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PROBABILITY DISTRIBUTION OF PEAK GROUND ACCELERATION RATIOS

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SUMMARY

In seismic design of lifeline structures, the spatial variation of the earthquake force is as important as the mean value itself. In this study, spatial variation of peak ground accelerations (PGAs) is examined using strong motion records for a large number of events from dense accelerometer arrays at Chiba in Japan, SMART–1 in Lotung, Taiwan, and a realtime city gas network damage estimation system (SIGNAL) in Japan. We defined PGA ratios as spatial intra-event variations of PGAs and examined their probability density functions (PDFs), mean values, standard deviations and percentiles estimated using accelerometer arrays of the Chiba, SMART-1, and SIGNAL databases. Then, the relationship between these statistics and the station separation distances was analyzed. We found that there is a very large scatter of PGAs in the results, especially for SMART–1 array, and that the PGA ratios in Chiba array tend to be higher than those for SMART–1 and SIGNAL. It was also revealed that the means and standard deviations have an almost linear relationship with the logarithm of the station separation distances ranging from several meters to one hundred kilometers, and this relationship can be attributed to the dependence of the correlation between intra-event PGAs on the separation distances.

INTRODUCTION

It has been reported in a large number of field studies of earthquake damage that for any earthquake event, the degree of damage suffered by similar structures varies significantly from one location to another, even though the separation between two structures may be reasonably small. Also, in the design of spatially extended structures, spatially distributed seismic forces have to be applied along the structure. Taking into account these effects of variation in seismic forces along spatially extended structures will help us better design such structures.

In this study, we examined the spatial variation of intra-event peak ground accelerations (PGAs) by using their ratios (PGA ratios) as presented in Kawakami [1]. The PGA ratio was defined as (smaller value)/(larger value) and it was calculated for all possible station pairs for each event. First, we analyzed the probability distribution of the PGA ratios. Assuming that the PGAs were lognormal random variables, we derived the probability density function, the mean values, standard deviations, and percentiles of the PGA ratios. Second, the PGA ratios were calculated using records of strong motion arrays: the Chiba

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array, SMART-1 array, and SIGNAL. The estimated mean values and standard deviations of the ratios were analyzed to determine their dependence on the station separation to account for wave scattering, and the wave-passage effects. This investigation revealed that the statistics had an almost linear relationship with the logarithm of the separation distance ranging from several meters to one hundred kilometers. Finally, the 50th and 95th percentiles were calculated to examine the possible difference of the PGAs between the two sites in future earthquake.

DATA

Chiba array

The Chiba dense array system, located about 30 km east of Tokyo, was put into operation in April, 1982. The current instrument arrangement is shown in Katayama [2]. Eight boreholes are grouped around borehole C0. Boreholes C1-C4 are only five meters away from C0, and each of P1-P4 are 15 meters away from C0. The Chiba database contains PGA ratios with separation distances ranging from five to 300 meters.

SMART-1 array

The SMART-1 strong motion array is located in the northeast corner of Taiwan near the city of Lotung (Bolt [3] and Oliveira [4]). The array was put into operation in 1980, and the installation of instruments was completed in August, 1982. At that time, the array consisted of 37 force-balanced triaxial accelerometers arranged in three concentric rings with a 200-, a 1000-, and a 2000-meter radius with one station, C-00, in the center (shown in Bolt [3]). Each of the three rings had twelve equally spaced stations. Later, in June l983, two more stations, E-01 and E-02, were added.

SIGNAL

SIGNAL is a system developed by Tokyo Gas Co., Ltd., to monitor ground motions in a wide area to enhance the safety of the city gas network (Shimizu [5]). It consists of 331 SI-sensors, five accelerometers, and network apparatus. The records containing PGAs, SI values, and site locations, are available to the public at the web site of Tokyo Gas Co., Ltd. [6]. The PGA value is a vector sum of two horizontal components. In Figure 1, the locations of the sensors are indicated by circles. The sensors are installed in many places throughout the Tokyo metropolitan area as shown in the figure. The sizes of the circles indicate the amplification factors of PGAs estimated from the attenuation relationship by Joyner [7] as shown later.

ANALYTICAL DISCUSSION FOR PGA RATIO

PGA ratio

The peak ground acceleration (PGA) ratio represents a spatial intra-event difference between two PGAs. The ratios were calculated as (smaller value) / (larger value) for all station pairs. Apparently, the closer the values of the ratio are to one, the higher is the correlation between the two concerned PGAs.

Probability density function of PGA ratio

A PGA can be ideally represented as a lognormal random variable, so that the joint probability density function of the logarithms of two intra-event PGAs is given by

$$f_{Z_1,Z_2}(z_1,z_2)=\frac{1}{2\pi\sigma_Z^2\sqrt{1-\rho^2}}\exp\left[-\frac{1}{2\sigma_Z^2(1-\rho^2)}\left\{(z_1-\mu_Z)^2-2\rho(z_1-\mu_Z)(z_2-\mu_Z)+(z_2-\mu_Z)^2\right\}\right] \quad (1)$$

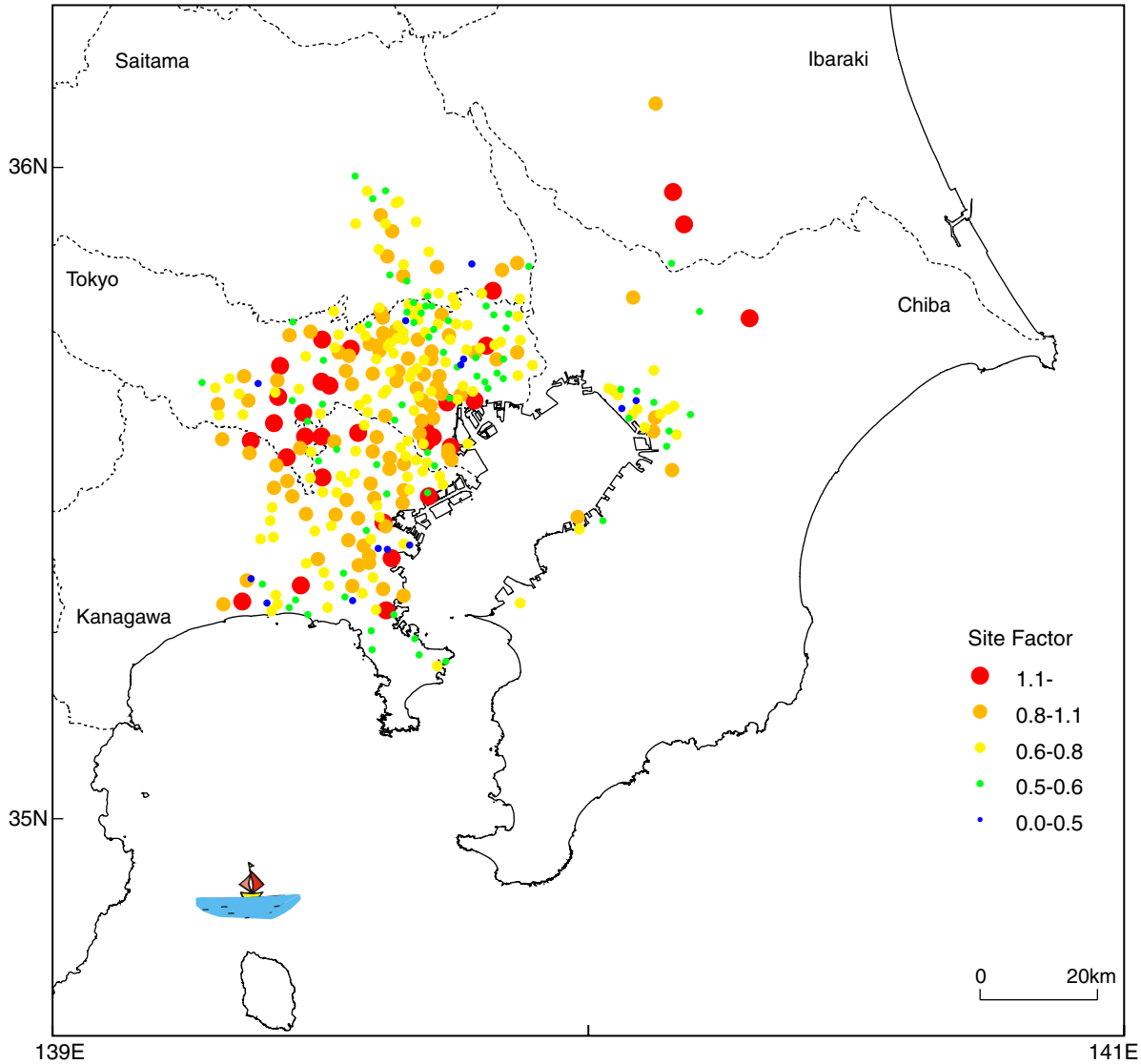

Figure 1: Instruments in the SIGNAL system (Tokyo Gas Co., Ltd., [6]) and site factors estimated from the attenuation relationship in equation (8) proposed by Joyner [7].

where Z_1 and Z_2 are the logarithms of PGAs X_1 and X_2 observed at the two sites, σ_Z is the standard deviation, and ρ is the correlation coefficient between Z_1 and Z_2. Since the PGA ratio R is defined as

$$R = \begin{cases} X_1 / X_2 & (X_1 < X_2) \\ X_2 / X_1 & (X_1 \geq X_2), \end{cases} \tag{2}$$

the probability density function of R can be obtained from equation (1) as

$$f_R(r) = \frac{2}{\sqrt{2\pi}\sigma_{P'} r} \exp\left(-\frac{\ln^2 r}{2\sigma_{P'}^2} \right), \quad 0 < r \leq 1. \tag{3}$$

We call this probability distribution "one-sided lognormal distribution".

Statistics of PGA ratios

Mean value

The mean value of PGA ratios, μ_R, can be obtained by using equation (3) as

$$\mu_R = \lim_{\varepsilon \to +0} \int_{\varepsilon}^{1} r\, f(r)\, dr = \exp\left(\frac{\sigma_{P'}^2}{2}\right)\left\{1 - \mathrm{Erf}\left(\frac{\sigma_{P'}}{\sqrt{2}}\right)\right\}, \tag{4}$$

where Erf(·) denotes the error function. Mean value μ_R is related to standard deviation $\sigma_{P'}$ as shown in equation (4). Thus, the dispersion of PGAs can be represented either by mean value μ_R or by standard deviation $\sigma_{P'}$.

Percentile

Let us define the γth percentile of a PGA ratio, r_γ, as the value of the ratio in which R in the range of $0 < r_\gamma \le R \le 1$ has the probability of γ percent. This condition can be expressed as

$$\frac{\gamma}{100} = \int_{r_\gamma}^{1} \frac{2}{\sqrt{2\pi}\sigma_{P'} r} \exp\left(-\frac{\ln^2 r}{2\sigma_{P'}^2}\right) dr = \int_{-\tau_\gamma}^{\tau_\gamma} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt, \tag{5}$$

where $\ln r = -t\sigma_{P'}$ and $\ln r_\gamma = -\tau_\gamma \sigma_{P'}$.

Local-site-effect correction for SIGNAL data

In the analysis of SIGNAL, the site factors affecting the PGAs were considered to eliminate the local site effects. We introduced the following definition of a site factor:

$$\ln a_j = \frac{1}{I}\sum_{i=1}^{I}\left(Z_{ij} - Z_{ij}^e\right) \tag{6}$$

where a_j is the site factor at the j-th station, I is the number of earthquakes, Z_{ij} is the logarithm of the PGA observed at the j-th station during the i-th earthquake, and superscript e denotes the value of an empirical estimate from the attenuation relationship given by equations (8) and (9). By using a_j, local-site-effect correction can be carried out as follows,

$$X_{ij}^c = X_{ij} / a_j \quad \text{or} \quad Z_{ij}^c = Z_{ij} - \ln a_j \tag{7}$$

where superscript c denotes the corrected value.

In this study, we used the following relationships proposed by Joyner [7] and Katayama [8], respectively:

$$\begin{aligned} \log_{10} A &= -1.02 - \log_{10} r - 0.00255\, r + 0.249\, M \\ r &= \sqrt{d^2 + 7.3^2} \end{aligned} \tag{8}$$

where A is the larger PGA of the two horizontal components in g, and d is the closest distance from the site to the surface projection of the fault rupture in km, and

$$\log_{10} A = 2.308 - 1.637 \log_{10}(R + 30) + 0.411 M \tag{9}$$

where A is the mean horizontal PGA in $\mathrm{cm/s^2}$, R is the focal distance in km, and M is the earthquake magnitude.

STATISTICAL ANALYSIS OF PGA RATIOS

PGA ratio

Scatter plots of the PGA ratios are shown in Figure 2. For SIGNAL, three sets of PGA ratios were analyzed (Cases A to C). Three cases included one set of ratios without the local-site-effect correction (Case A) and two sets with the local-site-effect correction based on the attenuation relationships proposed

by Katayama (Case B) and by Joyner and Boore (Case C). In the calculations of the PGA ratios, only pairs with differences in epicentral distances of less than one kilometer were used.

In these figures, a large scatter can be seen especially in the results of the SMART-1 array and SIGNAL. The ratios in the Chiba array tend to be closer to 1 than those in the SMART-1 array and in SIGNAL. A large number of PGA-ratio values as low as 0.1 or lower was observed in the SMART-1 array and SIGNAL. The PGA ratios were divided into station-separation groups a to r as shown in Figure 2. Then statistical analyses of the PGA ratios were carried out for each station-separation group.

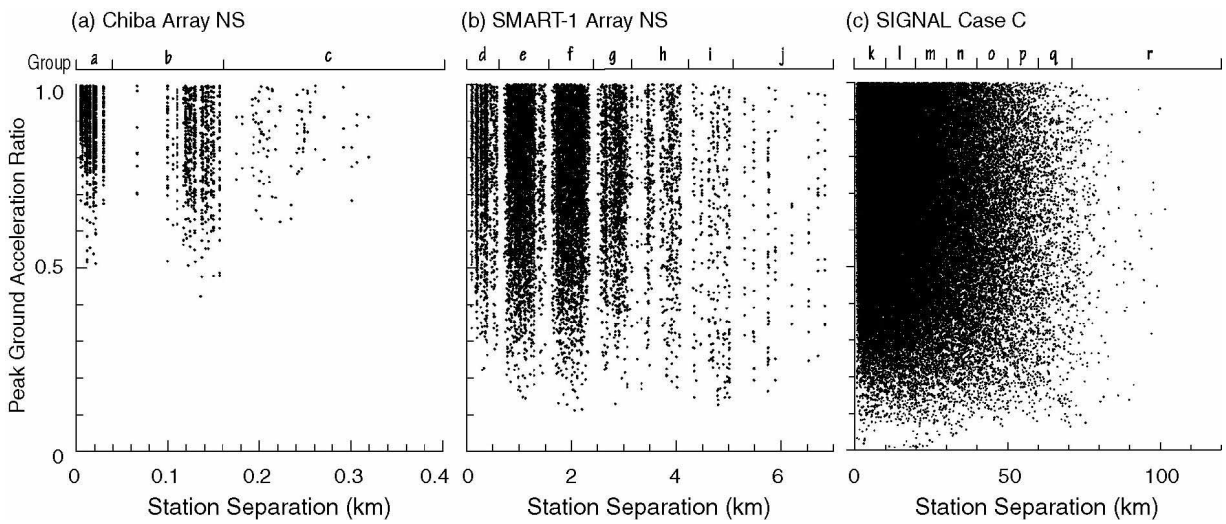

Figure 2: Scatter plots of PGA ratios vs. station separation for Chiba array (NS), SMART-1 array (NS), and SIGNAL (Case C)

Mean values and standard deviations of PGA ratios

The mean values and standard deviations of the PGA ratios and acceleration response spectral ratios (Kawakami [1]) against the station separations are shown in Figures 3 and 4, respectively. The station separation distance in the abscissa is the average value of separation for each of the station-separation groups.

The mean value, μ_R, and standard deviation, $\sigma_{P'}$, for group a (Chiba array) were about 0.9 (0.87 for EW, 0.90 for NS and UD) and 0.15-0.2 (0.19 for EW, 0.15 for NS and UD), respectively. Interestingly, a scatter of PGAs could be found even for the station pairs with a small separation such as those in group a. The mean value, μ_R, decreased gradually with an increase in the station separation. In contrast, the standard deviation, $\sigma_{P'}$, increased to 0.2-0.3 (0.24 for EW, 0.20 for NS, and 0.29 for UD) for group c.

In the SMART-1 array, mean value μ_R was 0.8 (0.78 for EW and UD, 0.77 for NS) for group d, and decreased to 0.7 (0.71 for EW, 0.72 for NS, and 0.70 for UD) for group h with an increase in the station separation distance. Standard deviation $\sigma_{P'}$ increased from 0.37 for group d to 0.5-0.55 (0.55 for EW, 0.50 for NS, and 0.49 for UD) for group h.

The mean values and standard deviations for the three cases of SIGNAL are also shown in Figure 4. By comparing the statistics of Cases B and C with those of Case A, we found that the local-site-effect correction reduced the dispersion of the PGAs similarly for each station separation group. On the other

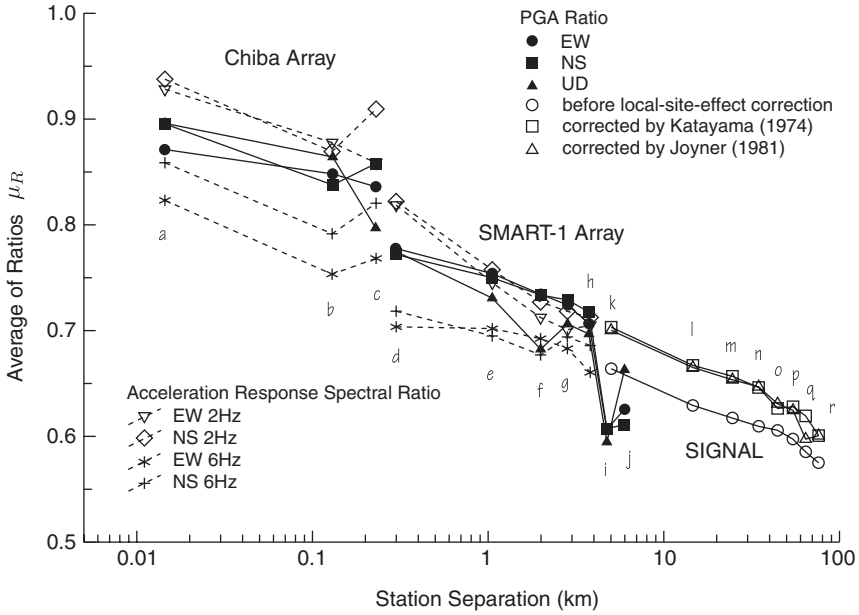

Figure 3: Mean value of PGA ratios and acceleration response spectral ratios vs. station separation for Chiba array, SMART-1 array, and SIGNAL.

hand, similar statistics were obtained for Cases B and C. This result indicates that the difference in the attenuation relationships does not significantly affect these statistics.

By comparing the statistics of Cases B and C of SIGNAL with those of the Chiba and SMART-1 arrays, we found that the mean values and standard deviations were almost continuous and linear functions of the logarithm of the station separation distances. From these results, we found that the scatter of the PGA ratios is characterized by a linear relationship between the mean values and standard deviations and the logarithm of the station separation distances ranging from several meters to one hundred kilometers. This relationship seems to come from the dependence of the correlation ρ on the separation distances. Also, Figure 4 shows that the standard deviation changed and so did the correlation ρ even at large separations of about 100 km. Therefore, we cannot assume the independence of the PGAs even for such large separation distances.

Percentiles of PGA ratios

Percentiles are useful statistics for analyses of PGA ratios, not only because of the clarity of the definition but also because the percentiles of PGA ratios and their logarithms are convertible into each other merely by using logarithms or exponents. We focused on the 50th and 95th percentiles (Figure 4). The former is used because it is a typical value of a ratio, and the latter is used because it is a minimum expected value (i.e., 5% significance level is assumed).

From these results, the expected differences between PGAs for the 50% and 95% probabilities can be summarized as follows:

(1) For two stations separated by 40 meters or less, there is a 50% probability that the difference in their PGAs will be less than 10-20% and a 95% probability that it will be less than 40%;
(2) if the stations are separated by almost one kilometer, there is a 50% probability that the difference in their PGAs will be less than 30% and a 95% probability that it will be less than 100%; and
(3) if the stations are separated by more than 70 kilometers, there is a 50% probability that the difference in their PGAs will be less than 70% and a 95% probability that it will be less than 400%.

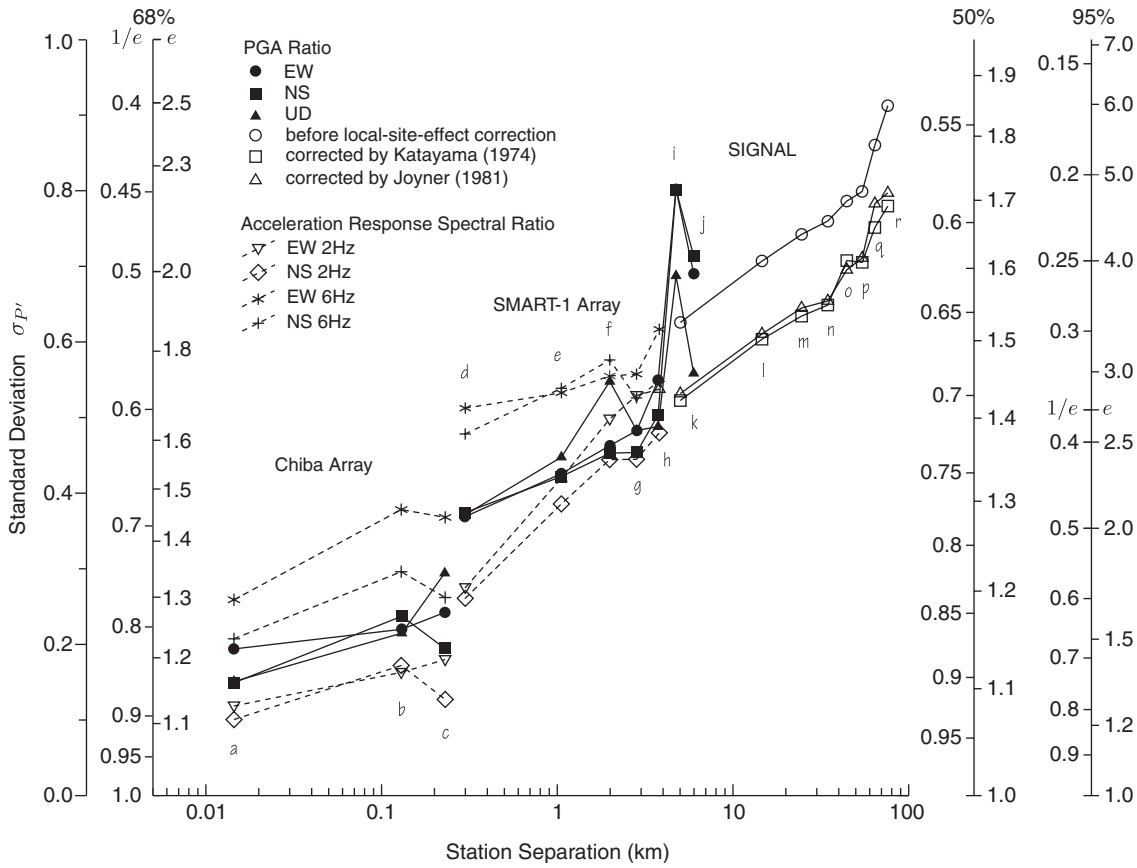

Figure 4: Standard deviation vs. station separation for Chiba array, SMART-1 array, and SIGNAL. The 50th, 68th, and 95th percentiles of the ratio are shown.

Probability density functions of PGA ratios

Figure 5 shows the probability density functions (PDFs) of the PGA ratios and acceleration response spectral ratios. In this figure, the PDFs estimated from the frequencies of occurrence are shown by the lines with symbols, and the analytical functions calculated from the observational standard deviations using equation (3) are shown by the curved lines.

In these figures, it can be seen that the peaks become lower and the shapes generally flatter as the station separation increases, which indicates a reduction in the correlation between the PGAs. The results for SMART-1 array and SIGNAL show flatter PDFs than those of the Chiba array, and the clear peaks of the PDFs vanished as a result of their large station separations. This figure also shows that the one-sided lognormal distribution is a good approximation of the probability distribution of the PGA ratios.

CONCLUSIONS

The probability distribution of the PGA ratios was investigated, and the following results were obtained.

(1) Analytical expressions of the probability density functions, mean values, standard deviations, and relationships between the mean values and standard deviations of the ratios were shown.

(2) Mean value μ_R linearly decreased as the logarithm of the station separation increased. Standard deviation $\sigma_{P'}$ increased as the logarithm of the station separation increased. These relationships can be attributed to the dependence of the correlation ρ between intra-event PGAs on the separation distances.

(3) Mean value μ_R and standard deviation $\sigma_{P'}$ for the three arrays were almost continuous and linear functions of the logarithm of the station separation ranging from several meters to one hundred kilometers.

(4) For the station pairs separated by 40 meters or less, there was a 50% probability of the difference in their PGAs being less than 10-20% and a 95% probability of it being less than 40%. For the station pairs separated by almost one kilometer, there was a 50% probability of the difference in their PGAs being less than 30% and a 95% probability of it being less than 100%.

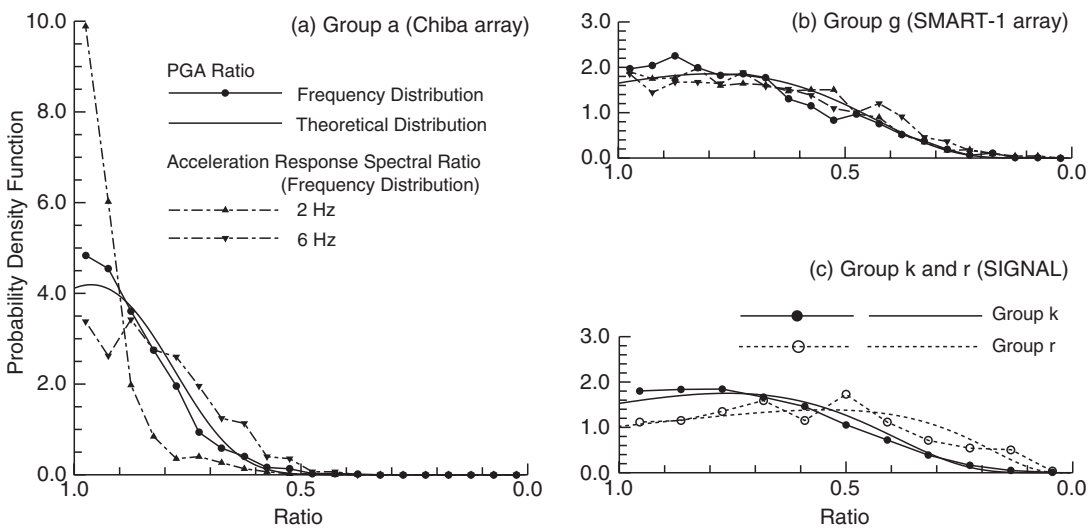

Figure 5: Probability density functions of PGA ratios and acceleration response spectral ratios for (a) group a of Chiba array (NS), (b) group g of SMART-1 array (NS), and (c) SIGNAL group k and r. Only the PDFs of PGA ratio are shown for SIGNAL.

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REFERENCES

1. Kawakami H, Sharma S. "Statistical study of spatial variation of response spectrum using free field records of dense strong motion arrays." Earthq. Engng. Struct. Dyn. 1999; 28: 1273-94.
2. Katayama T, Yamazaki F, Nagata S, Lu L, Turker T. "Strong motion databases for the Chiba seismometer array and its engineering analysis." Earthq. Engng. Struct. Dyn. 1990; 19: 1089-106.
3. Bolt BA, Loh CH, Penzien J, Tsai YB, Yeh YT. "Preliminary report on the SMART-1 strong motion array in Taiwan." Report No. UCB/EERC-82/13, EERC, UCB, 1982.

4. Oliveira CS, Penzien J. “Spatial variation of seismic ground motions based on SMART-1 array data.” Proc. Trilateral Seminar-Workshop on Lifeline Earthquake Engineering, Taipei, 1985; 59-75.
5. Shimizu Y, Yamazaki F. “Realtime city gas network damage estimation system SIGNAL.” 11th European Conference on Earthquake Engineering, Rotterdam: Balkema, 1998: CDROM.
6. Tokyo Gas Co., Ltd. “What’s SIGNAL”, World Wide Web.
7. Joyner WB, Boore DM. “Peak horizontal acceleration and velocity from strong-motion records including records from the 1979 Imperial Valley, California, earthquake.” Bull. Seism. Soc. Am., 1981; 71: 2011-38.
8. Katayama T. “Statistical analysis of peak accelerations of recorded earthquake ground motions.” Seisan-Kenkyu, 1974; 26: 18-20.