$ sec and a yield displacement of $\Delta'_y=25.6$ mm, the radial line of 1.0 sec intersects the ductility $\mu=4$ curve. The peak displacement is estimated by the equal displacement rule as equal to $\Delta'_u=102.4$ sec and the yield strength coefficient as $C_y=0.103$.) The fact that the method
gives a graphical solution renders it extremely efficient because apart from the simplicity, the designer has
the direct control of the changes in stiffness, strength and ductility. Thus, the YPS can be used as a tool in
a retrofit strategy that aims to control the deformation-demand of the upgraded system. In the retrofit
strategy the yield displacement may be limited by the designer according to the performance objectives the
rehabilitation procedure complies with. The applicable retrofit design scenario can be selected from a
wide range of combinations between strength, stiffness and ductility.

**Criteria for the selection of the retrofit strategy**

The retrofit strategy should deal with the deficiencies of the structural system accordingly modifying the
properties that mostly control the target objectives. In some cases local interventions aiming to increase
ductility suffice, but in most cases a coordinated use of global methods is required along with local
interventions. To identify the need for global considerations or local interventions or a combination
thereof it is necessary to use pertinent diagnostic tools in assessing the global adequacy of the structural
morphology at least for the pre-yield stage of response. To date the most familiar tool used to diagnose
lack of stiffness is the cracked-stiffness estimate of the fundamental period of vibration of the structure.
Departure of its magnitude from the established empirical upper bounds indicates the need for global
stiffening (e.g.: $T<0.1N_s$ for R.C. frames, and $0.05N_s<T<0.075N_s$ for R.C. frame-walls, where $N_s$
the number of storeys.) Another familiar index is the minimum floor area ratios of structural or masonry
walls; note that such ratios are an indirect control over the structural period, since they quantify the lateral
stiffness of the system. Yet, the signature of structural behavior is contained in its fundamental
translational mode-shape that may reveal the existence of soft-storeys. For linear elastic behavior the
fundamental mode shape may be a sufficient test, because any discontinuities in stiffness or mass will be
captured by relative normalized drift ratios that significantly exceed the mean value of $1/N_s$.

Depending on the outcome of the retrofitting criteria various options may be selected. The graphical
solution of each case is presented in the following paragraphs. For all three groups some additional
limitations may be imposed and refer to the ductility level and the lateral drift of the building. In particular
for the cases presented below, a limit of 4 has been set for the ductility whereas given the limiting drift at
the life safety performance objective (2% of the building height), the target shape of response (triangular)
and the height of the building (four-storey: 12m), the maximum top displacement of the elastic ESDOF
oscillator has been limited to $\Delta_y=240/1.33=180$ mm. The participation factor for the triangular shape of
response is equal to $\Gamma=L/M^*=1.33$. The shaded areas in Figures 4, 5, and 6 satisfy those limitations. The
YPS used is for the El Centro ground motion (1940, Imperial Valley).

**Scenario 1: Stiffness, Strength and Ductility modification**

The simultaneous modification of stiffness distribution (altering the vibration shape), strength and
ductility defines a design region as that drawn in Figure 4. Any point located in the region encircled by the
red line comprises an acceptable combination. Let’s assume that the blue dot shown in Figure 4 represents
the characteristics of a four-storey building with uniform mass and stiffness distribution along the height.
The period of the ESDOF oscillator is equal to $T^*=0.5$ sec and the participation factor $\Gamma=1.24$. For the
elastic response ($\mu=1$) the yield displacement of the oscillator is $\Delta_y=57.12$ mm. The top yield displacement
of the original building is $\Delta_{y, MDOF}=57.12\cdot 1.24=70.83$ mm. In case that the selected retrofit scenario imposes
a uniform distribution of the interstorey drift (triangular target shape of vibration) for ductility level of 2,
then the green dot depicted in Figure 4 represents the characteristics of the retrofitted four-storey building.
Due to the change in the vibration shape the participation factor, $\Gamma$, increases by 7.2%. The period and the
yield displacement of the oscillator for ductility level 2 are $T^*=0.35$ sec and $\Delta_y=11.55$ mm, respectively.
The top yield displacement of the building is $\Delta_{y, MDOF}=11.55\cdot 1.33=15.36$ mm.
**Scenario 2: Stiffness and Strength modification**

In case that the ductility level remains unchanged in the rehabilitation procedure, then various combinations of strength and stiffness are acceptable for different yield displacement limits. The strength-stiffness pairs lie on the red lines drawn in Figure 5, with each line representing a different ductility level. The green dot depicted in Figure 5 is the result of a retrofit scenario applied to the building used in the previous section when the response remains elastic and the triangular target shape of vibration is selected. The resulting period and yield displacement for the aforementioned oscillator are $T^* = 0.35$ sec and $\Delta'_y = 22.92$ mm, respectively. The top yield displacement of the building is $\Delta'_y_{\text{MDOF}} = 22.92 \times 1.33 = 30.48$ mm. The stiffness modification for a constant ductility level and a selected target shape of vibration results in shifting the response of the ESDOF oscillator to the left.

**Scenario 3: Strength and Ductility modification**

The retrofit scenario may aim to strengthen and enhance the ductility level of the structural system, while keeping the stiffness unchanged. In this case, there is only one pair of strength and yield displacement for each ductility level. Each radial line drawn in Figure 6 represents a path along which the stiffness is constant and strength and deformation at yield may change according with the retrofit design requirements.
Figure 5: Stiffness and strength modification
IV40ELCN180, 5% Damping, 0% Post yield stiffness

Figure 6: Strength and ductility modification
IV40ELCN180, 5% Damping, 0% Post yield stiffness
PARAMETRIC INVESTIGATION ON THE MAGNITUDE OF WEIGHTING FACTORS FOR RESPONSE MODIFICATION

Parameters of the investigation - Results
To demonstrate the different scales of influence that may be affected globally by different choices in the stiffness distribution of the upgraded system, a parametric investigation was conducted for low-, mid- and high-rise shear buildings. According to their mass and stiffness distribution, the buildings may be classified into three groups. The first group (UKM) comprises structures with uniform mass and stiffness distribution throughout the height. The second group is representative of structures where the upper storey is a setback. Hence, the upper storey characteristics (mass and stiffness) are reduced by a percentage compared to those of the remaining stories. This reduction refers to 20%, 50% and 80% for the UKMR80, UKMR50 and UKMR20 cases, respectively. The third group refers to structures that present a gradual reduction of mass and stiffness along their height. Two cases were studied; in the first the reduction reaches 50% of the ground floor characteristics (NUKMG50), while in the second 80% of the ground floor characteristics (NUKMG20).

This parametric study aims to unfold the influence of the stiffness of each floor to the displaced shape of the structure. The target shape chosen was the triangular profile, which assigns uniform distribution of interstorey drift along the height of the building. A basic assumption made is that mass remains unchanged. The procedure followed made use of the Improved Rayleigh method. The underlying concept in the Rayleigh method is conservation of energy throughout the process of its conversion from strain energy to kinetic and vice-versa, which occurs during free vibration. The results of this type of analysis are directly related to the selection of the vibration shape; the true vibration shape is obtained by iteration whereas the work-equivalent stiffness comprises contributions of the various floors multiplied by differing weight factors. These factors emerge from the analysis as $w_i$, as presented in Table 1. The examples that follow demonstrate the usage of the results of the parametric investigation.

First example
In the first example, the case of a four-storey structure with the following mass and stiffness distribution is studied:

\[ K = K_{\text{storey}} \cdot \{1, 1, 1, 1\}^T; \quad K_{\text{storey}} = K_{\text{tot}}/4 = 57500 \text{ kN/m} \]  
\[ M = m_{\text{storey}} \cdot \{1, 1, 1, 1\}^T; \quad m_{\text{storey}} = 44.6 \text{ kN} \cdot \text{sec}^2/\text{m} \]

Using the Improved Rayleigh method (iterative procedure), the dynamic characteristics of the building are estimated: $T^* = 0.50$ sec; $\Phi = \{1.00, 0.879, 0.653, 0.347\}^T$.

In case that the retrofit design aims to control the vibration shape by converting it to a triangular profile the stiffness of each floor is modified, so that the new stiffness distribution may result as the product of the weighting factor of each floor (Table 1) and the total stiffness of the intact building. The resulting post-retrofit distribution complies with the demands for a triangular displaced shape.

\[ K_{\text{new,storey}} = K_{\text{storey}} \cdot \{1.000, 1.700, 2.252, 2.452\}^T; \quad K_{\text{storey}} = 57500 \text{ kN/m} \]  
$T^* = 0.35$ sec; $\Phi = \{1.000, 0.750, 0.500, 0.250\}^T$

The change in the vibration shape induces a shift in the period of the building and the response becomes stiffer. Using the YPS for the El Centro ground motion (1940, Imperial Valley) for a ductility level 2, the yield strength coefficient and the yield displacement of the ESDOF system before and after the stiffness modification are $C_y = 0.34$, $\Delta_y = 21.20$ mm and $C'_y = 0.38$, $\Delta'_y = 11.55$ mm, respectively. Note the familiar tradeoff, where lowering the yield displacement demand results in increased strength demand.

Second example
The second example refers to a seven-storey structure that belongs to the third group of structures NUKMG20. The mass and stiffness distribution is:

\[ K = K_{\text{storey}} \cdot \{1.000, 1.667, 2.333, 3.000, 3.667, 4.333, 5.000\}^T; \quad K_{\text{storey}} = K_{\text{tot}}/21 = 13000 \text{ kN/m} \]  
\[ M = m_{\text{storey}} \cdot \{1.000, 1.667, 2.333, 3.000, 3.667, 4.333, 5.000\}^T; \quad m_{\text{storey}} = 18 \text{ kN} \cdot \text{sec}^2/\text{m} \]
The dynamic characteristics of the building are estimated with the use of the Rayleigh iterative analysis: 
\[ T^* = 0.85 \text{ sec}; \Phi^T = (1.000, 0.924, 0.808, 0.664, 0.501, 0.330, 0.160)^T \]
The building deforms according to the triangular profile in case that stiffness is distributed according to the corresponding weighting factors of Table 1. The weighting factors multiplied by the total stiffness of the intact structure yield the following stiffness profile.
\[ k_{\text{new}} = k_{\text{storey}} \cdot (1.000, 2.462, 4.128, 5.862, 7.446, 8.667, 9.385)^T; \ k_{\text{new,storey}} = w_i \cdot k_{\text{tot}} \]
\[ T^* = 0.76 \text{ sec}; \Phi^T = (1.000, 0.857, 0.714, 0.571, 0.429, 0.286, 0.143)^T \]

Using the YPS for the El Centro ground motion (1940, Imperial Valley) and a ductility level of 2 the yield strength coefficient and the yield displacement of the ESDOF system before and after the stiffness modification are \( C_y = 0.28, \Delta_y = 49.35 \text{ mm} \) and \( C'_y = 0.25, \Delta'_y = 36.38 \text{ mm} \), respectively. The stiffness modification stiffens the structure and both yield strength coefficient and yield displacement decrease. This observation is not compatible with the observation made in the preceding example. This inconsistency is attributed to the fact that YPS are sensitive to the ground motion characteristics and there is variability in response, a point that underlines the need for using smoothened design spectra in upgrading.

**Third example**
The weighting factors can be alternatively used in the case that the retrofit scenario imposes a limitation on the period of the ESDOF system, \( T^* \). Using equation (6) the stiffness of the ESDOF, \( K^* \), can be estimated. The contribution of each floor to the stiffness of the ESDOF system is estimated by:
\[ k_{\text{storey,i}} = k_{\text{tot}} \cdot w_i \cdot \Delta \Phi_i \]
where \( k_{\text{tot}} = K^*/(\sum w_i \cdot \Delta \Phi_i^2) \). The stiffness of each storey of the MDOF system is given by the \( k_{\text{storey,i}} = k_{\text{tot}} \cdot w_i \).

Let us examine the case of a five-storey structure of the NUKMG50 group. The mass of the ESDOF system for a triangular shape is \( M^t = 67.5 \text{ kN} \cdot \text{sec}^2/\text{m} \). If the retrofit strategy imposes the period of the ESDOF system to be limited to \( T^* = 0.45 \text{ sec} \), then the stiffness of the ESDOF system is equal to \( K^* = 13159 \text{ kN/m} \). The contribution of the stiffness of each floor to that of the ESDOF is:
\[ K^* = K^*_{\text{storey}} \cdot [1.000, 1.940, 2.932, 3.511, 4.008]^T; \ K^*_{\text{storey}} = 982.71 \text{ kN/m} \]

The distribution of the stiffness of the MDOF system is given by:
\[ K = K_{\text{storey}} \cdot [1.000, 1.940, 2.932, 3.511, 4.008]^T; \ K_{\text{storey}} = 25084 \text{ kN/m}, \ K_{\text{storey}} = K^*_{\text{storey}} / \Delta \Phi_i^2 \]
Evidently, the ratio is constant and in order to obtain the values of the stiffness term of the MDOF system a division by the term \( \Delta \Phi_i^2 \) is necessary, where \( \Delta \Phi_i = 0.20 \) (for a triangular distribution and a five-storey building).
Table 1: Weighting factors for fundamental mode shape

<table>
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<tr>
<th>Building Type</th>
<th>Floor</th>
<th>Weighting factor, $w_i$</th>
<th>UKM</th>
<th>NUKMR20</th>
<th>NUKMR50</th>
<th>NUKMR80</th>
<th>NUKMG20</th>
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UKM: Uniform distribution of mass and stiffness throughout the height of the building.
NUKMR20-50-80: Mass and stiffness reduction of the top floor by 80%, 50% and 20% respectively.
NUKMG20-50: Gradual distribution of mass and stiffness throughout the height of the building ending at the top floor to 20% and 50% of the mass and stiffness of the first floor.
CONCLUSIONS

In the above, criteria for the formulation of upgrading strategies for seismic rehabilitation of substandard construction are investigated. The modification of the predominant shape of vibration aims to achieve a target distribution of interstorey drift, thus controlling the extent of damage. The framework of displacement-based design and assessment is employed and use is made of the Yield Point Spectra to establish the possible alternative redesign scenarios. An extensive parametric study on representative example frame structures is conducted in order to assess the performance of the proposed framework. The derived weighting factors may be used to successfully control the response modes of structures, thus optimizing their seismic behavior.

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REFERENCES