DISPLACEMENT-BASED DESIGN OF CONCRETE TILT-UP FRAMES
ACCOUNTING FOR FLEXIBLE DIAPHRAGMS

Perry ADEBAR1, Zhao GUAN2, and Kenneth ELWOOD3

SUMMARY
This paper presents a simplified procedure to estimate the amplification of inelastic drifts in concrete tilt-up frames due to steel deck roof diaphragms designed to remain elastic during the design earthquake. The procedure assumes that roof diaphragm displacements relative to the ground are independent of wall strength, while roof diaphragm displacements relative to the walls are proportional to wall strength. Results obtained using this simplified procedure are in good agreement with results obtained from nonlinear dynamic analysis. A rational basis to decide when concrete tilt-up frames must meet seismic design requirements for cast-in-place frames is also presented.

INTRODUCTION
The technique of casting concrete panels flat and then lifting (tilting) them upright to form walls originated in California about 50 years ago as a method to construct solid reinforced concrete walls for industrial buildings. Today, tilt-up concrete walls are commonly used throughout the US and Canada to construct warehouses, shopping centers, office buildings, schools and many other types of buildings. The walls that are used in modern tilt-up buildings often have very large openings for windows and doors, resulting in wall panels that are actually multi-story frames.

Solid tilt-up wall panels or wall panels with small openings are inherently stiff and strong. The seismic design of these elements involves connecting enough wall panels together to prevent excessive rocking, and providing adequate connection between the wall panels and the roof diaphragm. Experience from recent California earthquakes has shown that the weak link is often the out-of-plane connection between the walls and the roof diaphragm, particularly timber roofs (Hamburger and McCormick [5]).

It is common for modern tilt-up buildings to have large openings along an entire side of the building, or on more than one side. This occurs, for example, when there are offices along the front of warehouses, and with store fronts in shopping centers. The concrete tilt-up frames around these openings must resist in-plane seismic forces in the same way as cast-in-place reinforced concrete frames; however, there are two significant differences between typical tilt-up frames and typical cast-in-place frames.

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The first difference is that the seismic design/detailing procedures used for tilt-up frames is much less stringent than procedures used for cast-in-place frames. Table 1 compares the Canadian code requirements for moderately ductile cast-in-place frames and common practice in western Canada for tilt-up frames (Weiler [9]). As a result of these differences, concrete tilt-up frames will have much less inelastic drift capacity than cast-in-place frames.

Table 1: Comparison of Canadian code requirements for moderately ductile cast-in-place frames and common practice in Canada for tilt-up frames.

<table>
<thead>
<tr>
<th>Issue</th>
<th>Canadian concrete code requirements for moderately ductile frames</th>
<th>Common practice in Canada for tilt-up frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inelastic mechanism</td>
<td>Plastic hinging limited to beams; columns must be about 20% stronger than beams.</td>
<td>Plastic hinging not restricted; normally in columns.</td>
</tr>
<tr>
<td>Columns</td>
<td>Anti-buckling ties (hoops) spaced at $6d_e$ at column ends.</td>
<td>No max. tie spacing; min. spacing equal to panel thick, regardless of $d_e$</td>
</tr>
<tr>
<td>Beams</td>
<td>Hoops spaced at $8d_e$ each end over length equal to twice beam depth.</td>
<td>Hoops spaced at $d/2$ (plastic hinging not expected).</td>
</tr>
<tr>
<td>Beam–Column Joints</td>
<td>Joint shear stress limited to $0.85 \sqrt{f_c'}$ (MPa); horizontal joint rein. designed for shear.</td>
<td>No joint shear stress limit; column ties provided.</td>
</tr>
</tbody>
</table>

The second significant difference between tilt-up and cast-in-place frames is that cast-in-place building systems include diaphragms that are essentially rigid, while tilt-up building systems often have flexible diaphragms. In western Canada, steel deck diaphragms are normally used, and these diaphragms are constructed in such a way that the diaphragms will likely remain elastic during the design earthquake. As a result of the diaphragm remaining elastic when the concrete walls yield, the tilt-up frames will be subjected to much larger drift demands than a linear analysis would suggest. In buildings with very large diaphragms, the inelastic displacements may be as much as 10 times the displacement determined by a linear analysis, and this increase must be accounted for.

The current paper presents a rational basis to decide when concrete tilt-up frames must meet the seismic design requirements for cast-in-place frames rather than common practice for tilt-up walls, and presents a simplified method to estimate the increased inelastic drift demands on concrete tilt-up frames due to flexible steel deck diaphragms.

**INELASTIC DRIFT CAPACITY OF TILT-UP FRAMES**

Tests on beam-column assemblies of typical concrete tilt-up frames have recently been conducted by Dew et al. [2]. Five tests were completed on three types of specimens with varying column tie spacing (100, 200, 300 mm). In all tests, the inelastic mechanism involved a plastic hinge in the column. A review of the original experimental data indicates the capacities summarized in Table 2.

Table 2: Measured capacities of tilt-up columns by Dew et al., [2].

<table>
<thead>
<tr>
<th>Column tie spacing (mm)</th>
<th>100</th>
<th>200</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Disp. Capacity (mm)</td>
<td>130</td>
<td>140</td>
<td>75</td>
</tr>
<tr>
<td>Inelastic Disp. Capacity (mm)</td>
<td>109</td>
<td>119</td>
<td>54</td>
</tr>
<tr>
<td>Inelastic Rotation(^\d) Capacity (rad)</td>
<td>0.040</td>
<td>0.044</td>
<td>0.020</td>
</tr>
</tbody>
</table>

\(^\d\) determined using an assumed hinge length of $h/2 = 200$ mm.
The columns had 20M (20 mm dia.) vertical reinforcing bars at the compression face that buckled at failure. Thus the tie spacing corresponds to $5d_h$, $10d_h$ and $15d_h$. The traditional tie spacing to prevent buckling is between $6d_h = 120$ mm and $8d_h = 160$ mm, thus the 100 mm spacing is clearly within this limit, and the 200 and 300 mm spacing clearly exceed the limit. This explains why the rotational capacities of the specimens with 200 and 300 mm tie spacing are considerably less.

The experimental results can be summarized very simply as the inelastic rotational capacity is 0.04 radians when the tie spacing is within the anti-buckling limit, and the inelastic rotational capacity is 0.02 radians when the tie spacing exceeds the anti-buckling limit (i.e., do not conform to the requirements for cast-in-place frames). No axial compression was applied in the tests because of limitations in the test set-up; however, typical columns in tilt-up frames are not subjected to high levels of axial compression. The addition of significant axial compression would reduce rotational capacities.

It is interesting to compare the results from the current tests with the acceptance criteria given in FEMA 356 for reinforced concrete columns. The inelastic rotation limit for collapse prevention performance level varies between 0.02 and 0.01 radians for columns with conforming ties, and between 0.01 and 0.005 radians for columns with nonconforming ties. The limits within FEMA 356 are intended to be conservative for columns with significant axial compression.

The inelastic drift capacity of the tilt-up frame (measured at the roof level) can be determined from the inelastic rotation capacity of individual panel components (columns and beams) using a push-over analysis of the particular wall panel. Often, the inelastic mechanism is hinging in the columns at the top of the lower level (the base is assumed to be pinned), in which case the inelastic drift capacity of the entire frame is simply the inelastic rotation capacity of the column (0.02 or 0.04 rad) times the effective height of the column (distance from the base to the center of the hinge).

2004 Canadian Concrete Code
The 2004 Canadian Concrete Code (Clause 21.7.1.2) requires that tilt-up wall panels with openings be designed to the requirements for Moderately Ductile Frames (ductility force reduction factor $R_d = 2.5$) when the maximum inelastic rotational demand on any part of the panel exceeds 0.02 radians, and in no case shall the inelastic rotational demand exceed 0.04 radians. As these are the first provisions to be incorporated in the code for tilt-up frames, the inelastic rotational limits were purposely made less restrictive. It is expected that these limits will be reduced somewhat in future editions of the code.

STEEL DECK DIAPHRAGMS

The roof diaphragms that were assumed in the current study correspond to typical western Canadian practice; Weiler [9]. The depth of the steel deck was 1½ in. (38 mm), and the joists were spaced at 6 ft (1.83 m). 22 gauge (0.76 mm), 20 gauge (0.91 mm) or 18 gauge (1.22 mm) were used depending on the size of the roof. Two different gauges were used for each roof size, with 20% of the roof area near the ends of the diaphragm (high shear zone) having the thicker gauge. The chord angles along the walls also depended on the roof size, and were either 3 x 3 x 1/4 in., 4 x 4 x 3/8 in. or 5 x 5 x ½ in. The side laps in the deck units were connected by button punching, and the deck was arc-spot welded to the frame. The spacing of the button punching and welding varied with the deck gauge. The mass on all roofs was assumed to consist of 20 psf (0.94 kPa) dead load, and 10 psf (0.47 kPa) live load (25% snow load).

In Canada, the in-plane stiffness of steel deck diaphragms is usually determined according to CSSBI [1] which is based on the procedures in the 1982 “Seismic Design for Buildings (Tri-Services Technical Manual).” The method assumes that a steel deck diaphragm is analogous to a plate girder having its web in a horizontal plane to resist the lateral forces applied to the building. The chord angles provided at the
edge of the deck are the flanges of the girder. The deflection of the steel deck diaphragm is assumed to be made up of a flexural component \( \Delta_f \), and a web (shear) component \( \Delta_s \). The flexural component is determined using conventional beam deflection formulae with the flexural rigidity \( EI \) calculated from the cross-sectional areas of the chord angles. The web deflection depends on the shear deformation of the steel deck, the flexibility of the deck-frame connections (arc-spot welds), and the amount of slip of the side lap connections between deck units. Table 3 summarizes the diaphragm deflections due to a lateral load equal to the dead load plus 25% snow load on the roof for some example roof sizes. In all cases shown in Table 3, the deck was assumed to be 20/22 gauge, and the chord angle had a cross-sectional area of 2.9 in.²

Table 3: Flexural and shear deflections in example roof diaphragms.

<table>
<thead>
<tr>
<th>Roof Size (W x L) (ft)</th>
<th>( \Delta_f ) (in.)</th>
<th>( \Delta_s ) (in.)</th>
<th>Building Period¹ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 x 100</td>
<td>0.23</td>
<td>0.85</td>
<td>0.42</td>
</tr>
<tr>
<td>100 x 200</td>
<td>3.60</td>
<td>3.41</td>
<td>0.85</td>
</tr>
<tr>
<td>300 x 300</td>
<td>6.07</td>
<td>7.67</td>
<td>1.16</td>
</tr>
<tr>
<td>200 x 300</td>
<td>9.11</td>
<td>7.67</td>
<td>1.26</td>
</tr>
<tr>
<td>100 x 300</td>
<td>18.22</td>
<td>7.67</td>
<td>1.52</td>
</tr>
</tbody>
</table>

¹ Calculated using Eq. [9].

A series of tests were conducted by Tremblay et al. [7] to investigate the ductility of steel deck diaphragms. They found that when side lap connections between deck units consist of button punches and the deck is arc-spot welded to the frame, the diaphragm fails in a very brittle manner. The residual strength reduces to zero with minimal energy dissipation. They recommend that such diaphragms be designed to remain elastic. The Canadian steel design code CSA S16.1-2001 requires a capacity design check to verify that the shear strength of steel deck diaphragms is greater than the actual lateral capacity of the vertical bracing system to ensure that inelastic action is avoided in the steel deck diaphragm.

**DYNAMIC ANALYSIS OF TILT-UP BUILDINGS**

A study was undertaken to investigate the behavior of some typical western Canadian tilt-up buildings in order to better understand these systems, and to develop a simplified procedure to estimate the inelastic demands on tilt-up frames. The buildings were assumed to be single-storey with steel deck diaphragms of varying sizes. Throughout this paper, roof length denotes span length of diaphragm or length of out-of-plane walls; and roof width denotes depth of diaphragm (plate girder) or length of in-plane walls. The simplified models used for each component of the tilt-up buildings are described below.

**In-plane Walls With Openings**

A typical wall panel with openings is shown in Fig. 1 (Weiler [9]). Sectional analysis of the beams and columns of this frame indicated that an effective flexural rigidity \( E_c I_c \) equal to 0.25 \( E_c I_g \) is appropriate to account for the reduction in stiffness due to flexural cracking. A pushover analysis of the frame indicated that the member shear forces are small enough to ignore the effect of shear cracking (Guan [4]).

A linear analysis of the frame, accounting for the joint size, indicates that the in-plane force at the level of the roof to cause a unit in-plane deflection at the frame is 8.46 kN/mm. Thus a single spring with a stiffness of 8.46 kN/mm can be used to represent the elastic response of the two-story frame in a simple model. Two other similar size wall panels with different opening sizes were also included in the current study and the corresponding stiffness of these is 1.27 and 13.47 kN/mm. The total mass of the wall panel shown in Fig. 1 is 16 tonnes, and the masses of the other two panels were 10 and 26 tonnes. In the single
spring model of the tilt-up panel, the distributed mass of the wall is represented by a lumped mass at the end of the spring equal to 50% of panel mass.

Out-of-Plane Walls
In a detailed model of tilt-up buildings, the out-of-plane walls could be modeled as vertical beams pin-connected to the floor slab and the roof diaphragm. Such a model would be necessary, for example, to study the out-of-plane connection force between the walls and roof diaphragm. As the natural frequency of the roof diaphragm spanning between in-plane walls is much lower than the natural frequency of the out-of-plane bending of the wall panels, the influence of the out-of-plane walls can be accounted for simply by adding half the mass of the out-of-plane wall to the roof diaphragm. In the current study, a mass of 890 kg/m was added for each out-of-plane wall.

Steel Deck Diaphragm
The level of complexity needed to model a steel deck diaphragm depends on the purpose of the analysis. Tremblay et al. [7] used a large number of truss elements with a stiffness degrading hysteresis response based on their shear element test results in order to simulate the nonlinear shear behavior of steel deck diaphragms. As the focus of the current study was to determine the influence of flexible roof diaphragms on the inelastic wall demand, a simpler model could be used. The roof diaphragm was modeled using the “Tri-Services plate girder analogy.” This approach was also used by Tremblay [8] to investigate the influence of flexible diaphragms on vertical bracing systems in one-story steel buildings. A model with two springs representing the tilt-up walls on either end of the diaphragm, and a beam (plate girder) representing the diaphragm is shown in Fig. 2(a).
Figure 3 shows the modal displacements for the 2005 NBCC spectrum (Vancouver Site Class E) of a tilt-up building with a roof that is 200 ft wide by 300 ft long. For this symmetric structure, only the symmetric (odd numbered) modes are present, and the first mode clearly dominates the response of the system. Linear and nonlinear time history analyses indicate that at the point of maximum displacement of the walls, the first mode clearly dominates. Thus certain aspects of the inelastic demands on the walls can be studied with an even further simplified model of the diaphragm. The first mode response of the diaphragm can be represented by a single spring as shown in Fig. 2(b). If the roof deflections are due to flexure, the mass participation in the first mode will be 81% of the total uniformly distributed mass. On the other hand, if shear deformations are significant, a larger portion of the uniformly distributed mass will participate in the first mode. Thus 100% of the roof mass $M_r$ was used in the simple two-spring model.

Results from the plate girder model of the diaphragm indicate that the large inelastic demands on the tilt-up walls occur when the roof span is large and the flexural deformations of the roof are dominate. Thus it is reasonable to develop a simplified model of the diaphragm by first converting the total first mode deformations of the diaphragm into an equivalent flexural mode. The stiffness of the spring is determined
so that the period of the spring, with a lumped mass equal to 100% of the roof mass, is equal to the first mode period of a simply supported beam with uniformly distributed mass, and with an equivalent flexural stiffness that results in the correct total (flexure plus shear) mid-span displacement:

\[ k_r = \frac{\pi^4 EI}{L^3} \left( \frac{\Delta_f}{\Delta_f + \Delta_s} \right) \]  

where \( EI \) is the flexural rigidity of the roof calculated from the chord angles, \( L \) is the span of the roof, and the typical ratios of flexural displacement to total displacement of the roof \( \Delta_f / (\Delta_f + \Delta_s) \) is shown in Fig. 4 for different roof spans and roof span-to-width ratios.

\[ \Delta f / (\Delta_f + \Delta_s) \]

**Fig. 4: Flexural portion of roof diaphragm deflections for different size steel deck diaphragms.**

**Nonlinear Dynamic Analysis**

The plate girder plus wall spring model shown in Fig. 2(a) and the two-spring model shown in Fig. 2(b) were used to conduct nonlinear dynamic analysis. In the first phase of the study (Guan [4]), five earthquake records were used. Two of the records, LA24 (1989 Loma Prieta) and SE32 (1985 Valparaiso – Seattle area), were from Phase 2 of the FEMA/SAC Steel Project, and were scaled to a 2% in 50-year probability of occurrence. The remaining three earthquakes VAN1, VAN2 and VAN3 were scaled to the 2005 NBCC design spectrum for Vancouver Site Class C (very dense soil).

In the second phase, four additional earthquake records, modified to fit the 2005 National Building Code of Canada design spectrum for Vancouver Site Class E (soft soil), were used. This includes two Loma Prieta records (LPEW and LPNS) and two San Fernando area records (SFCT316 and SFCT317). The acceleration spectrum and displacement spectrum for one of these records is shown in Fig. 5.

Computer program CANNY-99 (Li [6]) was used for all nonlinear analyses. 5% Rayleigh damping was used for the first two modes. The hysteresis model that simulated the pinched nonlinear behavior of the steel deck diaphragm was used in the two-spring model. A comparison of the theoretical hysteresis curves and the test results of Essa et al. [3] showed good agreement (Guan [4]). The nonlinear force-displacement relationships of the concrete tilt-up frames were simulated using a hysteresis model that assumes elastic-perfectly plastic behavior with the reloading slope reduced depending on the maximum displacement in the previous cycles.
Fig. 5: Acceleration and displacement spectra for earthquake record SFCT316 fit to 2005 NBCC Vancouver Site Class E design spectrum (shown as bold line).

ANALYSIS RESULTS: EQUAL ROOF DISPLACEMENT

Figure 6 shows a typical result obtained with the plate girder model of the roof diaphragm. The deflected shape of the linear structure at the instance of maximum demand is shown. That is, the maximum roof deflection/force and maximum wall deflection/force occurred simultaneously. The deflected shape of the same structure when the strength of the walls was limited to one-half the maximum elastic force demand is also shown in Fig. 6. It can be seen that the average deflection of the distributed mass on the roof diaphragm is approximately equal in the two cases.

Fig. 6: Typical maximum displacements in a tilt-up building with elastic walls (linear analysis) and inelastic walls (nonlinear analysis).

To further investigate this phenomenon, non-linear analyses were done using the two-spring model shown in Fig. 2(b) with the spring stiffnesses and the masses based on actual building properties. In these analyses, the ratio of roof mass to wall mass \((M_r/M_w)\) varied from 1 to 20, and the ratio of roof-spring stiffness to wall-spring stiffness \((k_r/2k_w)\) varied from 1 to 40. For each “building,” a linear time-step analysis and nonlinear time-step analysis was done. In the latter, the strength of the walls was set at half the maximum force during the corresponding linear analysis. That is, the nonlinear analysis was done for “buildings” with walls designed using a force reduction factor of 2.0. The maximum displacements of the roof masses during the nonlinear analyses and the maximum displacements of the roof masses during the linear analyses are compared in Fig. 7(a). In most cases, the roof displacements when the walls are inelastic were within 75 to 100% of the roof displacements determined from a linear analysis. That is, the roof displacements were generally smaller when the walls were inelastic.
Fig. 7: Comparison of maximum displacements determined from nonlinear analysis and linear analysis: (a) roof displacements, (b) wall displacements.

The maximum total displacements of the inelastic walls are compared with the wall displacements determined from a linear analysis in Fig. 7(b). In this case, the displacements of the inelastic walls are always larger than the linear analysis indicates. Depending on the flexibility of the diaphragm, the maximum displacements of the inelastic walls may be much larger than the elastic analysis suggests. While in reality the increase is a fixed amount related to the displacements of the diaphragm, if the increase is expressed as a ratio, the number would be very large for cases that the linear analysis indicates small wall displacements. This is consistent with the very large ductility ratios observed by Tremblay [8] in the analysis of vertical bracing in single-storey steel structures with flexible roof diaphragms.

SIMPLIFIED PROCEDURE TO ESTIMATE INELASTIC WALL DRIFT

The displacement of the walls can be calculated from the following expression:

$$\Delta_w = \Delta_r - (\Delta_r - \Delta_w)$$  \[2\]

where $\Delta_r$ is the deflection of the roof relative to the ground, and $(\Delta_r - \Delta_w)$ is the deflection of the roof relative to the top of the wall. The objective is to estimate the maximum $\Delta_w$ when the walls are inelastic. As discussed above, the maximum roof displacement determined from a linear analysis of the structure, denoted as $\delta_r$, can be used as a safe (larger) estimate of $\Delta_r$ at the instance of maximum inelastic wall displacement. Since the roof is assumed to remain linear, the displacement of the roof relative to the wall $(\Delta_r - \Delta_w)$ can be determined if the force in the roof is known.

The roof force at the instance of maximum inelastic wall displacement will be less than the maximum roof force when the walls are linear-elastic. Yielding of the walls will isolate the roof from the ground excitation and prevent significant force increase. A simple approach is to assume that the reduction in roof force due to yielding of the walls is proportional to the reduction in wall force. This can be expressed as:

$$\frac{V_{ri}}{V_{re}} = \alpha \frac{V_{wy}}{V_{we}}$$  \[3\]

where $V_{ri}$ is the force in the roof at the instance of maximum inelastic displacement of the walls, $V_{re}$ is the maximum force in the roof based on a linear analysis, $V_{wy}$ is the yield strength of the walls, $V_{we}$ is the...
maximum force in the walls based on a linear analysis, and $\alpha$ is a constant. Thus the actual displacement of the roof relative to the wall at the instance of maximum wall displacement is given by:

$$\Delta_r - \Delta_w = (\delta_r - \delta_w) \frac{V_r}{V_{re}} = (\delta_r - \delta_w) \frac{V_{wy}}{V_{we}}$$  \[4\]

where $\delta_r$ and $\delta_w$ are the roof displacement and wall displacement determined from a linear analysis.

Results from numerous nonlinear analysis using the two-spring model summarized in Fig. 8 for different wall types indicate that $\alpha$ varies from about 0.5 to 1.5, and that the average value of alpha is about 0.8. A value of $\alpha = 1.0$ was used for simplicity and to compensate for the difference between $\Delta_r$ and $\delta_r$. Thus the maximum (total) displacement of the inelastic walls can be estimated as:

$$\Delta_w = \Delta_r - (\delta_r - \delta_w) \frac{V_{wy}}{V_{we}}$$  \[5\]

Fig. 8: $\alpha$ factors in Eq. [3] determined from nonlinear analysis.

The total displacement of the inelastic walls is made up of an elastic portion, equal to the yield displacement of the wall, and an inelastic portion:

$$\Delta_w = \Delta_{wy} + \Delta_{wi}$$  \[6\]

The yield displacement of the wall is given by:

$$\Delta_{wy} = \delta_w \frac{V_{wy}}{V_{we}}$$  \[7\]

Substituting Eqs. [6] and [7] into Eq. [5] and rearranging, gives the following simplified expression for the maximum inelastic wall displacement:
The displacement of the roof diaphragm $\delta_r$ in Eq. [8] can be determined from a number of different linear analyses, such as the equivalent single degree of freedom model shown in Fig. 2(c), or can be determined directly from the spectral displacement $S_d$ if the fundamental period of the building is known. FEMA 356 suggests the following expression for the fundamental period of a one-story building with a single span flexible diaphragm:

$$T = \sqrt{0.1\delta_w + 0.078\delta_r}$$  \[9\]

where $\delta_w$ and $\delta_r$ are the in-plane wall displacement and mid-span roof diaphragm displacement in inches, due to a lateral load equal to the weight of the diaphragm. A comparison of the periods determined from Eq. [9] and the plate girder model shown in Fig. 2(a) indicates very good agreement. Table 3 presents some results obtained using Eq. [9] for some example buildings with different roof sizes and wall panels as shown in Fig. 1.

In practice, the ratio of wall strength to maximum elastic demand ($V_{wy}/V_{we}$) is determined from the force reduction factor $R$ used to design the walls and any over-strength of the walls, normally expressed as the ratio of nominal wall strength to factored load $\gamma_w$. The minimum value of $\gamma_w$ is the inverse of the resistance factor, and any factored resistance greater than the factored load will increase this ratio further. Thus the maximum inelastic wall displacement for a tilt-up building with a flexible diaphragm can be expressed directly as:

$$\Delta_{wi} = S_d(T)\left(1 - \frac{\gamma_w}{R}\right)$$  \[10\]

where $T$ is the fundamental lateral period of the building accounting for the flexible diaphragm, $R$ is the force reduction factor used to design the walls, and $\gamma_w$ is the nominal over-strength ratio.

The concept of estimating the inelastic displacement as the total displacement times $(1 - \gamma_w/R)$ is well known. The subtle but very important difference in Eq. [10] is that the inelastic displacement is in the walls only, while the total displacement $S_d(T)$ is of the wall and flexible diaphragm. According to Eq. [10], as the strength of the walls are reduced ($\gamma_w/R$ is reduced), the elastic displacements of both the walls and roof decrease, and the inelastic displacement of the walls must increase to make up the entire difference. This effect can be described as an amplification of the inelastic displacement in the walls due to the elastic diaphragm, as the wall strength is reduced.

**Unsymmetrical Structures**

Tilt-up buildings often have large openings on one side (e.g., front) of the building and fewer smaller openings along the opposite side (e.g., back) of building. Yielding of the wall panels will be limited to one side of the diaphragm in these unsymmetrical structures. The influence of walls yielding at only one end of the diaphragm was investigated using the beam (plate girder) model shown in Fig. 2(a).

When only one wall yields, the inertial force in the roof shifts away from the center of the roof – towards the yielding wall. As a result, the inertial force in the roof reduces for a given wall strength and the displacements of the roof correspondingly reduce. This effect can be accounted for in a simple rational way by reducing the $\alpha$ factor in Eq. [4].
For one example structure, the results from four different earthquake records fit to the same spectrum indicated that when only one wall yields, the maximum displacement of the inelastic wall varied from 70 to 180% (average of 140%) of the maximum wall displacement when both walls yielded simultaneously.

EXAMPLE

Figure 9 summarizes predictions from the simplified procedure for one-story tilt-up buildings with the wall panel frames shown in Fig. 1, and 300 ft long by 200 ft wide steel deck diaphragms were compared with predictions from nonlinear dynamic analysis using the beam (plate girder) model shown in Fig. 2(a). Eight buildings that were identical except for wall strength were investigated using the 2005 NBCC design spectrum for Vancouver – Site Class E. Two earthquake records fit to that spectrum were used to make the nonlinear analysis predictions.

Fig. 9: Comparison of results obtained from proposed simplified method and nonlinear dynamic analysis for two records fit to the same spectrum.
The simplified method predictions are shown as solid lines in Fig. 9, while the results from the nonlinear dynamic analysis using the plate girder model shown in Fig. 2(a) are shown as discrete points. In the simplified method, the displacement of the roof is assumed to be independent of the wall strength and is equal to $S_d(T)$, while the nonlinear analysis indicates that the roof displacement is influenced somewhat by the strength of the walls; see Fig. 9(a). In general, the simplified method predicts an upper-bound to the roof displacements.

The predictions of the inelastic wall displacements from Eq. [10] are compared with the nonlinear dynamic analysis results in Fig. 9(b). The two agree remarkably well. The inelastic displacements of the walls were used to determine the corresponding inelastic rotation in the columns, and these are shown in Fig. 9(c). For the frame shown in Fig. 1, the inelastic rotation (inelastic drift) is equal to the inelastic displacement divided by 2.55 m, which is the height from the column base to the center of the plastic hinge assumed to be 0.9/2 = 0.45 m down from the underside of the lower beams.

CONCLUDING REMARKS

The simplified method presented in this paper can be used to quickly predict the inelastic wall displacement in a variety of buildings with different size steel deck diaphragms. Figure 10 shows the results for three additional buildings with 100 ft wide roof diaphragms that have different spans. With the slender roof diaphragm, the inelastic displacements, and hence inelastic rotations of the tilt-up walls, increase very quickly if the walls are permitted to yield. Also shown in the figure (as dashed lines) are the results if the amplification due to the flexible diaphragm is ignored. That is, if the total displacement of the wall is assumed to be independent of the strength of the walls and equal to the value predicted by a linear analysis. The increase in inelastic rotation shown in Fig. 10(b) as dashed lines for that case is due only to the reduction in elastic wall displacements with reducing strength.

For each of the roof sizes shown in Fig. 10, and many other roof sizes, the elastic demand to wall strength ratio ($R/\gamma_w$) at which the inelastic rotation reaches the 0.02 rad and 0.04 rad limits given in the 2004 Canadian code are summarized in Fig. 11. That is, for values of $R/\gamma_w$ and roof span that plot to the left of the line for the particular roof width (W), the inelastic rotations will be less than 0.02 radian in Fig. 11(a) and less than 0.04 radians in Fig. 11(b).

The results in Fig. 11 indicate that in general, there are three ranges: (1) short roof diaphragm spans where the tilt-up frames do not need to be designed for significant inelastic rotations. In this case, common practice for tilt-up construction would suffice. (2) Long roof diaphragm spans where the tilt-up frames must be designed to remain elastic, and (3) intermediate roof diaphragm spans where an adequate combination of strength and ductility must be provided depending on the roof diaphragm dimensions.

REFERENCES

Fig. 10: Example predictions from proposed simplified procedure for different roof widths (w) and spans in ft.
Fig. 11: Example results from proposed procedure - relationship between elastic demand to wall strength ratio and roof span for different roof widths (W) in ft for: (a) inelastic rotation of 0.02 rad, and (b) inelastic rotation of 0.04 rad.