LIMITS IN THE GRADATION CURVES OF LIQUEFIABLE SOILS

Atsunori NUMATA\textsuperscript{1} and Shinichiro MORI\textsuperscript{2}

SUMMARY

The authors have paid attention to sand boils, which are ejected due to liquefaction, have conducted a series of site investigation on liquefaction and have collected over 800 samples of ejected soils on liquefied sites during 13 earthquakes in Japan, the United States, the Philippines and Taiwan since the 1987 Chibaken-toho-oki earthquake. Over 800 samples are deemed appropriate for constituting a database for statistical research. This paper elucidates the limits in the gradation curves of liquefiable soil based on this database. First, examples of the curves of soils ejected during an earthquake with typical features are shown for understanding the fundamental nature of the database, as well as the entire figure of the database. Second, the peculiarity of the ejected soils is clarified by the difference between the soil ejected on reclaimed land and that on other kinds of ground in terms of the relationship between fines and clay contents, and ranges of mean grain size. Third, a model of grain size distribution of ejected soil is derived by the relationship between 50\% diameter (mean grain size) and modified coefficient of uniformity, which is newly defined in this paper. Finally, statistical analysis of grain size distributions of certain percentages of fines is carried out, and new limits in the gradation curves of liquefiable soils are proposed based upon the results of this statistical analysis.

INTRODUCTION

It is generally considered that liquefaction resistance increases as the grain size becomes coarser due to improved drainage, and it increases as the grain size becomes finer due to increased cohesion. Consequently, clarifying the gradation curve of liquefiable soil is an important approach to liquefaction susceptibility of a ground. Tsuchida \cite{Tsuchida1970} already showed the ranges of grain size distribution of liquefiable soil in 1970. These ranges are used in the Technical Standards for Port and Harbour Facilities \cite{JapanPortAndHarbourAssociation1977} published by the Japan Port and Harbour Association. These are also used in the ATC-32 \cite{ATC1997}, the earthquake-resistant design code for bridges in the USA. Although a large number of earthquakes and liquefactions have occurred since Tsuchida showed these ranges, not enough verification and reconsideration have been performed about the ranges of grain size distribution of liquefiable soil.

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Most liquefaction events leave ejected soils on the ground after the earthquakes. Since ejected soil derives from the liquefied layer and appears on ground surface, the ejected soil must provide plenty of information on the liquefied layer useful for investigating the characteristics of the liquefied soil. For this reason, the authors have paid attention to the ejected soil, which can be easily sampled, and carried out site investigation at many liquefaction sites since the 1987 Chibaken-toho-oki earthquake in Japan, collecting and analyzing a large number of ejected soils.

OUTLINE OF INVESTIGATION

Table 1 shows a summary of 13 earthquakes with investigated liquefaction and the number of samples of ejected soils. The data of the 823 samples in this table are use in this paper.

Fig 1 shows the gradation curves of soils ejected during the 1987 Chibaken-toho-oki earthquake in Japan as an example of typical gradation curves of ejected soil. “Boundaries for most liquefiable soil (range A)” and “Boundaries for potentially liquefiable soil (range B)” defined in the Technical Standards for Port and Harbour Facilities in Japan [2] are also shown in this figure. Fig 1(a) shows ejected soils on reclaimed ground and Fig. 1(b) shows ejected soils on other kinds of ground. The grain size distributions of ejected soils include those consisting of nearly 100% fines, while most include less than 10% clay. Most comprises a uniform grain size, with the gradation curves being almost linear in the range between 30% diameter and 80% diameter. In addition, whereas ejected soils on other kinds of ground are within range A, some of ejected soils on reclaimed ground are outside range A and finer than the lower boundary of range A. The reclaimed ground where liquefaction occurred during the Chibaken-toho-oki earthquake was reclaimed after 1965 [4], being young with an age of around 20 years. These sites involving fine-grain ejected soils are not necessarily located near the epicenter subjected to strong ground motion, but are located among other kinds of ejected soils along the coastal region facing the Tokyo Bay. It should be noted that all of the ejected soils used in this paper are fundamentally non-plastic.

Table 1. Summary of each earthquake investigated liquefaction and the number of samples

<table>
<thead>
<tr>
<th>Name of earthquake</th>
<th>Date (Local time)</th>
<th>Magnitude</th>
<th>Epicenter Latitude deg</th>
<th>Epicenter Longitude deg</th>
<th>Depth km</th>
<th>Corrected previous JMA scale inside the parentheses</th>
<th>Publisher</th>
<th>Maximum epicentral distance of liquefied R(km)***</th>
<th>The number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chibaken-toho-oki EQ</td>
<td>1987.12.17</td>
<td>Mj=6.7</td>
<td>34.35N</td>
<td>140.48E</td>
<td>58</td>
<td>V*</td>
<td>JMA</td>
<td>36</td>
<td>89</td>
</tr>
<tr>
<td>Loma Prieta EQ (USA)</td>
<td>1989.10.17</td>
<td>Ms=7.1</td>
<td>37.04N</td>
<td>121.88W</td>
<td>19</td>
<td>V (V)</td>
<td>USGS</td>
<td>62</td>
<td>42</td>
</tr>
<tr>
<td>Luzon EQ (Philippines)</td>
<td>1990.7.16</td>
<td>Ms=7.8</td>
<td>15.66N</td>
<td>121.23E</td>
<td>25</td>
<td>Ⅷ<del>Ⅸ(Ⅷ</del>Ⅷ)</td>
<td>USGS</td>
<td>195</td>
<td>9</td>
</tr>
<tr>
<td>Kushiro-oki EQ</td>
<td>1993.1.15</td>
<td>Mj=7.8</td>
<td>42.85N</td>
<td>144.38E</td>
<td>107</td>
<td>V</td>
<td>JMA</td>
<td>255</td>
<td>109</td>
</tr>
<tr>
<td>Notohanto-oki EQ</td>
<td>1993.2.7</td>
<td>Mj=6.6</td>
<td>37.65N</td>
<td>137.30E</td>
<td>25</td>
<td>V</td>
<td>JMA</td>
<td>30</td>
<td>48</td>
</tr>
<tr>
<td>Hokkaido-nansei-oki EQ</td>
<td>1993.7.12</td>
<td>Mj=7.8</td>
<td>42.78N</td>
<td>139.20E</td>
<td>58</td>
<td>V</td>
<td>JMA</td>
<td>255</td>
<td>140</td>
</tr>
<tr>
<td>Northridge EQ (USA)</td>
<td>1994.1.17</td>
<td>Ms=6.7</td>
<td>34.21N</td>
<td>118.54W</td>
<td>19</td>
<td>V (V)</td>
<td>USGS</td>
<td>30</td>
<td>12</td>
</tr>
<tr>
<td>Hokkaido-toho-oki EQ</td>
<td>1994.10.4</td>
<td>Mj=8.1</td>
<td>43.37N</td>
<td>147.67E</td>
<td>30</td>
<td>Ⅷ</td>
<td>JMA</td>
<td>430</td>
<td>121</td>
</tr>
<tr>
<td>Sanriku-haruka-oki EQ</td>
<td>1994.12.28</td>
<td>Mj=7.5</td>
<td>40.43N</td>
<td>143.75E</td>
<td>0</td>
<td>V</td>
<td>JMA</td>
<td>150</td>
<td>7</td>
</tr>
<tr>
<td>Hyogoken-nambu EQ</td>
<td>1995.1.17</td>
<td>Mj=7.2</td>
<td>34.61N</td>
<td>135.00E</td>
<td>14</td>
<td>Ⅷ</td>
<td>JMA</td>
<td>88</td>
<td>142</td>
</tr>
<tr>
<td>Tottori-ken-seibu E</td>
<td>1997.5.13</td>
<td>Mj=6.3</td>
<td>31.95N</td>
<td>130.30E</td>
<td>65</td>
<td>Ⅷ</td>
<td>JMA</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>Chi-Chi EQ (Taiwan)</td>
<td>1999.9.21</td>
<td>Mj=6.3</td>
<td>23.85N</td>
<td>120.81E</td>
<td>78</td>
<td>Ⅷ</td>
<td>CWB</td>
<td>140</td>
<td>13</td>
</tr>
<tr>
<td>Tottori-ken-seibu EQ</td>
<td>2000.10.6</td>
<td>Mj=7.3</td>
<td>35.4N</td>
<td>133.4E</td>
<td>116</td>
<td>Ⅷ</td>
<td>JMA</td>
<td>105</td>
<td>74</td>
</tr>
</tbody>
</table>

Mj : JMA magnitude
Ms : Surface-wave magnitude
Ml : Local magnitude

* Previous JMA seismic intensity scale
** Modified Mercalli Intensity Scale
*** Adapted Rossi-Forel seismic intensity scale
**** Seismic intensity scale in Taiwan
***** Calculated by corrected JMA magnitude
COMPARISON BETWEEN EJECTED SOILS ON RECLAIMED GROUND AND THOSE ON OTHER KINDS OF GROUND

50% diameter (mean grain size)

The 50% diameter is a representative index to the mean grain size of the gradation curve and it is especially convenient for comparing gradation curves with similar uniformity coefficients. Consequently, the ranges of gradation curves of ejected soils on both reclaimed and other kinds of grounds are described using the 50% diameter to clarify the differences between the two.

Fig. 2 shows the ranges of 50% diameter of soils ejected during each earthquake. The ejected soil data of the 1964 Niigata earthquake [5], 1968 Tokachi-oki earthquake [6], 1978 Miyagiken-oki earthquake [7] and 1983 Nihonkai-chubu earthquake [8, 9] are added to this figure. Range A for a uniformity coefficient \( U_c \) of less than 3.5 is also superimposed. Many ejected soils generally fall within range A, but there are ejected soils having finer grains than range A found on reclaimed ground. The finer boundary of ejected soils on other kinds of ground is almost the same as the finer boundary of range A. Most ejected soils coarser than range A were ejected during the 1995 Hyogoken-nanbu earthquake. Both ejected soils on reclaimed ground and on other kinds of ground are coarser than range A during this earthquake.

In Fig. 2, one ejected soil with a small or large 50% diameter widens the range, necessitating a statistical analysis. As a solution to this problem, Fig. 3 shows frequency distributions and cumulative relative frequencies of 50% diameters on a \( \phi \) scale. Ejected soils on reclaimed ground and on other kinds of ground are expressed in this figure. The \( \phi \) scale, which is defined as Eq.(1), is convenient for describing frequency distributions.

\[
\phi = -\log_2 D
\]  

where D is grain size in mm. Table 2 shows a comparison between grain size D, which is used in the geotechnical field, and \( \phi \) scale, which is used in the geological field. The values of the \( \phi \) scale in this figure indicate the minimum value in each range of the \( \phi \) scale. For instance, a range between 0 and -1 is expressed as -1. Generally speaking, most ejected soils fall in the range of sand. The number of ejected soils on reclaimed ground is approximately twice the number of those on other kinds of ground. Reclaimed ground is therefore more liquefiable than other kinds of ground, such as Holocene ground, from the aspect of the number. The cumulative relative frequency of reclaimed ground is finer than other kinds of ground. Whereas the fractions for 4 and more on the \( \phi \) scale account for 0\% of other kinds of

\[\text{(a) Reclaimed ground} \quad \text{(b) Other kinds of ground}\]

Fig. 1. Gradation curves of ejected soils during the 1987 Chibaken-toho-oki earthquake

\[\text{Table 2: Comparison between grain size D, which is used in the geotechnical field, and } \phi \text{ scale, which is used in the geological field.} \]
Fig. 2. Ranges of 50% diameter of ejected soils during each earthquake

Table 2. Comparison between grain size and $\phi$ scale

<table>
<thead>
<tr>
<th>$\phi$ (mm)</th>
<th>D (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.00</td>
<td>0.008</td>
</tr>
<tr>
<td>6.00</td>
<td>0.016</td>
</tr>
<tr>
<td>5.00</td>
<td>0.031</td>
</tr>
<tr>
<td>4.00</td>
<td>0.063</td>
</tr>
<tr>
<td>3.00</td>
<td>0.125</td>
</tr>
<tr>
<td>2.00</td>
<td>0.25</td>
</tr>
<tr>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>0.00</td>
<td>1.0</td>
</tr>
<tr>
<td>-1.00</td>
<td>2.0</td>
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<tr>
<td>-2.00</td>
<td>4.0</td>
</tr>
<tr>
<td>-3.00</td>
<td>8.0</td>
</tr>
<tr>
<td>-4.00</td>
<td>16.0</td>
</tr>
<tr>
<td>-7.64</td>
<td>0.005</td>
</tr>
<tr>
<td>-3.74</td>
<td>0.075</td>
</tr>
<tr>
<td>-2.00</td>
<td>0.25</td>
</tr>
<tr>
<td>-1.25</td>
<td>0.42</td>
</tr>
<tr>
<td>-1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>-2.25</td>
<td>4.78</td>
</tr>
</tbody>
</table>

Fig. 3. Frequency distribution of 50% diameters in terms of $\phi$ scale
ground, these account for 8% of reclaimed ground. Therefore, it is also statistically clarified that a large amount of fine soil classified in silt erupted on reclaimed ground.

Relationship between percentages of fines content and clay content

Fig. 4 shows the relationship between the percentages of fines content and clay content of ejected soils. Fig. 4(a) shows ejected soils on reclaimed ground and Fig. 4(b) shows those on other kinds of ground. Ejected soils containing nearly 100% fines exist on reclaimed ground in figure (a), but the upper limit of fines on other kinds of ground is approximately 50%, and most are lower than 40%. Meanwhile, nearly all percentages of clay content of ejected soils are lower than 10%. Fig. 5 shows the relationship between the percentages of fines content and clay content of the upper and lower parts of the Yurakucho formation, a typical Holocene ground in Japan. This figure contains 2825 data from the upper part of the Yurakucho formation mainly consisting of sand and the lower part mainly consisting of clay. Despite the large dispersion in this relationship, the average relationship between them is nearly linear. A linear regression equation from these relationship is described as Eq.(2).

$$P_c = 0.4P_f \quad (r=0.71)$$

(a) Reclaimed ground

(b) Other kinds of ground

Fig. 4. Relationship between percentages of fines content and clay content of ejected soils

Fig. 5. Relationship between percentages of fines content and clay content of the upper and lower parts of the Yurakucho formation in Japan
where $P_f$ is the fines content in % and $P_c$ is the clay content in %. In general, clay exists in Holocene ground at a ratio of approximately 40% of the fines content. Accordingly, a fines content of nearly 100% and a clay content of less than 10% are a characteristic grain size distribution of ejected soil on reclaimed ground.

**PROPERTIES OF GRADATION CURVES OF EJECTED SOILS**

**Relationship between 50% diameter and central uniformity coefficient of ejected soils**

The uniformity coefficient of ejected soils is generally low, being below 10 and tends to increase to become well-graded as the grain size increases. The authors pay attention to 50% diameter and the steepest value of gradation curve as representative values of gradation curves and attempt to formulate a general gradation curve of ejected soils. The central uniformity coefficient newly defined in this paper is used instead of the uniformity coefficient. The central uniformity coefficient $U_{cc}$ is defined as Eq.(3).

$$U_{cc} = \frac{D_{60}}{D_{30}}$$  \hspace{1cm} (3)

where $D_{60}$ is 60% diameter in mm and $D_{30}$ is 30% diameter in mm. Assuming that the gradation curve is linear between 10% diameter and 60% diameter on semi-log graph paper, a uniformity coefficient of

Fig. 6(a) shows the relationship between the 50% diameter and Ucc of ejected soils. The Ucc values are small at the small 50% diameter part and increase as the 50% diameter increases. Most of Ucc values are smaller than 2.12, especially at 50% diameters smaller than about 0.2mm. The solid line in the figure represents the average of Ucc, which are calculated at equally divided intervals on the log axis. Fig. 6(b) shows the average and standard deviation of ejected soils. It is more clearly understood that most Ucc values are smaller than 2.12 when the 50% diameter is smaller than 0.2mm, and the average of Ucc increases linearly as the 50% diameter increases. Fig. 6(c) shows the relationship between the 50% diameter and Uc of Holocene ground based on the same database in Fig. 5. It is found that there is no correlation between D50 and Uc, and the relationship of ejected soil significantly differs from that of Holocene ground.

Since the average of Ucc in Fig. 6(b) may be regarded as two lines, the grain size distribution of ejected soil may be modeled as Eq.(4).

\[
U_{cc} = 1.5(0.02\text{mm} \leq D_{50} < 0.2\text{mm})
\]

\[
U_{cc} = 2.36D_{50}^{0.27}(0.2\text{mm} \leq D_{50} < 5\text{mm})
\]

(4)

Assuming that the gradation curve between D10 and D60 on semi-log graph paper is linear, the relationship between Ucc and Uc can be expressed as:

\[
U_{cc} = U_c^{0.6}
\]

(5)

Then Eq.(4) can be expressed as Eq.(6) in terms of a function of Uc.

\[
U_c = 2.0(0.02\text{mm} \leq D_{50} < 0.2\text{mm})
\]

\[
U_{cc} = 4.18D_{50}^{0.45}(0.2\text{mm} \leq D_{50} < 5\text{mm})
\]

(6)

Since most of the gradation curves of ejected soils are linear between the finer percentages of 30% and 80% on semi-log graph paper, the main body of a grain size distribution around 50% diameter can be modeled as Eq.(4) in terms of the trend of gradation curve. Therefore Eq.(4) is a grain size distribution model of ejected soil between the finer percentages of 30% and 80%.

Fig. 7 shows a comparison between gradation curves of the model of ejected soil and of the Technical Standards for Port and Harbour Facilities in Japan. The gradation curves for both Uc<3.5 and Uc\geq3.5 of the Technical Standards for Port and Harbour Facilities are shown in this figure. The models of ejected soil correspond to the Technical Standards in the case of Uc<3.5 on the finer side of the figure. On the other hand, the models of ejected soil correspond to the Technical Standards in the case of Uc\geq3.5 on the coarser side of the figure. As shown in the figure, Eq.(4) means that the grain size distribution is poorly-graded on the fine side and well-graded on the coarse side. The reason for this is considered as follows: When the grain size of general soil decreases, the soil contains much clay because of Eq.(2), having high cohesion, thereby achieving high liquefaction resistance. However, when the grain size of poorly-graded soil decreases, the liquefaction resistance dose not increase, because poorly-graded soil is scarcely cohesive with little clay. On the other hand, when the grain size increases, the liquefaction resistance increases because the drainability generally increases. However, in the case of well-graded soil,
liquefaction resistance does not increase even if the grain size increases, because of the low drainability due to fines present in well-graded soil.

**Limits of gradation curves**
In this section, the limits of gradation curves of ejected soils are statistically considered. Fig. 8 shows the frequency distribution of ejected soil. Each figure shows the frequency distribution of grain size in terms of the $\phi$ scale at each step of percentage finer, e.g., $D_{10}$, $D_{20}$, ..., $D_{90}$. The frequency distribution of $P=50\%$ in this figure is the same as Fig. 3. In this figure, dotted lines represent the minimum and maximum of the data, while broken lines represent the values $\pm 3\sigma$ away from the average of the data, and bold dotted curves express the normal distribution of the data.

Fig. 8(a) shows the histograms of ejected soils on reclaimed ground. The values of the maximum, $-3\sigma$, the average (the top of the normal distribution curve), the minimum in terms of $\phi$ scale increase (decrease in terms of grain size) as the percent finer decreases from 70% to 40%. However, the values of $+3\sigma$ remain unchanged even if the percentage finer varies between 20% and 90%, and are larger than the maximum (smaller than the minimum in terms of grain size). Since the frequency distribution dose not indicate the normal distribution at the large end of the $\phi$ scale (small grain end), an upper limit (a lower limit in terms of grain size) exists on the fine side of the grain size distribution. On the other hand, no obvious limit exists on the coarse side of the grain size distribution. Fig. 8(b) shows ejected soils on other kinds of ground. The tendencies of this figure are nearly the same as Fig. 8(a), but each value shifts toward the smaller side (coarser side in terms of grain size).

Based on the analysis mentioned above, proposed limits in the gradation curves of ejected soils are shown in Fig. 9. As stated above, gradation curves are generally linear on semi-log graph paper in the range of percentage finer from 30% to 80%, and the relationship between 50% diameter and the central uniformity coefficient can be modeled as Eq.(4). For this reason, the limits of ejected soils in the gradation curves on semi-log graph paper are assumed to be linear in the range of percentage finer from 30% to 80%, and the slope of each limit is calculated by the 50% diameter at the limits of gradation curves using Eq.(4). In this figure, the broken lines represent the Technical Standards for Port and Harbour Facilities in Japan [2] to compare the results with the current standard. From the consideration of Fig. 7, the Technical Standards for $U_c<3.5$ and $U_c\geq3.5$ are adopted on the fine and coarse sides, respectively. The limits from “a” to “e” are described as follows:

“a” is the minimum value observed in this study. This gradation curve is a singular instance, because only this curve is apart from other data. The reason for this singularity may be attributed to the fact that this curve was derived from reclaimed ground comprising volcanic ash. The possibility of liquefaction for very fine volcanic ash like this cannot be denied. The liquefiability of volcanic ash is a problem to be solved. Limit “a” was therefore indicated for reference.

“b” is the minimum value of ejected soil excepting “a.” No ejected soil finer than “b” is found except for such special soil as volcanic ash. It is thus considered that there is little possibility that soil finer than “b” liquefies.

“c” is the minimum value of ejected soil on the ground such as Holocene ground except reclaimed ground. It is considered that there is little possibility that soil finer than “c” liquefies in the ground other than reclaimed ground. Recently deposited ground, such as reclaimed ground, exists between “b” and “c.” For this reason, it is considered that the difference between “b” and “c” may be associated with the deposit time.
Fig. 8. Frequency distribution of ejected soil
“d” is the value $3\sigma$ away from the average. Most ejected soils are finer than “d.”

“e” is the maximum value observed in this study. Since the frequency distribution of ejected soils is similar to the normal distribution on the coarse side, the possibility of liquefaction for soil coarser than “e” cannot be denied. However, the probability of liquefaction of soil coarser than this is considered marginal. Most gradation curves between “d” and “c” represent soils ejected during the 1995 Hyogoken-nanbu earthquake, and samples within this range account for approximately 1% of all data. This earthquake was the type of direct hit earthquake. The seismic motion during this earthquake was very strong, recording 7 by JMA seismic intensity scale. Most liquefaction sites were reclaimed lands comprising decomposed granite soil, “Masa-do” in Japanese. Consequently, it is considered that the limit on the coarse side depends on the intensity of seismic motion or peculiarity of soil.

CONCLUSIONS

Ejected soils are non-plastic and of uniform grain size. Its clay content is lower than 10%, but its fines content widely ranges from 0% to 100%. The gradation curves of ejected soils are generally linear in the range of percentage finer from 30% to 80% on semi-log graph paper.

The relationship between the fines content and the clay content of ejected soils on reclaimed ground significantly differs from that of ejected soils on other kinds of ground. Whereas the fines content of ejected soils on reclaimed ground is widely distributed from 0% to 100%, that of soils ejected on other kinds of ground is limited to less than 50%. In addition, the clay content of all ejected soils is smaller than 10%. On the other hand, the relationship between clay content and fines content of Holocene ground is generally expressed as $P_c=0.4P_f$, which significantly differs from that of ejected soils.

The gradation curves of ejected soils are poorly-graded on the fine side and become well-graded as the grain size increases. For this reason, the relationship between the 50% diameter and the central uniformity coefficient, which is newly defined in this paper, is modeled as the grain size distribution of ejected soil. This relationship cannot be recognized in Holocene ground.

As the limits of gradation curves for ejected soils, the minimum value in this study for a special case, the minimum value, the minimum value excepting ejected soils on reclaimed ground, the limit values in which most of ejected soils distribute, and the maximum value are proposed.

Fig. 9. Limits in the gradation curves of ejected soil
Minimum limits of gradation curves for ejected soils exist in recently deposited ground such as reclaimed ground and in other kinds of ground such as Holocene ground. Therefore it is considered that the liquefaction possibility for soils finer than each limit is very low, excepting special soil like volcanic ash.

REFERENCES