



## **ASSESSMENT OF MODAL PUSHOVER ANALYSIS PROCEDURE AND ITS APPLICATION TO SEISMIC EVALUATION OF EXISTING BUILDINGS**

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### **SUMMARY**

Nonlinear static analysis procedures (or pushover analyses with an invariant lateral force pattern) have been developed for routine application in the practice of performance-based earthquake engineering due to their conceptual simplicity and computational effectiveness. Nonlinear static procedures, however, are limited in their ability to consider higher mode effects and possible redistribution of inertial forces in a structure due to yielding. An improved static procedure termed the Modal Pushover Analysis (MPA) (Chopra and Goel, 2001) was developed to consider explicitly higher mode effects. The MPA Procedure assumes that uncoupling of modal responses for a building system is still valid in its inelastic stage. The seismic response of each mode is determined from pushing the structure to its modal target displacement with an invariant modal lateral force distribution. Overall building peak response is obtained by combining the seismic response of each mode per the appropriate modal combination rule. In the first part of this paper, the reliability and accuracy of the MPA procedure are evaluated through nonlinear static analyses and nonlinear response-history analyses (NL-RHA) of an existing 13-story symmetrical Steel Moment Frame building. Seismic response parameters including the magnitude and spatial distribution of plastic hinge rotations and story drift ratios are considered, and results are used as benchmarks to compare nonlinear static procedures. Three variations on the MPA procedure are studied to observe additional trade-offs between accuracy and practicality. In the second part of this paper, the MPA procedure is extended to evaluate asymmetric buildings in three dimensions with lateral force patterns including both lateral forces and torsional moments. The MPA procedure is implemented to assess an existing 15-story composite steel and concrete pier-spandrel building with an irregular plan configuration.

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## INTRODUCTION

Among many issues essential to the implementation of performance-based earthquake engineering, reliable and accurate estimation of seismic deformations in a structure is critical. Recent performance-based guidelines for rehabilitating existing buildings in seismic regions have developed the nonlinear pushover analysis to be a viable method to evaluate seismic performance of existing buildings. Several practical methodologies involving nonlinear pushover analysis using an invariant height-wise lateral force distribution, such as the FEMA356 Nonlinear Static Procedure (ASCE 2000), have routinely been used in structural engineering practice for estimation of global and local structural deformations. Static procedures, however, are limited in their ability to consider higher mode effects and possible redistribution of inertial forces in a structure due to yielding. To overcome some of these limitations, general methodologies have been proposed including a scheme using adaptive lateral force distributions (Bracci et al. 1997; Gupta and Kunnath, 2000) and another considering more than just the fundamental mode of vibration (Paret et al., 1996; Sasaki et al., 1998).

Chopra and Goel (2001) developed another improved static procedure considering the contribution of several modes of vibration, termed the Modal Pushover Analysis (MPA). In a parallel NSF-funded project, Degenkolb Engineers implemented this procedure to an existing 13-story steel moment resisting frame building (Yu et al. 2001). The relative accuracy of the MPA Procedure and the FEMA 356 Nonlinear Static Procedure (NSP) was evaluated through comparison with nonlinear response history analyses of this existing structure. The research findings are summarized in the first part of this paper. These nonlinear static procedures are restricted to planar (2D) analyses of building structures with plan symmetry. Analysis on this basis is strictly valid only for buildings with coincident centers of mass and rigidity. However, a great amount of existing buildings are plan asymmetric and thus, cannot be effectively represented by a 2D model. To evaluate plan asymmetric structures, a three-dimensional (3D) model is required to capture coupled lateral-torsional response. Some attempts have been made to apply simplified NSP methods with triangular force distribution to low- and mid-rise 3D structural models (Moghadam and Tso, 1996; Faella and Kilar, 1998). To evaluate the seismic behavior of complex tall asymmetric buildings with significant higher mode effects, a 3D nonlinear response history analysis is in principle the most rigorous method used to estimate building response, but it is often not practical and perhaps impossible with a very complex and large building. The MPA procedure becomes an attractive option because it achieves a satisfactory balance between accuracy and practicality. In the second part of this paper, the MPA procedure is first extended to three-dimensional models to incorporate lateral-torsional response and then implemented to assess the seismic performance of an existing 15-story plan-asymmetric building.

## DESCRIPTION OF SELECTED EXISTING BUILDINGS

### **13-story Symmetric Steel Moment Frame Building**

This 13-story steel Moment Resisting Frame building is located in the Southern San Fernando Valley. It was designed for standard office occupancy, and was built in 1975. It is 160 feet square in plan and approximately 191 feet tall. The lateral force resisting system consists of four identical steel moment frames around the perimeter of the building as shown in Figure 1. Typical framing consists of A36 wide-flange sections, with built-up box columns in the corners of the building. Typical floors consist of concrete fill on metal deck. The floor plan broadens at the first floor above grade to form a plaza level that terminates on three sides into a hillside. Reinforced concrete walls are provided around the perimeter at the basement level, and the building foundation system consists of piles, pilecaps, and grade beams. Because of its symmetry and regularity, this building is an excellent candidate for assessing the relative accuracy of the FEMA356 Nonlinear Static Procedure (NSP) and the MPA procedure as compared to

nonlinear response history analyses (NL-RHA). To further avoid unnecessary complication, ground motions recorded on typical firm soil sites are used.

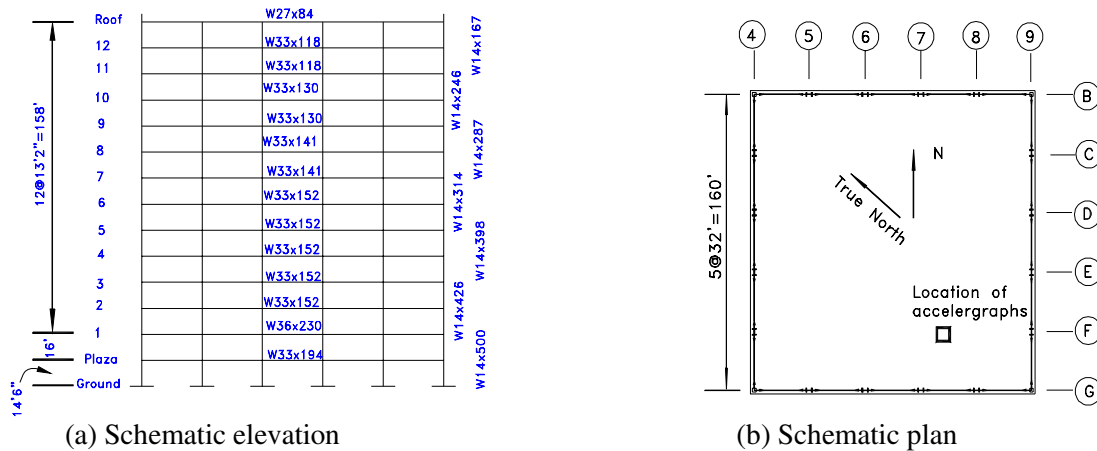


Figure 1 Schematic elevation and plan

### 15-Story Asymmetric Composite RC-Steel Pier-Spandrel Building

This 15-story composite steel and concrete building is located in Northern California. This building was originally designed in 1950, and it is in a cruciform shape (as shown in Figure 2) measuring 234 feet by 260 feet with a roof elevation of 195 feet. The building has a setback at the 7th floor. Figure 3 shows the building elevation at its north face. The building has a complete steel frame system. Typical of large buildings constructed during this era, the steel frame was completed first and followed with the concrete wall construction. The lateral force resisting system consists of two lines of composite pier-spandrel frames on the perimeter in each direction with composite shear walls (steel columns form the boundary elements) at the ends of the building (see Figures 2 and 3). The spandrels are typically 6-ft deep steel truss girders encased in 12-inch thick concrete (see Figure 4b). The concrete piers are typically T-shape in plan, with the building steel wide flange section column encased at the center (see Figure 4b). The gravity system consists of reinforced concrete slabs and steel beams encased in concrete supported on steel columns encased in concrete. Concrete used in these structural components is light-weight concrete. The piers on the building perimeter become solid normal-weight concrete retaining/shear walls at the basement level, which are supported by isolated spread footings located at the column gridlines. The soil at the site is classified as  $S_D$ . A Site Specific Response Spectrum for the BSE-1 earthquake level was developed in accordance with FEMA 356 Section 1.6.2.

This building was first evaluated with a linear dynamic analysis of a 3D model to determine its seismic performance in the beginning of 2002. One of the major deficiencies identified was that some of the composite piers were shear critical. Since a steel column is embedded inside the pier web, after the concrete develops diagonal cracks, the steel column would become effective and provide strength and significant energy dissipation. Therefore, to address this deficiency and better understand the actual behavior of the building with such composite piers, it is necessary to investigate the building deformation through the nonlinear analysis. Encouraged by research findings presented in Part I, Degenkolb Engineers (Yu et al. 2003) extended the original 2D-MPA procedure from the seismic evaluation of a planar structure to a 3D asymmetric structure, with the help of structural dynamics theory [e.g., Dynamics of Structures by Chopra (2001)], and then its seismic performance was determined using the 3D-MPA procedure.

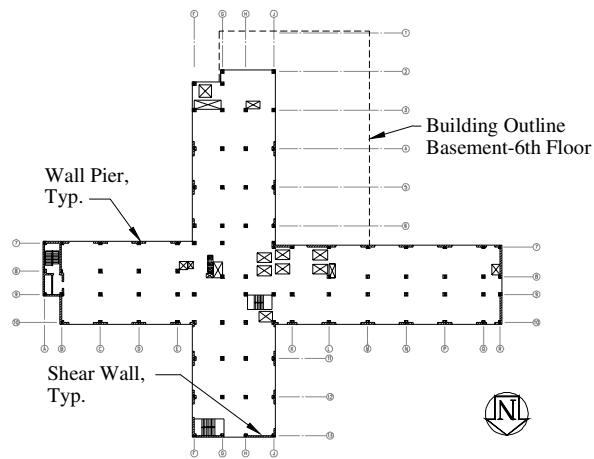


Figure 2 Building Plan View

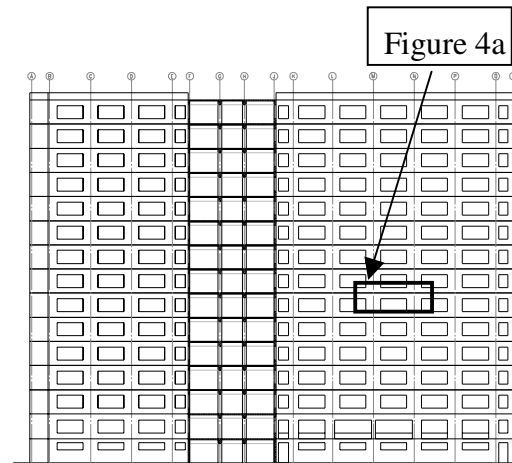
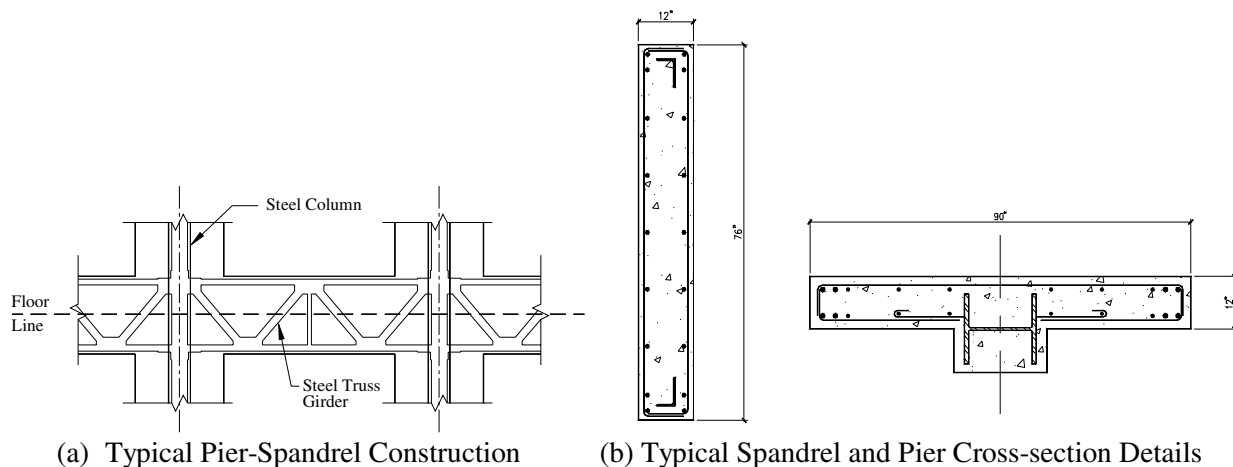


Figure 3 Building Elevation at Line 10



(a) Typical Pier-Spandrel Construction

(b) Typical Spandrel and Pier Cross-section Details

Figure 4 Pier and Spandrel Section Details

## PART I—ASSESSMENT OF THE 2D-MPA PROCEDURE

The 13-story steel Moment Frame building was analyzed using the DRAIN-2DX computer program (Prakash et al. 1993). A two-dimensional model is judged to be adequate due to its symmetric plan. The computer model consists of beam and columns modeled by DRAIN-2DX Element 2, 100% rigid-end offset, 2% strain hardening for the beams, the LRFD P-M interaction curve for column strength, panel zones (except at box columns) modeled as semi-rigid with DRAIN-2DX Element 4, a (vertical) line of fictitious columns linked to the moment frame for P-delta effects, and Rayleigh damping of 5% for the first and third modes. The columns are assumed to be fixed at the ground, with the plaza level modeled with infinitely stiff springs to prevent lateral translation. A yield strength of 47.3 ksi is used for the A36 steel beams and columns. Since the building was instrumented at the time of the 1994 Northridge Earthquake, the computer model was validated by comparing the calculated response to the measured response, and was judged to be a good representation of the actual building response (Uang et al. 1995). By calculation, the natural periods for the first three translational modes are 3.05, 1.06 and 0.63 seconds. Ground motions chosen for the nonlinear response history analyses are 20 accelerograms developed at a 2%/50 year hazard level (on soil type  $S_D$ ) for the Los Angeles basin used in SAC Phase II (Sommerville et al. 1997). The records have been scaled by a factor of 0.83 to better correlate with the 1997 NEHRP 2%/50 design spectrum, modified to soil type  $S_D$ .

## Nonlinear Static Analysis Procedures

### *Modal Pushover Analysis (MPA) Procedure*

In the MPA procedure, the seismic response of the building is determined by pushing the structure in each mode to its “modal” target displacement using an invariant “modal” lateral force distribution. Overall building response is obtained by combining the response in each “mode” by an appropriate modal combination rule so that higher mode effects are explicitly considered. The fundamental assumption of this procedure is that uncoupling and superposition of modal responses in an inelastic building system is accurate enough for approximation of peak building response, given the assumptions inherent in nonlinear static procedures. In this study, two modifications to the original MPA procedure were developed for determining the peak displacement in each mode in order to evaluate additional trade-offs between accuracy and practicality. These modifications represent a transition from “accurate”, at one extreme, to “practical”, at the other. The following process is used to evaluate the MPA procedure:

1. Construct a 2D model and compute the natural periods,  $T_n$  and modes,  $(\Phi)^n$ , for linear vibration of the building;
2. Develop the pushover curve for each of the first two modes using lateral force distributions equal to  $m_i(\Phi_i)^n$ , where  $m_i$  is floor mass at  $i$ th floor, and  $(\Phi_i)^n$  is the modal coordinate of  $n$ th mode at  $i$ th floor for linearly elastic vibration of the building;
3. Idealize the pushover curve for each mode into a bilinear curve, and convert it into a force-deformation relationship for an equivalent inelastic single-degree-of-freedom (SDOF) system (note that the bilinear relationship is target displacement dependent);
4. Determine the peak displacement of each modal SDOF system using each of the following methods;
  - Method 1: Accurate Approach (original MPA)  
A nonlinear response history analysis of each modal SDOF system is performed to calculate the peak displacement in each mode, the bilinear relationship is updated, and the nonlinear response history analysis is re-run until the solution converges. This calculation is repeated for each ground motion.
  - Method 2: Semi-accurate Approach (modified MPA)  
The peak displacement in each mode is taken as the median value of the displacement of the equivalent SDOF system for the 20 ground motions.
  - Method 3: Practical Approach (modified MPA)  
The peak displacement in each mode is estimated using the FEMA 356 coefficient method (with  $C_0$  be  $\Gamma_n \phi_m$  for the  $n$ th mode) and the median elastic response spectrum for the suite of ground motions.
5. Push the structure to the target displacement in each of the first two modes;
6. Determine the total peak response by combining the peak “modal” responses using an appropriate modal combination rule.

### *FEMA356 Nonlinear Static Procedure (NSP)*

In the FEMA356 NSP analysis, three patterns of lateral force distribution, (1) a triangular distribution proportional to  $m_j(h_j)^k$ , (2) a modal distribution proportional to the story shear calculated by combining modal responses capturing at least 90% participating mass, and (3) a uniform distribution proportional to the total mass at each level, are used to obtain a solution that bounds possible changes in lateral force distribution during nonlinear earthquake response. Because the force patterns are based primarily on the fundamental mode of response, the procedure is limited in its application when higher mode effects are significant. When the fundamental mode underestimates the story shear at any level by 30%, FEMA 356 requires the use of a supplemental dynamic analysis in addition to the NSP. For the building in this study, the first mode response underestimates the story shears in the top three stories, so higher mode response is significant. The target displacement is calculated using the coefficient method and the median response spectrum for the suite of 20 ground motions. The target displacement is compared to the peak roof

displacement determined using MPA, which is taken as the SRSS combination of the peak roof displacements in each mode.

## **Comparison of Calculated Seismic Response**

### *Assessment Methodology*

The building is first analyzed for the suite of LA 2%/50 ground motion records using nonlinear response history analyses (NL-RHA). Seismic response parameters considered in the NL-RHA included spatial distribution of the plastic hinge rotations (to reflect the present trend of comparing computed plastic hinge rotations against rotation limits established in FEMA 356) and spatial distribution of the story drift ratios. Taken as “exact”, the NL-RHA results are then used as benchmarks to evaluate the relative accuracy of the more “approximate” FEMA356 NSP and MPA procedures.

### *Record-by-record comparison of MPA Method 1, FEMA 356 NSP, and NL-RHA*

MPA Method 1 is intended to represent a more accurate approximation to NL-RHA at the expense of additional effort. In Method 1, the target displacement in each mode is determined by NL-RHA of an equivalent modal SDOF system, and overall building response is predicted by combining modal results. Because this process was repeated for each record, there are 20 values for each response parameter. Because the coefficient method is not applicable to individual records, the FEMA 356 NSP in this comparison is used with target displacements determined from MPA results. This results in three sets of values for each response parameter for each ground motion, and a record-by-record comparison between the methods can be made.

Due to space limitations, the results for LA 30, which produces a maximum SDR of between 3% and 4%, are shown in Figures 5 through 6 to illustrate typical observations. Figure 5 shows MPA Method 1 plastic hinge location and relative magnitude over the height of the building as compared to NL-RHA. MPA Method 1 appears to predict hinge location well, with some over-prediction in the magnitude of beam and column plastic hinge rotations. The FEMA 356 NSP appears to predict hinge location, but the magnitude of beam and column plastic hinge rotation is substantially larger than NL-RHA results. In Figure 6, the results are plotted numerically, showing the deviation from NL-RHA results. MPA Method 1 or FEMA 356 NSP values that match NL-RHA results will fall on the 45-degree line. Points above the line are conservative approximations while points below the line are unconservative approximations. Figure 6(a) illustrates the FEMA 356 NSP over-prediction of magnitude. The FEMA 356 uniform force pattern, which is intended to envelope changes in the dynamic characteristics of the system as yielding occurs in structure, overwhelms the results and is overconservative in predicting the building response. When the results from the uniform force pattern are taken out, the outliers are removed, and the MPA Method 1 and FEMA 356 NSP yield consistent deviations from NL-RHA results as shown in Figure 6(b). The MPA Method 1 overall more closely approximates the NL-RHA results for this building.

### *Comparison of MPA Method 2, FEMA 356 NSP, and NL-RHA*

MPA Method 2 is intended to represent an intermediate level of accuracy and effort. In Method 2, the target displacement for each mode is taken as the median value of the displacement of the equivalent SDOF system for the 20 ground motions. For the FEMA 356 NSP, the responses are calculated at a target displacement determined using the coefficient method and the median response spectrum for the suite of 20 ground motions. These response parameters are compared to the median (estimated by the geometric mean) of all values of the responses calculated by NL-RHA for the 20 ground motions.

Both MPA Method 2 and FEMA 356 NSP appear to predict panel zone hinge location and magnitude reliably. While both also appear to predict beam and column hinge locations fairly well, the FEMA 356 uniform force pattern again overwhelms the response prediction of the building as shown in Figure 7(a). Since the results without the uniform force pattern more closely approximate the NL-RHA results for this building [see Figure 7(b)], the uniform force pattern was not considered in the comparisons that follow.

However, Figure 7(b) shows that FEMA 356 NSP, even without uniform lateral force pattern, significantly over-predicts the magnitude. This is primarily caused by differences in the estimation of peak roof displacement. The target displacement estimated using the coefficient method (41 inches) is larger than the target displacement calculated from MPA (36 inches) and the median NL-RHA peak roof displacement (33.5 inches). To disintegrate the source of the difference, the maximum displacement from MPA Method 2 is used as the target displacement for FEMA 356 NSP. Figure 8(a) shows that when this portion of the difference is eliminated, the scatter in the results for beam and column plastic hinge rotation is reduced, as expected. It also can be shown that when the magnitude of plastic rotation is greater than 0.01 radians, both the MPA and FEMA 356 NSP provide conservative estimation of plastic rotation demand with comparable consistency. Similar observation can be made for the magnitude and spatial distribution of story drift ratio. Although MPA Method 2 somewhat underestimates story drift ratios in mid-upper stories where story drift ratios are typically less than 2% and beam plastic hinge rotations are less than 0.01 radian, overall it more closely approximates the NL-RHA results for this building [see Fig. 8(b)].

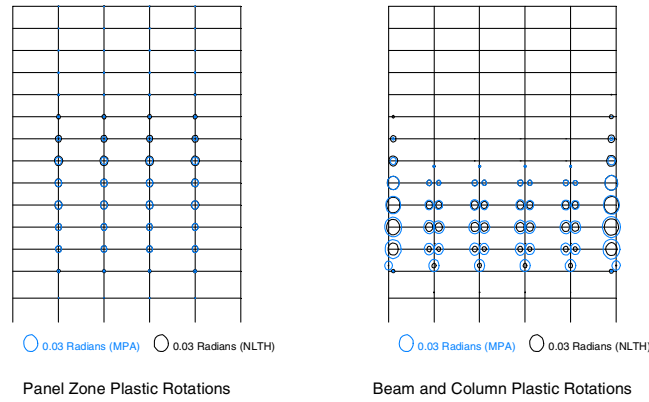
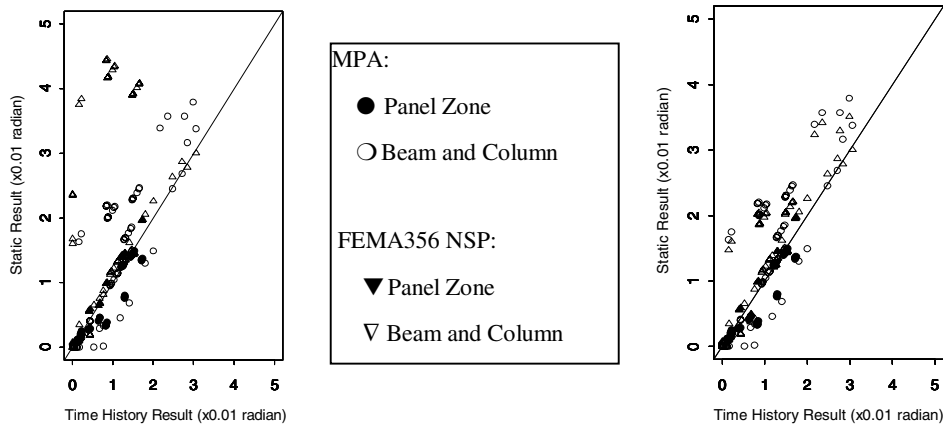


Figure 5. Plastic hinge locations – MPA Method 1 vs. NL-RHA



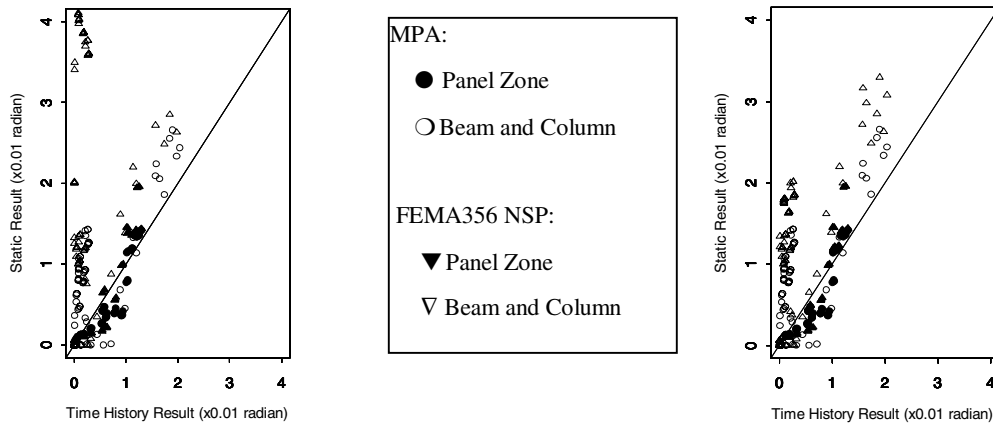
(a) NSP with uniform lateral force distribution (b) NSP without uniform lateral force distribution

Figure 6. Correlation of plastic hinge rotation - MPA Method 1, FEMA356 NSP and NL-RHA

#### Comparison of MPA Method 3, FEMA 356 NSP, and NL-RHA

MPA Method 3 is intended to represent a practical alternative for approximating the results of a NL-RHA for routine application by structural engineers. In Method 3, the target displacement for each mode is calculated using the FEMA 356 coefficient method on the median response spectrum for the suite of 20

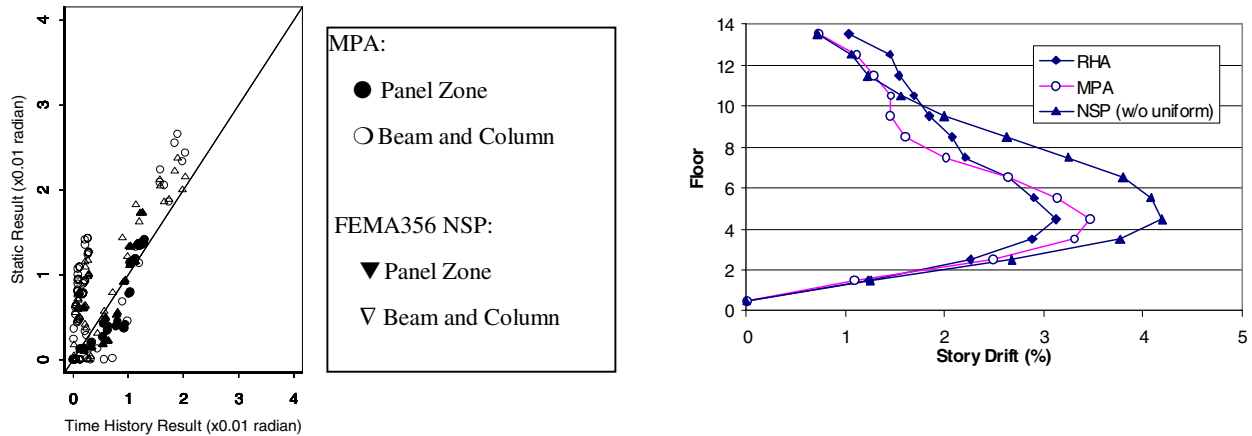
ground motions. The MPA Method 3 and the FEMA 356 NSP responses are compared to the median of all values of the responses calculated by NL-RHA for the 20 ground motions, and the uniform force pattern has been omitted.



(a) NSP with uniform lateral force distribution

(b) NSP without uniform lateral force distribution

Figure 7. Correlation of plastic hinge rotation - MPA Method 2, FEMA356 NSP and NL-RHA



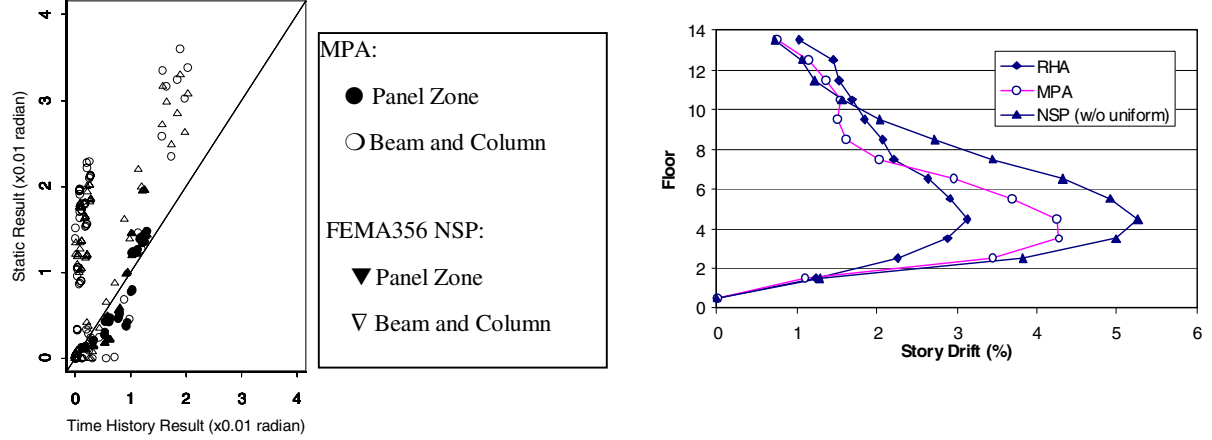
(a) Correlation of plastic hinge rotation

(b) Comparison of story drift ratio

Figure 8. Comparison of MPA Method 2, FEMA356 NSP (same displacement) and NL-RHA

The target displacements calculated for both the MPA Method 3 and FEMA 356 NSP are nearly identical, but larger than the median NL-RHA peak roof displacement. Differences from NL-RHA results can be partially attributed to these displacement differences, but any differences between MPA and FEMA 356 NSP are essentially due to pushover techniques alone. Both the MPA Method 3 and FEMA 356 NSP appear to predict the location and magnitude of panel zone plastic rotation fairly well. However, because of similar over-predictions in target displacement, both approaches over-predict the magnitude of beam and column plastic hinge rotations with comparable consistency [see Figure 9(a)]. In comparing Figures 7(b) and 9(a), the relative accuracy of MPA Methods 2 and 3 can be observed. With a reduction in effort, relative accuracy of the MPA procedure is reduced, yet with increased conservatism. Similar observation holds valid for magnitude and spatial distribution of story drift ratio through comparison of Figures 8(b) and 9(b). This can be used as a trade-off between level of effort and desired accuracy, and the MPA represents an effective alternative for the NSP to approximate full building NL-RHA results.





(a) Correlation of plastic hinge rotation (b) Comparison of story drift ratio  
Figure 9. Comparison of MPA Method 3, FEMA356 NSP and NL-RHA

This comparison study clearly shows that the MPA procedure was able to reliably predict spatial distribution of the plastic hinge rotations and spatial distribution of the story drift ratios. The MPA also provides further insight into the contribution of each mode to the total inelastic deformation of the structure under consideration.

## PART II — EXTENSION OF THE MPA PROCEDURE TO ASSESS 3D ASYMMETRIC STRUCTURE

### Basis of The 3D-MPA Procedure

For an  $N$ -story building shown in Figure 10, each floor diaphragm has three DOFs defined at the center of mass. The DOFs for the  $j$ th floor are: translation  $u_{jx}$  along the horizontal X-axis, translation  $u_{jy}$  along the horizontal Y-axis, and torsional rotation  $u_{j\theta}$  about the vertical Z-axis. The building is subject to two horizontal components [i.e.,  $u_{xg}(t)$  and  $u_{yg}(t)$ ] that are known at some point below the foundation level of the structure. The spatial variations in horizontal components of the ground motion cause the rotation (about the vertical axis) of the building's base,  $u_{\theta g}(t)$ . Such rotation input induces torsional motion of the building if the building has any purely torsional mode. The following equation governs the motions in  $3N$  DOFs of the system with rigid floor diaphragms.

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -\sum_{i=1}^3 [M]\{l\}_i \ddot{u}_{ig}(t) = -\{M\}_x \ddot{u}_{xg}(t) - \{M\}_y \ddot{u}_{yg}(t) - \{M\}_\theta \ddot{u}_{\theta g}(t)$$

where

$$[M] = \begin{bmatrix} M_x & & \\ & M_y & \\ & & I_\theta \end{bmatrix}, \text{ and } [M_x] \text{ and } [M_y] \text{ are diagonal sub-matrices of order } N, \text{ with } m_{jx} \text{ and } m_{jy}$$

the mass at the  $j$ th floor diaphragm in the X-direction and Y-direction, respectively, and  $[I_\theta]$  is a diagonal matrix of order  $N$  with  $I_{\theta j}$  the moment of inertia of the  $j$ th floor diaphragm about the vertical axis through the center of mass.

$\{u\}$  is the floor displacement vector of order  $3N$  and expressed as  $\{u\} = \begin{Bmatrix} u_x \\ u_y \\ u_\theta \end{Bmatrix}$ , where

$$\{u_x\} = \langle u_{1x} \quad u_{2x} \quad \dots \quad u_{Nx} \rangle^T, \quad \{u_y\} = \langle u_{1y} \quad u_{2y} \quad \dots \quad u_{Ny} \rangle^T, \text{ and } \{u_\theta\} = \langle u_{1\theta} \quad u_{2\theta} \quad \dots \quad u_{N\theta} \rangle^T.$$

$\{l\}_i$  is the influence vector of order  $3N$  representing the displacements of the masses resulting from static application of a unit ground displacement in the “ $i$ ” directional degree-of-freedom. For example,  $\{l\}_x$  is a vector with ones in the X directional degrees-of-freedom and zeros in all other positions.

$$\{M\}_x = [M]\{l\}_x, \{M\}_y = [M]\{l\}_y, \text{ and } \{M\}_\theta = [M]\{l\}_\theta$$

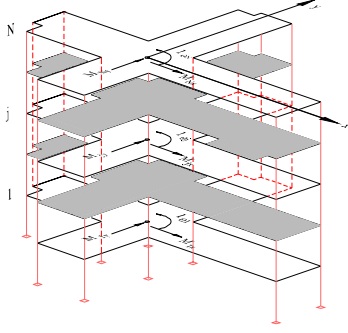


Figure 10  $N$ -story Asymmetric Building

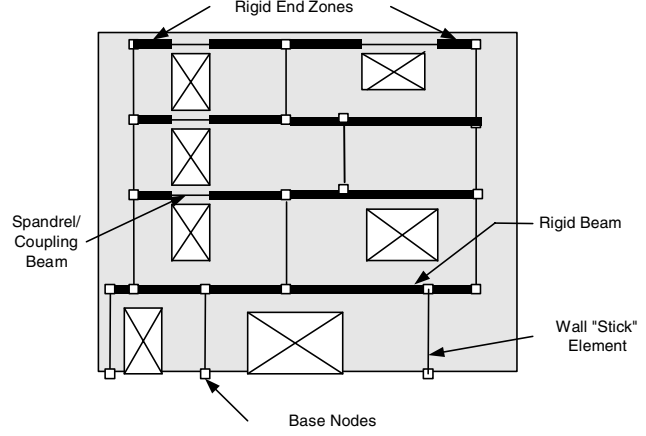


Figure 11 Perforated Wall Modeling

We can simplify the differential equation through a modal analysis using the mass- and stiffness-orthogonality property of modes, and the equation of motion for the  $n$ th-mode becomes

$$\ddot{q}_n(t) + 2\zeta_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) = -\Gamma_{nx} \ddot{u}_{xg}(t) - \Gamma_{ny} \ddot{u}_{yg}(t) - \Gamma_{n\theta} \ddot{u}_{\theta g}(t)$$

where

$$\Gamma_{ni} = \frac{L_{ni}}{M_n}, i = 1, 2, 3, L_{ni} = \{\phi_n\}^T [M] \{l\}_i, M_n = \{\phi_n\}^T [M] \{\phi_n\}, \{\phi_n\} \text{ describes mode shape for the}$$

$n$ th mode. If the translational response is coupled with the torsional response, the mode shape consists of non-zero components in three directions.

#### *Torsional Modes and Torsional Base Motion*

Components in X- and Y-directions of  $\{\phi_n\}$  are close to zero for a purely torsional vibration mode (or a torsional mode with minor lateral coupling). Accordingly,  $L_{nx}$  and  $L_{ny}$  are equal (or close) to zero. The governing equation for the  $n$ th-mode becomes

$$\ddot{q}_n(t) + 2\zeta_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) = -\Gamma_{n\theta} \ddot{u}_{\theta g}(t)$$

This equation indicates that a torsional base motion is required to excite purely torsional modes (or torsional modes with minor coupling). In structural engineering practice, the ground motion at any instant of time is assumed to be the same at all points of the foundation (i.e., the torsional base motion is simply not defined). Instead, such torsional excitation is indirectly accounted for through the inclusion of accidental torsion. Thus, purely torsional modes (or torsional modes with minor lateral coupling) are discarded in the MPA procedure. In the remainder of this paper, the torsional excitation is excluded from the discussion.

#### *Equivalent Static Forces and Effective Modal Masses*

Under horizontal seismic excitations, the spatial distribution of the effective earthquake forces  $[M]\{l\}_i$  can be expanded as a summation of modal inertia force distribution:

$$[M]\{l\}_i = \sum_{n=1}^{3N} \{s_n^i\} = \sum_{n=1}^{3N} \Gamma_{ni} [M]\{\phi_n\}$$

The contribution of the  $n$ th-mode to  $[M]\{\ddot{u}\}_i$  is

$$\{S_n^i\} = \Gamma_{ni} [M]\{\phi_n\}$$

The equivalent static forces associated with the  $n$ th-mode response are

$$\{f_n^i(t)\} = \Gamma_{ni} [K]\{\phi_n\} D_n(t) = \Gamma_{ni} [M]\{\phi_n\} A_n(t) = \{S_n^i\} A_n(t)$$

Thus, for the  $n$ th-mode, equivalent static force distribution  $\{f_n\}$  at the center of mass of the floors of the building is proportional to  $[M]\{\phi_n\}$ . The force distribution consists of components in three directions and includes not only lateral forces but also torsional moments. Such lateral force distribution is used to conduct a 3D pushover analysis and develop the base shear-roof displacement,  $V_{bn} - u_m$ , pushover curve for the  $n$ th mode. In the X-direction, the total resisting base shear for the  $n$ th mode is the sum of all the equivalent static forces in X-direction.

$$V_{nx} = \sum_{j=1}^N S_{jn}^x A_n(t) = \{\ddot{u}\}_x^T \{S_n^x\} A_n(t) = \{\ddot{u}\}_x^T \Gamma_{nx} [M]\{\phi_n\} A_n(t) = \frac{(L_{nx})^2}{M_n} A_n(t)$$

Thus, the effective modal mass in the X-direction for the  $n$ th mode can be defined as  $M_{nx}^* = \frac{(L_{nx})^2}{M_n}$ .

Similarly, the effective modal mass in the Y-direction for the  $n$ th mode can be defined as  $M_{ny}^* = \frac{(L_{ny})^2}{M_n}$ .

#### *Peak Roof Displacements of Inelastic MDOF System*

If the multistory building undergoes inelastic deformation, the governing equation becomes

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + \{R(\{u\}, \{\dot{u}\})\} = - \sum_{i=1}^2 [M]\{\ddot{u}_i\} \ddot{u}_{ig}(t)$$

The MDOF system can be decomposed into a number of  $3N$  inelastic SDF systems using the natural vibration modes of the corresponding linear system, but the modal coordinate  $q_n$  is found to be coupled with other modal coordinates, with the  $n$ th-“mode” typically dominant in the response to  $\{S_n^i\} \ddot{u}_{ig}(t)$ .

Therefore, it is reasonable to assume that uncoupling of modal responses is valid in the inelastic stage. With this assumption, the equation of motion for the  $n$ th-“mode” can be derived from the above governing equation for the MDOF system by utilizing the mass- and stiffness-orthogonality of the natural vibration modes of the corresponding linear system:

$$\ddot{q}_n(t) + 2\zeta_n \omega_n \dot{q}_n(t) + \frac{F_{sn}}{M_n} = -\Gamma_{nx} \ddot{u}_{xg}(t) - \Gamma_{ny} \ddot{u}_{yg}(t), n = 1, 2, \dots, 3N$$

where  $F_{sn}$  is a function of modal coordinate,  $q_n$ . For each specific translational mode, only the parallel ground motion significantly contributes to the structural response. Therefore, for ease of discussion, only the X-direction ground motion is used in the following discussion. With  $q_n(t) = \Gamma_{nx} D_n(t)$ , the governing equation becomes

$$\ddot{D}_n(t) + 2\zeta_n \omega_n \dot{D}_n(t) + \frac{F_{sn}}{L_{nx}} = -\ddot{u}_{xg}(t), n = 1, 2, \dots, 3N$$

This governing equation implies that the  $n$ th mode inelastic SDOF system is characterized with (1) natural frequency of  $\omega_n$  and damping ratio  $\zeta_n$ , (2) unit mass; and (3)  $F_{sn}/L_{nx} - D_n$ , the resisting force-displacement, relation for the  $n$ th -“mode”. This resisting force-displacement relationship can be obtained from a pushover analysis for the  $n$ th-mode by utilizing:

$$\frac{F_{sn}}{L_{nx}} = \frac{V_{bn}}{M_{nx}^*} \quad D_n = \frac{u_m}{\Gamma_n \phi_m}$$

The peak value of  $D_n$  can be either determined by conducting a response history analysis of this inelastic SDOF system or estimated from a response spectrum using the  $R_y-\mu-T_n$  relationship (e.g., FEMA356 Coefficient Method). Then, the peak roof displacement for the  $n$ th mode is estimated using  $\phi_m \Gamma_{nx} D_n$ .

Kilar and Fajfar (2001) conducted an applicability study of pushover analyses for seismic evaluation of asymmetric structures, and found that the 3D pushover analysis is limited to a torsionally stiff building (i.e., the first two periods of vibration are predominantly translational). The static response of such a structure is qualitatively similar to the dynamic behavior. Thus, the MPA procedure is only applicable to a torsionally stiff structure.

### 3D Modal Pushover Analysis Procedure

The MPA procedure is summarized below in a series of steps used to estimate the peak inelastic response of 3D asymmetric, multistory buildings:

1. Construct a 3D model and compute the natural periods,  $T_n$  and modes,  $\{\phi_n\}$ , for linear vibration of the building;
2. Determine applicability of this procedure. The procedure is limited to a torsionally stiff structure. (i.e., the first two periods of vibration are predominantly translational) since the static response of such a structure is qualitatively similar to the dynamic behavior (Kilar and Fajfar 2001). Thus, consider only translational modes for the MPA discussed in steps 3 through 6 and ignore purely torsional modes (or torsional modes with minor lateral coupling).
3. For the  $n$ th-mode, apply the lateral force distribution  $\{S_n\} = [M]\{\phi_n\}$  at the center of mass of each floor to conduct a 3D pushover analysis and develop the base shear-roof displacement,  $V_{bn}-u_m$ , pushover curve. The lateral force distribution pattern,  $\{S_n\} = [M]\{\phi_n\}$ , consists of components in three directions and includes not only lateral forces but also torsional moments.
4. Idealize the pushover curve as a bilinear curve in accordance with the FEMA 356 procedure.
5. Convert the idealized pushover curve to obtain  $F_{sn}/L_{nx} - D_n$ , the resisting force-displacement, relation for the  $n$ th-“mode”, and calculate peak roof displacement  $u_m$  using  $\phi_m \Gamma_{nx} D_n$  for the  $n$ th-“mode” using the FEMA 356 coefficient method and the site-specific response spectrum.
6. From the pushover database (Step 3), extract values of desired responses  $r_n$  (floor displacements, story drifts, plastic hinge rotations, etc.) at the peak roof displacement  $u_m$  computed in Step 5.
7. Repeat Steps 3-6 for as many translational modes as required for sufficient accuracy.
8. Determine the total response (demand) by combining the peak “modal” responses using appropriate combination rule.

### Computer Model and Modeling Assumptions

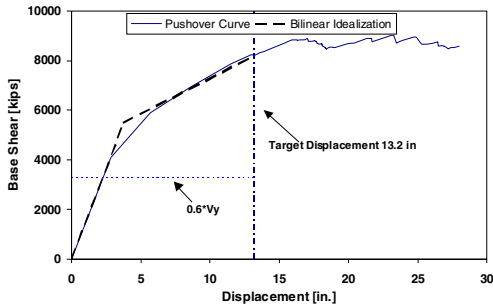
#### General

The 3D-MPA procedure is then implemented to analyze this 15-story composite pier-spandrel building using SAP 2000 (CSI 2002). Perimeter pier-spandrel systems and shear walls located on the ends of the building wings are modeled with frame elements, and the model is laid out on the centerlines of the piers and the floor elevations on the spandrels. 100% rigid end offsets are used to account for the stiffness of the panel zone. When there are openings in the shear walls, techniques shown in Figure 11 are used to simplify the computer model in lieu of using shell elements. Initial stiffness is modeled at 0.85 and 0.95 times the gross-sectional stiffness for the spandrel/pier and the shear wall, respectively (Yu et al. 2003). The M- $\phi$  diagrams are used to define the moment-rotation backbone curve for the spandrels. Both the Axial Force-Moment (P-M) diagrams and Moment-Curvature (M- $\phi$ ) diagrams are developed for the piers and composite shear walls to characterize their moment-rotation relationship. Nonlinear vertical soil springs are explicitly modeled to capture the rocking behavior occurring during uplift of the foundations. The diaphragm is assumed rigid and the seismic weight at each story is lumped at the center of mass

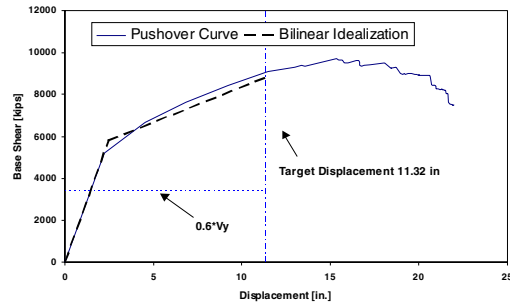
together with its mass moment of inertia. Light-weight concrete compression strength is modeled at 3000 psi and yielding strength of reinforcing steel (ASTM A15) and structural steel (ASTM A7) is taken to be 36.3 ksi.

### *Modeling of Composite Pier Shear Hinges*

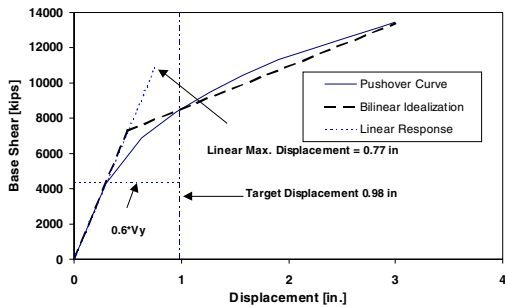
The shear capacity of the piers is the sum of the nominal shear strength of the structural shape plus the nominal shear strength that is provided by the tie reinforcement in the concrete encasement (AISC 1997). Once the shear force demand on the pier exceeds its shear strength, a pair of approximately 45° diagonal cracks are expected to form in an “X” shape under cyclic actions since the pier aspect ratio is approximately one-to-one. The steel section segment (embedded in the pier) near the mid-height would then act as a dowel. As it deforms under story shear, the trapezoid-shaped concrete at each side of the steel section is expected to separate from the steel section. Consequently, the steel section in the mid-height would behave much like a shear link with length approximately equal to the column depth. The majority of the pier shear deformation would concentrate at this location. Therefore, the pier shear hinge is modeled similar to a shear link per FEMA 356 Section 5.6.3. In accordance with the experimental tests by Roeder and Popov (1977), the axial effects on the deformation capacity of this short link are minor. Axial force in the pier is only accounted for in reducing the shear strength and flexural capacity.



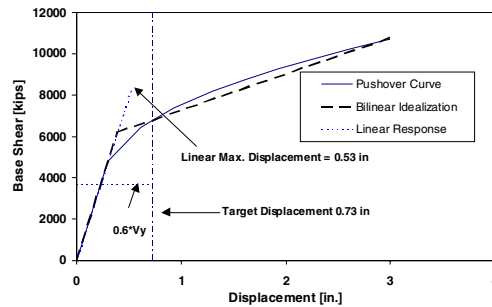
(a) Mode 1 (1<sup>st</sup> Mode in N-S direction)



(b) Mode 2 (1<sup>st</sup> Mode in E-W direction)



(c) Mode 4 (2<sup>nd</sup> Mode in N-S direction)



(d) Mode 5 (2<sup>nd</sup> Mode in E-W direction)

Figure 12 Pushover Curves for First two Translational Modes in Each Direction

## **Building Evaluation Results**

### *Dynamic Characteristics*

The fundamental periods of the building with expected soil spring stiffness are 1.07 sec and 0.95 sec in N-S and E-W directions, respectively. Since the first two periods of vibration are predominantly translational, the building can be classified as a torsionally stiff building, for which the MPA is a viable approach for seismic evaluation. Mode 3 is a torsional mode and, thus, ignored in the MPA procedure. Modes 4 and 5 are the 2<sup>nd</sup> translational modes with mass participation of above 15% and periods of 0.31

sec and 0.28 sec in the N-S and E-W directions, respectively. Due to the irregular plan configuration and setback at the 7<sup>th</sup> floor, the building fundamental translational modes are coupled with torsion. The lateral force patterns used in the MPA are calculated from the mode shapes, and include not only lateral forces in both horizontal directions but also torsion moments about the vertical axis.

### *Building Response*

Figure 12 shows the pushover curves of the first four translational modes (i.e., first two translational modes in each direction). The plastic hinge pattern of the building at the target displacement for each mode shows that many plastic hinges are formed at its target displacement in the pier-spandrel frames in the direction perpendicular to the primary vibration direction of that mode. This is partly due to inclusion of torsional moments in the lateral force patterns. Under lateral force patterns from higher modes, the plastic hinges are found to mainly concentrate at the building top stories. Plastic rotations from the pushover analyses of these four modes at their respective target displacement are finally combined with the SRSS combination rule and are compared with the performance acceptance criteria per FEMA 356. In addition, a series of parametric studies are conducted to investigate effects of soil variability and accidental torsion on the building effects. With explicit modeling of the actual behavior of the composite pier, together with this 3D-MPA procedure, the seismic performance of the building is effectively captured.

## **CONCLUSIONS**

This study first evaluated relative the relative accuracy of the FEMA 356 Nonlinear Static Procedure and the 2D-MPA using an existing 13-story SMRF in the Los Angeles region. Then, to evaluate tall complex buildings with higher mode effects, the 2D-MPA procedure was extended to three dimensional models and then implemented to evaluate an 15-story asymmetric building in Northern California. The following conclusions can be drawn:

- 1.The 2D-MPA procedure is a good predictor of building local (including plastic hinge rotation and story drift ratio) and global (target displacement) response.
- 2.The 2D-MPA procedure, with variations considered in this study, represents a practical alternative for approximating NL-RHA results. This procedure was able to reliably predict the NL-RHA response of the building. Since effort involved in this procedure is similar to that of the FEMA356 NSP, it is possible that the MPA could be incorporated into FEMA 356 as an alternative procedure to the existing NSP.
- 3.The MPA procedure that explicitly accounts for higher mode effects is further extended to analyze 3D models in order to evaluate a tall asymmetric building. Lateral force pattern for each mode is directly calculated from each mode shape, and it consists of lateral forces in both directions and torsional moments. Our case study involving an existing 15-story asymmetric building has demonstrated that this procedure is conceptually simple and can be easily implemented to evaluate the seismic performance of buildings using available commercial computer analysis programs.

## **ACKNOWLEDGMENTS**

The first part of this study is funded by the National Science Foundation under Grant CMS-9812531, as a part of the U.S.-Japan Cooperative Research in Urban Earthquake Disaster Mitigation. It is a result of discussion and collaboration with Anil K. Chopra and Chatpan Chintanapakdee of University of California at Berkeley, Rakesh K. Goel of California Polytechnic State University at San Luis Obispo, and Helmut Krawinkler of Stanford University. We also greatly appreciate the mentorship and technical

guidance provided by Jon Heintz and Chris Poland of Degenkolb Engineers. The second part of the study has benefited from discussions with Tom H. Hale, John D. Gillengerten, and Chris Tokas of the Office of Statewide Health Planning and Development (OSHPD), California.

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