SIMPLE SEMI-ACTIVE BASE ISOLATION USING
DISTURBANCE-ACCOMMODATING FREQUENCY-SHAPED
SLIDING MODE CONTROL

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SUMMARY

The base isolation of structures has received much attention and its effectiveness and usefulness have already been made clear for large earthquakes. In recent years, further improvement of base isolation performance has been demanded to cope with small and middle classes of earthquake, various restrictions and so on. In this study, in order to make base isolation system more applicable to various situations, a semi-active base isolation system designed by a new sliding mode control method is presented.

First, a multi-storied structural model with an active base isolator is made to examine the fundamental control characteristics and the optimal parameters for design. Second, a new method of the disturbance-accommodating sliding mode control is presented. This method enables a controller to take account of disturbance dynamics, and it is possible to utilize effectively limited control force. Third, the switching hyper-plane of sliding mode control is designed by the disturbance-accommodating bilinear optimal control theory. Finally, the simple semi-active base isolation using the disturbance-accommodating sliding mode control is designed through a frequency-shaped design in the criterion function in order to maintain the high performance even if the damper coefficient is switched into only two values.

The reduction performance of the proposed method is verified through numerical simulations for a structure with a semi-active base isolation system subjected to various earthquakes. In the simulations, the characteristics of control equipment is supposed so that the semi-active damper has a time-delay and the maximum control input is limited to an arbitrary value. Moreover, as an earthquake input, Hanshin-Awaji, Northridge, El Centro and Taft earthquakes are used respectively. As a result, the usefulness of the proposed method is demonstrated in comparison with the conventional methods.

INTRODUCTION

It has been well known that base isolation system is effective and useful for large earthquakes, particularly since the Hanshin-Awaji Earthquake occurred in Kobe City of Japan. Several types of dampers, steel, lead, oil, friction and so on have been utilized for base isolation system. For the first time in 2000, a semi-active

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oil damper was put into practice in the real building at Keio University in Japan. In this system, the coefficient of oil damper can be changed into 4 levels by manipulating on-off valve installed into the semi-active damper according to the bilinear optimal control theory which the author developed [1, 2]. This system is useful not only for large earthquake, but also middle size earthquake. Since each semi-active damper has on-off valves for four levels of damping coefficient and the total number of semi-active dampers is 8, however, semi-active base isolation system needs many relay circuits and it results in much cost. Therefore, it is desired to develop a simple mechanism of semi-active base isolation system.

In this study, in order to decrease the number of on-off valve for semi-active damper, it is considered to utilize two values of damping coefficient. Since the utilization of two values of damping coefficient is a kind of switching control, it is desirable to use the control method that is suitable for this kind of control. Hence, this study aims at establishing a simple semi-active base isolation system by developing a new method of disturbance-accommodating frequency-shaped sliding mode control. There are many studies on semi-active control for structural control [3, 4] and also there are many studies on the sliding mode control [5-7]. However, there are few studies to take into consideration the dynamics of disturbance and the theoretical bi-linearity of semi-active control. In this study, both of these characteristics are taken into consideration. Furthermore, in order to realize the simplicity of mechanism and the robustness of control, frequency-shaped hyper-plane is utilized in the sliding mode control. In order to investigate the effectiveness of the method, computer simulations are carried out by using the seismic waves of the recorded earthquakes.

**MODEL OF BASE ISOLATION SYSTEM**

In this study, 10 stories of building is dealt with as an object of base isolation as shown in Fig.1. Although multiple semi-active dampers are actually distributed in building, their characteristics are represented by single Maxwell model expressed by cascade combination of spring and damper in Fig.1. Since the semi-active oil damper used for building is a large scale of damper, it has delay property of response, which is modeled as a first order system of which transfer function is written as follows:

\[
G = \frac{1}{T \cdot s + 1},
\]

where \( T \) is a constant that is determined from the delay property as shown in Fig.2. The state equation of the base isolation system is formulated as follows:

\[
\dot{x}_p = A_p x_p + B_{wp} \begin{bmatrix} x_1 - x_0 \end{bmatrix} u + B_{wp} z
\]

\[
x_p = \begin{bmatrix} x_{10} & x_9 & \cdots & x_1 & x_0 \end{bmatrix}^T, \quad u = c^{-1}
\]

\[
A_p = \begin{bmatrix}
0_{10 \times 11} & I_{10 \times 10} \\
0_{1 \times 11} & 0_{1 \times 10} \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}, \quad B_{wp} = \begin{bmatrix} 0_{1 \times 10} \\
B_0 \end{bmatrix}, \quad z = \begin{bmatrix} \ddot{z} \\
k
\end{bmatrix}
\]

where \( x_p \) indicates the state variable vector of the building, the suffix of the element denotes the story of the building as shown in Fig.1, \( z \) denotes the displacement of ground and \( c \) denotes the damping coefficient of the semi-active damper which can be switched into two values. \( M, C \) and \( K \) are respectively mass matrix, damping matrix and stiffness matrix and their elements denote the corresponding parameters.
DESIGN OF DISTURBANCE-ACCOMMODATING FREQUENCY-SHAPED SLIDING MODE CONTROLLER

In the design of controller, the stiffness property of Maxwell model is ignored and the state equation is written by

\[
\dot{x}_s = A_x x_s + B_{ux} X u + B_{ux} \dot{z}
\]

\[
x_s = [x_{10} \cdots x_i x_0 \dot{x}_{10} \cdots \dot{x}_i]^T.
\]

Although the dynamics of earthquake disturbance is taken into consideration in the design, by considering the robustness against variation of earthquakes the acceleration of earthquake is assumed to be flat in the main frequency range as shown in Fig.3 and the frequency characteristics of the shaping filter of which output possesses the power spectral density as shown in Fig.3 is given by the transfer function:

\[
\frac{\tilde{Z}(s)}{W(s)} = \frac{\omega_d^2 s^2}{s^2 + 2\zeta_d \omega_d s + \omega_d^2}\frac{s^2}{s^2 + 2\zeta_h \omega_h s + \omega_h^2}.
\]

The state equation of the shaping filter is written as follows:

\[
\dot{z}_{	ext{dis}} = A_{	ext{dis}} z_{	ext{dis}} + D_{	ext{dis}} w
\]

\[
z_{	ext{dis}} = \begin{bmatrix} z \\ \dot{z} \\ \ddot{z} \end{bmatrix}, \quad A_{	ext{dis}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D_{	ext{dis}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_d^2 \end{bmatrix}
\]

\[
e = \omega_d^2 \omega_d^2, \quad d = 2\omega_d \omega_d (\zeta_d \omega_d + \zeta_d \omega_h)
\]

\[
c = \omega_d^2 + 4\zeta_h \omega_h \omega_d + \omega_h^2, \quad b = 2(\zeta_d \omega_d + \zeta_d \omega_h)
\]

where the input white noise has the following property.

\[
E[w(t)] = 0 \quad E[w(t)w^T(t)] = W(t)\delta(t - \tau)
\]

\(E\) denotes the mathematical expectation and \(\delta(t)\) is Dirac's delta function.
The augmented state equation of Eqs.(3) and (5) is rewritten by
\[ \dot{x}_m = A_{sm} x_m + B_{sum} X_m u + B_{wam} w \]
(6)

By considering the weight on the damping force generated by semi-active damper and frequency-shaped weights on the displacement and the acceleration of building, the following criterion function is adopted in this study.
\[
J(u) = \int_{-\infty}^{\infty} \left[ y^*(s) G_a(s) y(s) + \ddot{y}^*(s) G_a(s) \dot{y}(s) + u^*(s) X_m^T(s) R_m(s) X_m(s) u(s) \right]_{s=j\omega} d\omega
\]
(7)

where the asterisk denotes the conjugate transposed matrix and \( G_a(s) \) and \( G_b(s) \) are the following transfer functions.
\[
G_a(s) = \frac{\alpha \omega_0^2}{s^2 + 2\zeta_\alpha \omega_0 s + \omega_0^2}
G_b(s) = \frac{\beta s^2}{s^2 + 2\zeta_\beta \omega_b s + \omega_b^2}
\]
(8)

If we adopt the optimal bilinear control gain which minimizes the above equation as the gradient matrix of switching hyper-plane, we have
\[
S(x) = (X_m R_m)^{-1} B_{sm}^T P_m
\]
(9)

where \( P_m(t) A_m + A_m^T P_m(t) - P_m(t) B_m R_m^{-1} B_m^T P_m(t) + Q_m = 0 \). \( Q_m \) is easily obtained from Eq.(6). In the design of sliding mode controller, boundary layer \( \phi \) is utilized. Then, the control input is chosen as follows:
\[
u^0 = \begin{cases} u_e & |\sigma| > \phi \\ u_a & |\sigma| \leq \phi \end{cases}
\]
(10)

In the above equation \( u_e \) is a general sliding mode control input and \( u_a \) is given by
\[
u_a = -Kx = -(k_1 x_1 + k_2 x_2 + \ldots + k_n x_n)
\]
(11)

\[
k_i = \frac{1}{2\phi} \left[ \left( \phi + \beta_i(x) \right) \left( \sup \alpha_i + \delta_i \right) + \left( \phi - \beta_i(x) \right) \left( \inf \alpha_i - \delta_i \right) \right]
\]
(12)
where \( \beta_i(x) = \sigma S(x) B_{\text{asym}} X_i x_i = S(x) x_m^r S(x) B_{\text{asym}} X_i x_i \) 

(13)

If it is assumed that \( S(x) B_{\text{asym}} X_i \neq 0 \),

\[
\alpha = (S(x) B_{\text{asym}} X_i)^{-1} (S(x) A_{\text{asym}})
\]

(14)

The speed of convergence to the switching hype-plane is adjusted by \( \delta \). And the switching rule of damping coefficient of semi-active damper is given by

\[
C_x = \begin{cases} 
C_{\text{max}} & \left(u^0 \geq \frac{(C_{\text{max}} + C_{\text{min}})}{2}\right) \\
C_{\text{min}} & \left(u^0 < \frac{(C_{\text{max}} + C_{\text{min}})}{2}\right)
\end{cases}
\]

(15)

**COMPUTER SIMULATIONS**

In order to investigate the effectiveness of the disturbance-accommodating frequency-shaped sliding mode control for a simple semi-active control of base isolation system of building, computer simulations were carried out. As above-mentioned, although the delay characteristics of the semi-active damper is ignored in the design of controller, in the computer simulations they are taken into consideration and the influence of delay property on the control performance was examined as the robustness performance of the controller in the wide sense. As an earthquake input, Hanshin-Awaji, Northridge, El Centro and Taft earthquakes were used respectively.

Figure 5 shows the result of the acceleration at the 10th floor of the building subjected to Hanshin-Awaji earthquake input for the cases of the proposed method of frequency-shaped sliding mode control and the not frequency-shaped one. It is seen that the proposed method is superior to the not frequency-shaped on-off control in the reduction of acceleration response. Figure 6 shows the corresponding result of the displacement at the first floor. The response of the semi-active control is nearly equal to the passive one. It is seen from these results that the frequency-shaped sliding control method is effective in comparison with the ordinary disturbance-accommodating sliding mode control.

![Acceleration at the 10th floor under Hanshin-Awaji earthquake](image)
(a) Frequency shaped + on-off damping  
(b) on-off damping  
Fig. 6 Displacement at the first floor under Hanshin-Awaji earthquake

(a) Frequency shaped + on-off damping  
(b) on-off damping  
Fig. 7 Damping coefficient change of the base isolation system under Hanshin-Awaji earthquake

Table 1 Responses of maximum acceleration at the 10th floor under various earthquakes for case of variation of structural stiffness $[\text{m/s}^2]$

<table>
<thead>
<tr>
<th></th>
<th>Kobe $\downarrow K=+30%$</th>
<th>El Centro $\downarrow K=-30%$</th>
<th>Northridge $\downarrow K=+15%$</th>
<th>Taft $\downarrow K=-15%$</th>
<th>Hachinohe $\downarrow K=+0%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DA-SMC (Fre + on-off damping)</td>
<td>2.73</td>
<td>0.67</td>
<td>2.76</td>
<td>0.47</td>
<td>0.87</td>
</tr>
<tr>
<td>DA-SMC (on-off damping)</td>
<td>4.28</td>
<td>0.89</td>
<td>4.51</td>
<td>0.49</td>
<td>1.01</td>
</tr>
<tr>
<td>DA-SMC (normal)</td>
<td>3.07</td>
<td>0.84</td>
<td>3.99</td>
<td>0.50</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Table 2 Responses of maximum displacement at the first floor under various earthquakes for case of variation of structural stiffness $[\text{m}]$

<table>
<thead>
<tr>
<th></th>
<th>Kobe $\downarrow K=+30%$</th>
<th>El Centro $\downarrow K=-30%$</th>
<th>Northridge $\downarrow K=+15%$</th>
<th>Taft $\downarrow K=-15%$</th>
<th>Hachinohe $\downarrow K=+0%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DA-SMC (Fre + on-off damping)</td>
<td>0.28</td>
<td>0.13</td>
<td>0.43</td>
<td>0.08</td>
<td>0.18</td>
</tr>
<tr>
<td>DA-SMC (on-off damping)</td>
<td>0.26</td>
<td>0.14</td>
<td>0.37</td>
<td>0.09</td>
<td>0.18</td>
</tr>
<tr>
<td>DA-SMC (normal)</td>
<td>0.27</td>
<td>0.13</td>
<td>0.40</td>
<td>0.07</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Fig. 8 Responses under various earthquakes for the variation of structural stiffness
Figure 7 shows the time histories of damping coefficient of the semi-active base isolation system. It is demonstrated that high frequency switching is reduced in the frequency-shaped sliding mode control.

The responses of maximum acceleration at the 10th floor under various earthquakes are shown in Table 1 for case of variation of structural stiffness. And the responses of maximum displacement at the first floor under various earthquakes are shown in Table 2 for case of variation of structural stiffness.. In these tables, the normal DA-SMC means the disturbance-accommodating sliding mode control without frequency-shaped characteristics. It is seen from these tables that the disturbance-accommodating frequency-shaped sliding mode control method has the robustness against the variation of structural stiffness.

Figure 8 shows the responses at the 10th floor under various earthquakes for the variation of structural stiffness. The reduction rate of acceleration of the method depends on the input earthquake. As a whole, it is demonstrated that the proposed method is superior to the not frequency-shaped on-off semi-active damper and the normal continuous sliding mode control in both the reduction rate and robustness.

CONCLUSIONS

This study considered a structural control problem for simple semi-active control system of structural base isolation and it was devised to utilize on-off dampers as semi-active dampers. As a control method suitable for the mechanism of semi-active base isolation system, the disturbance-accommodating frequency-shaped control method was presented in this study. In order to investigate the effectiveness of the proposed method, computer simulations were carried out by using a ten degree-of-freedom structural model and various recorded earthquakes. As a result, it was demonstrated that the present method has the effectiveness and the robustness.

REFERENCES