

# A QUASISTATIC ANALYSIS METHOD TO IMPROVE COLLAPSE MECHANISM ANALYSES OF MULTISTORY BUILDINGS

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## SUMMARY

This study proposes a quasi-static analysis method to improve the reliability of conventional collapse mechanism analyses of multi-story buildings. This method is based on the hypothesis that the incremental deformations of the buildings subjected to earthquakes are proportional to the eigenvectors evaluated by using equivalent story stiffness and damping. In this method, the incremental displacements proportional to the eigenvectors are accumulated in the story drifts of the buildings; the eigenvectors are estimated by performing modal analyses whenever an inelastic event occurs in the stories. The analytical results indicate that the conventional pushover analysis generally overestimates the first story drift, while the quasi-static method tends to give good agreements with the results evaluated by the time history analysis.

## **INTRODUCTION**

Most buildings are expected to deform beyond the limit of linearly elastic behavior when subjected to strong earthquakes. Thus the earthquake response of buildings deforming into their inelastic range is of central importance in earthquake engineering. Therefore, the performance-based seismic design has been applied for various types of structures in recent years. Accordingly, a number of methods for implementing the performance-based seismic design have been proposed: Capacity Spectrum Approach (Freeman [1]); the N2 Method (Fajfar et al [2]); and Direct Displacement-based Design (Priestley and Calvi [3]), etc. In most of these methods, pushover analysis is used for identifying the collapse mechanism of structures as well as their ductility capacities. The pushover analysis have been considered simple and useful techniques in analyses of the static, inelastic response of structures, where the collapse mechanism can be determined through the stepwise formation of local mechanism or plastic hinges for a given lateral force distribution. The pushover analysis provide a capacity curve (or a pushover curve) that represents the structure's ability beyond the elastic limit to resist a seismic demand. Herein, the capacity curve is generally expressed by the force-displacement relation by tracking the base shear and the roof displacement of structures. The capacity curve can exhibit the performance of the structures, which includes global drift, story drift, inelastic element deformations, and other important performance parameters.

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FIG.1 Five-story building model and inelastic relation between story shear and story drift

Herein, many types of lateral force distribution for the pushover analysis have been employed or recommended in reports and codes: SEAOC (VISION2000 [4]) reports that the inverted triangle is a popular function for the lateral force distribution; IAEE (EUROCODE8 [5]) describes that the lateral force applied to structures can be assumed in proportion to the product of the mass of structures and the fundamental mode of vibration. In actual, however, the characteristics of the capacity curve may surely be influenced by the types of lateral force distribution. In addition, these types of lateral force distribution are commonly unchangeable through out the analysis. It is seen that the lateral force distribution may change due to changes in the member stiffness or the introduction of a inelastic hinge mechanism. It should be noted, therefore, that the assumption of the non-varying lateral force distribution makes significant differences in the performance of structures especially beyond the elastic limit as described in Krawinkler and Seneviratna [6].

The main objectives of the present study are: (1) to assess the reliability of the conventional pushover analysis with many types of lateral force distribution, by comparison with the collapse procedures of multi-story shear frame buildings evaluated by the pushover analysis and inelastic time history analysis.; and (2) to propose a new quasi-static analysis method to improve the performance of the conventional pushover analysis so that the collapse mechanism can be evaluated more precisely. It is noted that the higher modes of vibration may also influence the differences in the collapse procedure of structures as well as the unchangeable lateral force distribution. It is conceivable, however, the inelastic time-history analysis is more appropriate than static analyses based methods if the higher modes highly contribute to the total response of structures. It is assumed, therefore, that the fundamental mode of vibration is mainly dominated in the buildings used for the analyses in this study.

## ASSESSMENT OF CONVENTIONAL PUSHOVER ANALYSIS

#### Model studied

Consider the five-story building of Fig.1 with bay width L, height of each story  $H_i$ , elastic modulus E, and second moment of inertia about the axis of bending= $I_b$  and  $I_c$  for the beams and columns,

respectively. In this study, the collapse procedure of the five-story building is analyzed for assessing the reliability of the conventional pushover analysis. The beam-to-column stiffness ratio  $\rho$  is defined as follows:

$$\rho = \frac{\sum_{beams} EI_b / L_b}{\sum_{columns} EI_c / L_c} \qquad (1)$$

where  $L_b$  and  $L_c$  are the lengths of the beams and columns and the summations include all the beams and columns in the midheight story. In this study, the beam-tocolumn stiffness ratio  $\rho$  is



assumed to be infinity (the beams and the floor systems are rigid in flexure). So, the beams restrain completely the joint rotations, and the building behaves as a shear beam with double-curvature bending of the columns in each story. The effects of shear building idealization upon the dynamic behavior are discussed by Cruz and Chopra [7][8]. The shear building assumption is convenient to investigate overall structural behavior, while it is necessary to work with realistic idealizations with an appropriate value of  $\rho$  for practical results. Therefore, detailed discussion of the practical issues concerning the dynamic behavior of rotational joints and elements with a finite value of  $\rho$  is beyond the scope of this study. The mass of each story is lumped at the floor level with  $M_i$  denoting the mass at the i-th floor. The horizontal stiffness of each story  $K_i$  is identical to each other. The horizontal stiffness can be calculated by the following formula:

$$K_i = \sum_{columns} \frac{12EI_c}{L_c^3} \tag{2}$$

In general, the higher modes of vibration tend to contribute little to the response of structures with the fundamental period about less than 1.0s. Therefore, the mass and the stiffness of the model are adjusted so that the natural period of the model with initial stiffness becomes 0.7s ( $M_i = 200$  t and  $K_i = 2.5 \times 10^5$  kN/m). For the inelasticity, the tri-linear skeleton curve is assumed in the stories as shown in Fig.1. The yielding strength ratio and the corresponding ductility are k = 0.08 and  $\mu = 1.0$ , respectively. Herein, the strength ratio is defined as the ratio of the strength of a story to the product of the total mass of the building and the gravity acceleration (9.81m/s<sup>2</sup>). The strength ratio and the corresponding ductility at the maximum point are k = 0.12 and  $\mu = 6.0$ , respectively. The ratio of the initial secant stiffness to the second secant stiffness is  $\alpha = 0.1$ . The secant stiffness ratio of initial to the third is  $\beta = 0.01$ . For the time history analysis, the followings are assumed: the hysteresis model applied to the structural model is the Clough model, which would represent the hysteretic behavior of reinforced concrete elements; the material damping constant is assumed 0.05 and damping coefficient is assumed as Rayleigh damping.



FIG.3 Collapse procedures of five-story shear building subjected to earthquakes(a: sinusoidal wave, b: El Centro, c: Kobe, and d: Hachinohe)(Y: yielding , M: maximum, 1,2,3:the number of floors from the bottom)



FIG.4 Time histories of fifth story with collapse procedures of five-story shear building subjected to earthquakes (a: sinusoidal wave, b: El Centro, c: Kobe, and d: Hachinohe)



FIG.5 Collapse procedures of weak shear building subjected to earthquakes (a: sinusoidal wave, b: El Centro, c: Kobe, and d: Hachinohe)(Y: yielding , M: maximum, 1,2,3:the number of floors from the bottom)



FIG.6 Time histories of fifth story with collapse procedures of weak shear building subjected to earthquakes (a: sinusoidal wave, b: El Centro, c: Kobe, and d: Hachinohe)

#### **Rigorous inelastic solution**

In this study, the collapse procedure evaluated by the time history analysis is considered as a rigorous solution. It is presumed, however, that the behavior of the model after yielding might be changed by input earthquakes. Therefore, unique characteristics of the collapse procedure are tried to be summarized in this section. The time history analysis is carried out by using the Newmark  $\beta$ -method with the time interval of 0.001s. Four types of input motions are applied as follows: 1) sinusoidal motion (the fundamental frequency is 2.0s and the PGA is 0.2g), 2) El Centro NS (1940); 3) Kobe NS (1995), and 4) Hachinohe EW (1968). The PGA of these earthquakes (2), 3) and 4) ) is scaled to be 0.3g. The elastic acceleration response spectra of the earthquakes for 5% damping are shown in Fig.2.

Fig.3 shows the snapshots of the lateral ductility distribution over the height of the building whenever a story drift exceeds the yielding ductility ( $\mu = 1.0$ ) or the ductility of the maximum point ( $\mu = 6.0$ ). The lateral ductility distribution is almost identical to each other when the second story drift reaches the maximum point (in this study, the maximum deformation of the building is defined as the second story drift excesses the maximum point). The sequence of failure occurrence and the lateral ductility distribution in the process of reaching the maximum deformation of the building are similar to each other, except for the case of the sinusoidal earthquake. Fig.4 shows the time histories of the displacement at the fifth story relative to the base with failure events of the building. Fig.4 indicates that the yielding and the excess of the maximum point are not confined to a short time; they occur in an oscillating period of the dynamic response of the building. It is predicted, however, that the collapse procedure might become different from each case due to different earthquakes; especially if the yielding and the excess of the maximum point occur, not in such a oscillating period but in a monotonically drifting period of the response of the building within a short time. The sequent collapse with a monotonically drifting period may occur when the strength of the building is relatively small to the intensity of the earthquakes. So, a building with the same properties of the previous (standard) model except the strength of the stories decreased by a factor of 0.5 is considered. Hereinafter, this building is called "weak building" or "weak model". The collapse procedures of the weak building are analyzed with the same earthquakes. Fig.5 shows the snapshots of the lateral ductility distribution over the height of the weak building, and Fig.6 shows the time histories of the displacement at the fifth story relative to the base. Fig.6 indicates that the most of the yielding and the excess of the maximum point in each case are confined to a shorter time than in the standard model within less oscillating period (the case a and b are about the monotonically drifting period) of the response of the building. Fig.5 shows that the lateral ductility distribution at the yielding. does not seem to have unique characteristics among the cases. However, the large ductility distribution as the second story drift excesses the maximum point is almost identical to each case. This behavior is also identical to the cases in the previous standard model. Herein, it is noted that the drift of the first story tends to increase when the yielding and the excess of the maximum point occur within the shorter time: e.g., the maximum first story drift is about less than 10 in the standard model, while the drift in the weak model is larger than or equal to 10.

## **Pushover analyses**

The pushover analysis presented hereinafter is performed by the following procedure:

(1) First, the lateral force distribution is selected. In this study, three types of lateral force distribution are considered: a) rectangular; b) inverted triangle, and c) the distribution proportional to the eigenvector of the first mode of vibration evaluated with the initial story stiffness.

(2) Iterative procedure is performed to balance the static lateral forces and the internal forces based on the modified Newton-Raphson method.

(3) When an inelastic event (yielding or the excess of the maximum point) occurs at the stories, the displacements of the model are recorded.

(4) When the iterative procedure can not be converged, the analysis is stopped.



FIG.7 Collapse procedures of five-story shear building evaluated by pushover analysis (a: rectangular , b: inverted triangle, c: first mode)(Y: yielding , M: maximum, 1,2,3:the number of floors from the bottom)

Fig.7 shows the snapshots of the lateral ductility distribution over the height of the building evaluated with the various types of lateral force distribution. Fig.7 shows that the sequence of collapse procedure is similar to each other, except for the case of the rectangular lateral force distribution. Moreover, Fig.7 indicates that a unique behavior can be seen through the collapse procedure evaluated by the pushover analysis: the deformations are highly concentrated in the first story rather than those of the time history analyses. This behavior implies that the drift in the first story tends to be overestimated by the conventional pushover analysis. In addition, this behavior would not be affected by the types of the lateral force distribution. It is conceivable that this concentration is caused by the monotonic static loading since the first story drift tends to increase in the time history analyses when the collapse procedure is confined to an approximately monotonic drifting period of the response of the building.

## A quasi-static analysis method (IMDA method)

The extreme concentration of the deformations in the first story should be avoided for enhancing the reliability of static analyses. It is noted, however, the conventional pushover analysis may not avoid this concentration because of its monotonic loading. There is an alternative method that would avoid the concentration of the deformations. This method is based on the hypothesis that the incremental deformations of the building are proportional to the eigenvector of the fundamental mode of the building with equivalent story stiffness and damping. This method is called "IMDA (Inelastic Modal Deformation Analysis)" method. The analytical procedure is summarized as follows:

1) estimate the eigenvector by the conventional modal analysis, where equivalent story stiffness  $K_{eq}$  and equivalent story damping  $h_{eq}$  are used. The equivalent story stiffness is the tangential stiffness from the



FIG.8 The analytical procedure for IMDA (Inelastic Modal Deformation Analysis) Method (a: routine, b: equivalent stiffness and c: equivalent damping)

origin to the next limit state; i.e., the limit state is assumed to be the yielding point if the current story drift does not exceed the yielding point; and the limit state is assumed the maximum point if the drift exceeds the yielding point and does not exceed the maximum point; and the limit state is the point ( $\mu = 9.0$ ) if exceeds the maximum point: moreover the point ( $\mu = 30.0$ ) if exceeds the previous point ( $\mu = 9.0$ ). The equivalent story damping is a simulative hysteretic damping for the Clough model. The damping represents the dissipated energy due to hysteresis loop during a cycle of simple harmonic motion. Actually, the hysteretic damping has little effects upon the vibrating modes of the building whenever the deformations are so large that its damping constant exceeds 0.3. To the author's knowledge, however, if the initial damping is very large due to SSI (radiation damping) or base isolation systems, the vibrating modes of the building are highly affected by the damping. The stiffness and damping are given by the following formula:

$$K_{eq} = K_{i} \qquad \mu \leq 1$$

$$= K_{i} \frac{\alpha(\mu - 1) + 1}{\mu} \qquad 1 \leq \mu \leq \mu_{M} (= 6) \qquad (3)$$

$$= K_{i} \frac{\beta(\mu - \mu_{M}) + \alpha(\mu_{M} - 1) + 1}{\mu} \qquad \mu_{M} \leq \mu$$

$$h_{eq} = 0 \qquad \mu \leq 1$$

$$= \frac{(1 - \alpha)(\mu - 1)}{\pi\mu} \qquad 1 \leq \mu \leq \mu_{M} (= 6) \qquad (4)$$

$$= \frac{\mu - \beta(\mu - \mu_{M}) - \alpha(\mu_{M} - 1) - 1}{\pi\mu} \qquad \mu_{M} \leq \mu$$



FIG.9 Comparison with Collapse procedures of five-story shear building evaluated by (a) IMDA and (b) time history analysis in case of El Centro previously shown in Fig.3(b)(Y: yielding , M: maximum, 1,2,3:the number of floors from the bottom)

It is noted that the conventional modal analysis can not consider the damping effects. This study recommend the space state method (Hart and Wang [9]) for calculating the eigenvector with damping.

2) assume that the eigenvector is proportional to the incremental displacements of the stories3) accumulate the incremental displacements in the corresponding stories stepwise

4) perform the model englysis to calculate the sign vector when an inclusion event ecour in the

4) perform the modal analysis to calculate the eigenvector when an inelastic event occur in the stories.

5) assume that the revised eigenvector is proportional to the incremental displacements in the next stage, and accumulate the incremental displacements in the stories again

6) repeat the procedure from 2) to 5)

According to the analytical procedure of the IMDA, the standard model described above is analyzed. Fig.9 shows the snapshots of the lateral ductility distribution over the height of the building evaluated by the IMDA method. Fig.9 also shows the result of the time history analysis (in the case of El Centro earthquake previously shown in Fig.3(b)), which is the most compatible result with that of the IMDA. In contrast with the pushover analysis, the deformations of the building would never be extremely concentrated in the first story when evaluated by the IMDA method. This is attributed to the accumulation of the incremental displacements in the stories over the height of the building, instead of loading static lateral forces. This method would not largely reduce the numerical efforts since the IMDA requires the number of modal analyses instead of the iterative procedure, such as Newton-Raphson method. However, the precision of the collapse procedure based on the IMDA method would be higher than the conventional pushover analysis method.

## CONCLUSION

This study presented the discrepancies of collapse procedures of the five-story shear buildings evaluated by the time history analysis and the conventional pushover analyses with three types of lateral force distribution. For enhancing the reliability of collapse mechanism analyses, this study proposed a quasistatic analysis method. This method is based on the hypothesis that the incremental deformations of the buildings subjected to earthquakes is proportional to the eigenvectors evaluated by using equivalent story stiffness and damping. The analytical results indicate that the quasi-static method tends to give good agreements with the results evaluated by the time history analysis; especially, the extreme concentration of the deformations in the first story can be avoided by using this method. As a limited scope, the results presented in this study are restricted to multistory shear buildings, tri-linear story shear-drift relations, four types of earthquakes. For further development of the IMDA method, the effects of the number of stories, the beam-to-column stiffness ratio, and relative strength and stiffness among the stories should be investigated in a more realistic fashion.

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