



## **PROBABILISTIC RESPONSE SIMULATION AND REGIONAL VULNERABILITY DETERMINATION USING RESPONSE SURFACE METAMODELS**

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### **SUMMARY**

This research presents an alternative means of calculating vulnerability relations for buildings. Simulation-based fragility curves generated using a Monte Carlo simulation are usually not practical because of the large number of time-consuming analyses needed to obtain reliable statistics of the outcomes. This research implements a response surface metamodel to approximate complex analysis in a function-like manner. Monte Carlo simulation can then be efficiently performed using the metamodel to calculate fragility. The methodology is implemented in the rapid seismic fragility assessment of a hypothesized building portfolio. Resulting vulnerability relations for each building in the portfolio are presented.

### **INTRODUCTION**

Seismic fragility or vulnerability is defined as a conditional probability of exceeding certain limit states assuming a specific level of seismic intensity. Integration of fragilities over different levels of seismic intensities yields a fragility (or vulnerability) curve, which depicts the likelihood of damage to a building due to increasing levels of seismic hazard.

Seismic loss estimation for a portfolio of buildings requires fragility relations for individual buildings in the portfolio in order to assess probable damage due to seismic hazards. Conventional methods (NIBS [1]) use generic fragility curves to quantify damage to buildings of a similar type. However, buildings in the same classification may have significantly different vulnerability relations due to variability in geometric layout or other building aspects such as age, applicable design code, etc. This could lead to an inaccurate estimation of building portfolio losses. As a result, portfolio-specific vulnerability relations are needed for

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better estimation of the loss. While building-specific fragility curves can be determined from brute-force simulations based on complex structural analyses on each building, for portfolio assessment, it becomes impractical to produce a large number of seismic response analyses.

An alternative approach for carrying out rapid probabilistic response simulation for buildings in a portfolio of interest is described. A response surface metamodel is used to predict the seismic structural response of a building characterized by carefully selected macro-level parameters (e.g., variability in building geometry, age, etc) and micro-level parameters (e.g., material properties) in a simple functional form. Such metamodels are not appropriate for individual buildings because they cannot accurately represent discrete events such as member yielding or different limit states that will develop. However, when applied to a portfolio of buildings, they can provide a statistical average behavior. Use of metamodels makes it practical to employ Monte Carlo methods to carry out probabilistic response computations. The computational cost required in a Monte Carlo simulation is significantly reduced since the simulation is performed on a simple polynomial response surface function, rather than an explicit complex dynamic analysis.

## **RESPONSE SURFACE METAMODELS FOR VULNERABILITY ASSESSMENT**

Response Surface Methodology (RSM) originated in a study by Box [2] to determine the optimal conditions in a chemical investigation. Since then, RSM has been successfully applied in many different fields of study such as chemical engineering, industrial engineering, manufacturing, aerospace engineering, structural reliability, and computer simulation.

RSM begins with a simple screening process to identify those input variables that have the greatest effect on the output. Design of Experiments (DOE) techniques (Montgomery [3]) are then employed to efficiently construct samples of the input variables that can be used to define a set of high fidelity analyses (typically these are detailed computer analyses). Finally, regression analysis is used to create a polynomial approximation of the expensive, high fidelity computer analyses. Quadratic polynomials provide a reasonable balance between flexibility and numerical complexity so that a computed response function can be expressed in the following form.

$$\hat{y}(X) = b_0 + \sum_{i=1}^k b_i X_i + \sum_{i=1}^k b_{ii} X_i^2 + \sum_{i=1}^{k-1} \sum_{j>i}^k b_{ij} X_i X_j$$

where

$\hat{y}$	=	Response or dependent variable
$X_i, X_j$	=	The $i^{\text{th}}$ input or independent variables
$b_0, b_i, b_{ii}, b_{ij}$	=	Coefficients to be estimated
$k$	=	Number of input variables

The use of response surface methodology in connection with Monte Carlo simulation simplifies the process of generating fragility curves. A response surface function is sought to approximate an implicit building seismic response computation using an explicit polynomial function. The process for calculating seismic fragility based on the use of response surface metamodels is described as follows.

As shown in Figure 1 below, the first step is to define the input and output (or response) variables for the response surface. An appropriate building response or damage measure such as a peak inter-story drift is

defined as an output variable. Building parameters characterizing response calculation are used as input variables, and the range of each input variable applicable to the region of interest is defined based on the inventory data. When a large (more than a 5-10) number of input variables are identified, a screening process is generally used to determine the subset of variables that have the largest influence on the output (response). A DOE technique is utilized for selecting an efficient set of input variable combinations (experimental sampling). Finally, a seismic intensity measure is defined, and an ensemble of ground motion records are scaled such that they have the same level of intensity.

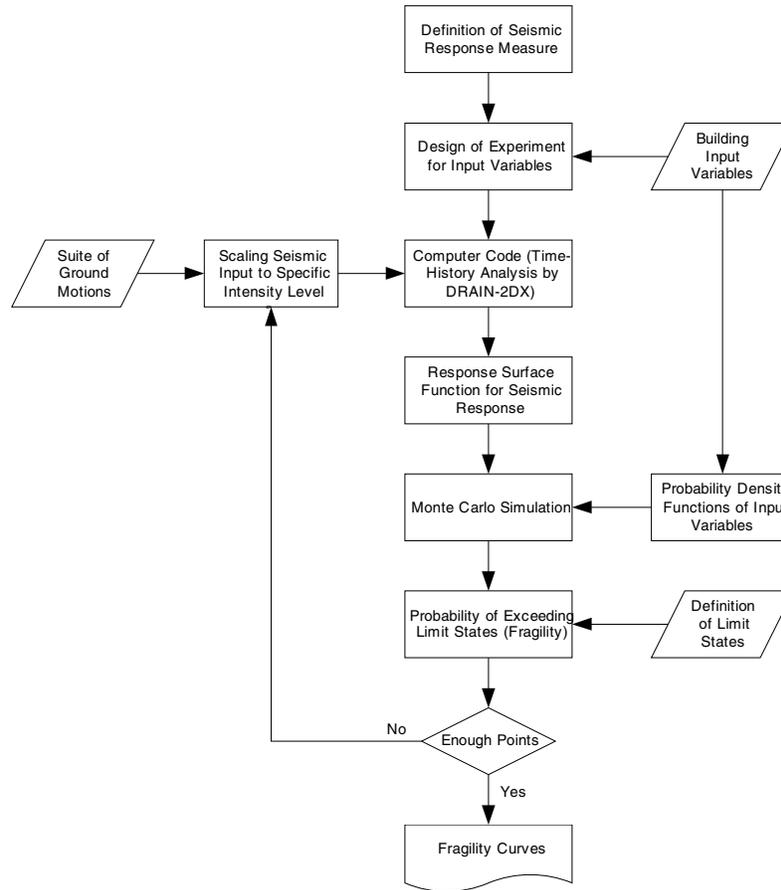


Figure 1: Process for Generating Fragility Curves using RSM

Next, detailed computational analysis is performed on a building model constructed to represent one combination of input variables defined by the DOE step. Each of the scaled acceleration records is used as input for this analysis, and the chosen seismic response (e.g., interstory drift, peak drift, peak acceleration, etc) is extracted from each analysis run. The process is repeated for each combination of input variables defined in the DOE step. Least-square regression analysis is then performed over the sampled input data points and corresponding outputs to form a polynomial response surface function that defines the RSM metamodel.

The main advantage of the response surface metamodel is that while it provides an admittedly simple functional relation between the most significant input variables and the output (response), the model is computationally very efficient. In particular, this computational efficiency makes the metamodel particularly attractive for use with Monte Carlo techniques in which a large number of simulations using the RSM metamodel can be carried out using assumed probability density functions for the input

variables. In particular, if the statistical characteristics of an ensemble of buildings are used to construct input distribution functions for a Monte Carlo simulation, the probability of the chosen response exceeding certain damage limit states can be extracted from the simulation outcomes. This probability value is conditioned on a specific earthquake intensity level and represents one point in a fragility curve. Repetition of the process over different levels of earthquake intensity provides exceedance probability values at other intensity levels, and the fragility curves can be constructed for the building ensemble.

## RAPID VULNERABILITY ASSESSMENT OF A BUILDING PORTFOLIO

Seismic loss estimation is of interest to governmental agencies or building owners who are responsible for developing and applying effective mitigation measures for their building portfolios. Seismic vulnerability assessment based on each specific building is essential to the process, and while the basic metamodel development described in the previous section is designed for evaluation of a specific building ensemble, it can also be applied to a smaller portfolio of individual buildings. The use of response surface metamodels provides an efficient means for rapidly deriving vulnerability relations specific to each building in the target portfolio. Parameters are defined such that geometrical and structural (macro) variability (including vertical irregularities) and randomness in material (micro) properties for buildings in a region can be described. Response surface metamodels must be generated as described in the previous section using building parameters from building inventory data for the larger region where the building portfolio of interest is located. As a result, the developed metamodels are applicable to a specific geographical region. Monte Carlo simulation is then performed on the metamodel to generate building specific fragility curves.

### A Hypothetical Building Portfolio

As an example application, a loss assessment of a hypothetical building portfolio consisting of 4 buildings located in a specific geographical region is considered. It is assumed in this study that all buildings in the portfolio are low-rise steel moment resisting frame buildings (the S1-L classification as defined by NIBS [1]). Assumed characteristics of each building are shown in Table 1.

Table 1: Characteristics of Buildings in a Hypothetical Portfolio

Building Name	Class	Year Built	# of Floors	Typical Floor Height	1 <sup>st</sup> -Floor Height	Bay Width
AAA	S1-L	1968	3	9.0 ft.	11.2 ft.	30.0 ft
BBB	S1-L	1993	3	10.0 ft.	16.8 ft.	30.0 ft
CCC	S1-L	2002	3	10.5 ft.	14.0 ft	36.0 ft.
DDD	S1-L	1967	3	10.0 ft.	14.0 ft.	24.0 ft.

After screening studies, output and input variables are defined for the response surface metamodel. Peak inter-story drift representing a global damage measure is chosen as an output variable for the response surface. Measures of damage states for steel moment frame buildings are also defined by FEMA [4] in terms of peak drift experienced during an earthquake excitation. Building input parameters are selected in such a way that each one of them strongly affects response calculations and, at the same time, is largely independent of the others. These building parameters (Table 2) can be divided into 2 groups: (1) control variables characterizing individual building (X1 to X7) and (2) random variables describing uncertainties in material properties (X8 to X11). Information on bounds of these the building parameters can be obtained from actual regional field survey, experimental results, and possibly design documents from the building owners. However, resources are not typically available to develop the necessary statistical survey of the structural characteristics for the target region. As a result, in this study, variability of building

characteristics is assumed such that it represents a wide variety of S1-L buildings in the region. Geometrically, this parameter set can represent low-rise steel moment frame buildings ranging from 96 to 144 feet in length and 29 to 39 feet in height. However, these parameters can only describe the 2-dimensional aspect of a building. The 3-dimensional effect of a building to seismic response calculation is presented in Dueñas-Osorio [5].

Table 2: Building Input Parameters and Bounds

	<b>Building Parameter</b>	<b>Lower</b>	<b>Median</b>	<b>Upper</b>
X1	Seismic design provision	None	Low	High
X2	First floor height (ft)	11.2	14.0	16.8
X3	Typical floor height (ft)	9.0	10.0	11.0
X4	Bay width (ft)	24.0	30.0	36.0
X5	Second floor mass (kips-sec <sup>2</sup> /ft)	26.2	32.8	39.3
X6	Third floor mass (kips-sec <sup>2</sup> /ft)	26.2	32.8	39.3
X7	Roof mass (kips-sec <sup>2</sup> /ft)	28.4	35.4	42.5
X8	Column yield strength (ksi)	44.0	55.0	66.0
X9	Beam yield strength (ksi)	44.0	55.0	66.0
X10	Column modulus (ksi)	27550	29000	30450
X11	Beam modulus (ksi)	27550	29000	30450

One disadvantage of a response surface model is the limitation on the number of independent input variables. In general, for standard DOE designs, 8 input parameters or fewer are considered practical for a response surface metamodel. The building parameters to be included in the metamodels should be those that have greatest impact on the output or peak inter-story drift calculation, but the selection is not always obvious for complex problems. As noted earlier, a screening process can be employed to identify the contribution of each parameter to the response of the system. One of the simplest methods is to systematically make reasonable increments in each input variable and compute the response for each case. A rank-ordered output yields what is often called a Pareto optimal solution.

Parameter screening tests for the assumed region show that five parameters contribute to about 80% of the response calculation. The five parameters, in the order of significance, are X1, X4, X8, X2, and X10. These parameters are used as basic input parameters for a response surface. It can be observed that this set of input parameters contains both control (X1, X2, and X4) and random variables (X8 and X10).

In order to incorporate an uncertainty in seismic loading, a total of 20 synthetic ground accelerations (Wen [6]) are used in the nonlinear time-history analyses of each building model. This suite of ground accelerations represents a variety of seismic magnitudes, sources, and distances representative of assumed earthquakes. Each of these ground acceleration records is scaled such that it has similar spectral acceleration value measured at the fundamental period of the building.

### **Response Surface Generation**

Design of Experiments techniques are used to efficiently select combinations of input parameters levels to define the response surface. Experimental designs for fitting a second order response surface must include at least 3 levels of each input variable. An appropriate design is selected on the basis that the resulting function provides good fit to the data points and, at the same time, requires minimal number of observations. An initial (Towashiraporn [7]) investigated the use of a Full Factorial Design (FFD), a Central Composite Design (CCD), and a Box-Behnken Design (BBD) for generating response surfaces. It was found that the CCD showed the best compromise between accuracy and computational effort required. A CCD with 5 input parameters contains 27 experimental or observation points. As a result, a

total of 27 building models having building parameters defined by the CCD are constructed. For simplicity in this initial study, only 2D building structural models are considered. Nonlinear time-history analysis is performed using DRAIN-2dx software on each building model, and the peak inter-story drift is extracted. Twenty seven sets of output (peak drift) and the corresponding combination of inputs are used to fit a response surface model.

A least-square regression analysis is applied to estimate the polynomial coefficients. The response surface obtained from this regression analysis approximates the peak inter-story drift for different buildings due to a suite of acceleration records scaled to a specific level of spectral acceleration. It is a function of the control variables as well as the random variables. This type of response surface model is referred to in this study as a global response surface. Statistical tests of this response surface must be made to ensure applicability and goodness-of-fit of the model (Towashiraporn [7]).

### **Building-Specific Fragility Estimation**

The response surface function obtained from the previous steps is appropriate for predicting peak drifts for an aggregation of low-rise steel moment frame buildings. Modification is needed in order to convert to a response surface applicable to a specific building. Control variables dictate how a metamodel for one building differs from the others. Building-specific response is derived by substituting values of the control variables specific for that building in the response surface of an aggregation. For example, actual values for the design code, the 1<sup>st</sup>-floor height, and the bay width of building AAA are converted and substituted for X1, X2, and X4, respectively, in the global response surface function to produce a response surface specific for building AAA. The response surface becomes a function of the remaining unspecified variables (column yield strength and modulus) and a fixed effect from the specified control variables. Probability density functions of the remaining variables are introduced into the model. Lognormal and uniform distributions are assumed for the column yield strength and modulus, respectively. Monte Carlo simulations are then carried out on each response surface using the assumed probability density functions for the remaining input variables. Probability of response exceeding a certain limit can be computed from a suitably large number of outputs obtained from the simulation. This, then, yields a value of fragility for a specific level of spectral acceleration. The process is repeated for other sets of ground acceleration records scaled to other levels of earthquake intensity. Fragility for various levels of intensity can be plotted in the form of fragility curve. Fragility curves specific to the 4 buildings in the hypothetical portfolio are depicted in Figure 2. It is obvious that, even though all 4 buildings are in the same classification, they exhibit different seismic fragility relations. The use of generic fragility curve for all building in the same class may produce erroneous estimates of potential seismic loss.

## **DISCUSSION AND CONCLUSION**

The ability to quickly estimate a vulnerability relation for an individual building in a target portfolio is a step toward more accurate seismic loss estimation. Nevertheless, the hypothetical building portfolio presented in this paper is composed of only a single building structural classification. This may be unrealistic because an actual portfolio of interest will likely contain buildings of several structural classifications. Ultimately, regionally derived metamodels for every major building classification must be formulated based on the building inventory data. A similar process to that used in this paper for the S1-L classification can be utilized to generate metamodels for all other building classes.

The greatest benefits from using metamodels are their computational efficiency and versatility. Considerable computational cost may be necessary in the process of actually generating the global metamodels, but utilizing them in the loss estimation process requires relatively little computational effort,

even for Monte Carlo simulation. After metamodels applicable for building inventory in a geographical region are developed, they can be implemented for any portfolio of interest located within the same region.

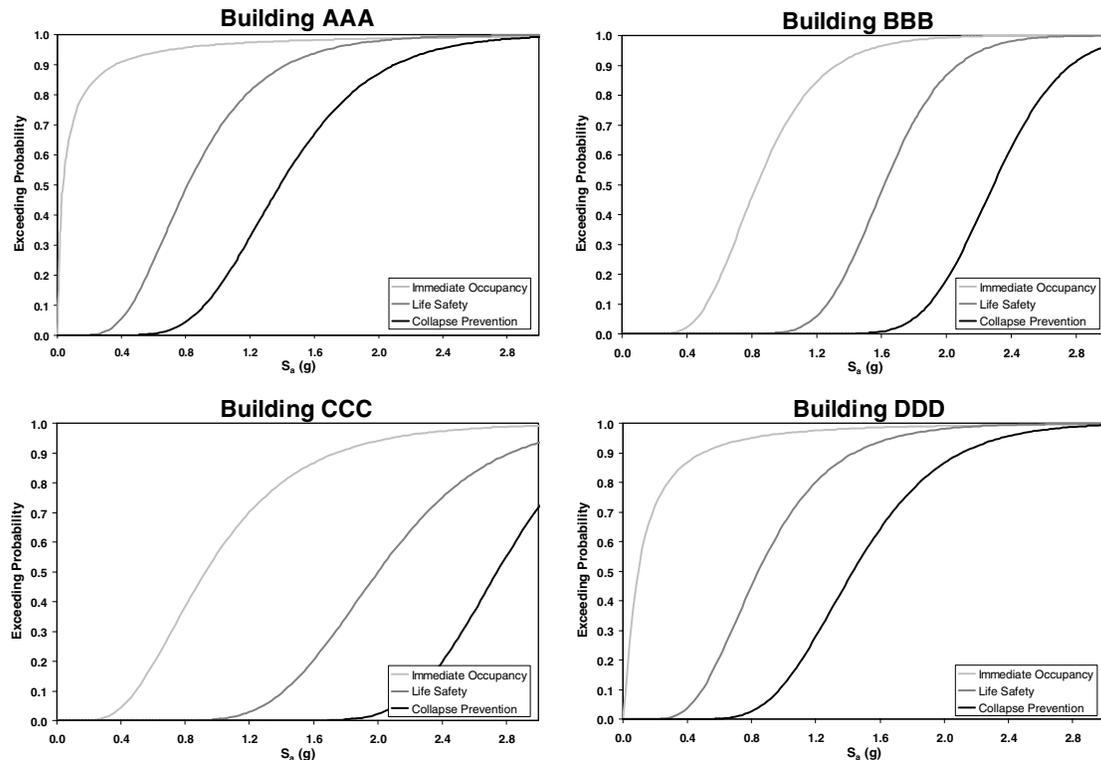


Figure 2: Fragility Curves for Buildings in the Hypothetical Portfolio

In summary, this paper presents a means for a rapid vulnerability computation. Response surface metamodels are applied in the prediction of seismic response for an aggregation of buildings. Results from the simulations can provide the seismic vulnerability of an individual building in a portfolio. The results from this paper must be considered hypothetical at this point, but the method is shown to be effective and can be directly applied to some of the available data sets.

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