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Evaluation and Advancement of a Reinforced Concrete Beam-Column Joint Model

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SUMMARY

A previously proposed two-dimensional reinforced beam-column joint model (Lowes and Altoontash [1]) is evaluated through comparison of simulated and observed response for building sub-assemblages tested in the laboratory under pseudo-static simulated earthquake loading. An extensive set of experimental data that characterize the response of joints with details typical of post-1970's construction is compiled. The results of the evaluation include 1) a modification to the original model formulation that results in improved accuracy in response simulation and 2) identification of range of joint geometries and design parameters for which the model is appropriate.

INTRODUCTION

Experimental investigations and post-earthquake reconnaissance (EERI [2]) suggest that failure of reinforced concrete beam-column joints may result in structural collapse. Experimental investigation (Leon [3], Walker [4]) indicates also that joint deformation may significantly impact the global structural performance. Thus, it is necessary that inelastic joint action be simulated explicitly in predicting the response of reinforced concrete structures under earthquake loading.

Previous research has resulted in the development of a number of models for use in simulating the inelastic response of reinforced concrete beam-column joints. Most of these previously proposed models are relatively simple and required engineers to make significant assumptions about response or to have experimental data in hand for use in calibrating the models. Examples include the following:

- Plastic hinge models used to represent inelastic joint action as well as inelastic flexural response of frame member (e.g. Otani [5], Anderson and Townsend [6]).
- Empirically calibrated rotational springs placed between beams and columns (e.g. El-Metwally and Chen [7]).

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- Empirically calibrated rotational springs placed between beams and columns as well as rigid offsets used for beams and columns to simulate the flexural rigidity of the joint (e.g. Alath and Kunnath [8], Deng et al. [9]).
- Two rotational springs placed in series between beams and columns. One spring is calibrated to simulate the inelastic response of the joint core under shear loading and one is calibrated to simulate inelastic action due to anchorage failure. (Biddah and Ghobarah [10]). The material model for the joint shear spring was based upon the softened truss theory (Hsu [11]) and a hysteretic model for bond springs.

Beyond these relatively simple models, a few models have been proposed in which the response of the joint region is simulated using continuum-type elements and a relatively fine discretization of the joint region. While these models require far fewer assumptions by engineers and eliminate the need for experimental data, they are typically too computationally intensive and are not sufficiently robust for use in simulating the response of muti-story, muti-bay frames. Examples include the following:

- Elmorsi et al. [12] model the joint-core concrete with inelastic plane-stress elements, reinforcing steel with truss elements, concrete steel bond zones with discrete bond-link elements and the plastic hinge regions of the beams and columns adjacent to the joint with quadrilateral transition elements.
- Ziyaeifar and Noguchi [13] introduce transition beam-column elements to a joint element that include higher order shape-functions and are capable of representing shear distortion in the vicinity of the joint. Both material and geometric nonlinearity is simulated in this model.
- Fleury et al. [14] use typical plane-stress elements to represent the joint core concrete and the smeared effect of transverse reinforcement, a mesh of quadrilateral elements for beam reinforcement steel, truss elements for column steel, and transition elements to connect the joint element to the beam and column elements.

In order to strike a balance between model objectivity, accuracy and computational efficiency, Lowes and Altoontash [1] developed a beam-column joint super-element that explicitly accounts for the mechanisms that determine response of beam-column joints observed in laboratory. This model provides a means of simulating the response of joints subjected to earthquake loading that requires relatively few assumptions on the part of the engineer using the model and provides sufficient computational efficiency to enable its use in simulation of relatively large structural systems.

RESEARCH OBJECTIVES

The model proposed by Lowes and Altoontash is intended for use in simulating the earthquake response of two-dimensional joints with moderate volumes of transverse reinforcement. Within these bounds, preliminary evaluation of the model presented by Lowes and Altoontash [1] indicates that the proposed model simulates well the response of beam-column building joints. A further study by Lowes [15] indicates that the model can be used to simulate the response of bridge joints in which a splitting-type bond failure occurs, if the bond strength is reduced to account for the splitting-type failure. Additional research was undertaken here to evaluate the proposed models for a wide range of building joint specimens and identify ways in which the model could be improved to better represent observed behavior of building joints.

EXPERIMENTAL DATASET

To evaluate thoroughly the joint model, experimental data characterizing the response of beam-column building joints with a wide range of material properties, geometric configurations and design parameters were required. To collect these data experimental investigation conducted by researchers in the US, Japan and New Zealand were evaluated using the following criteria:

1. Two-dimensional beam-column building joint specimens with no slabs, no beam eccentricity and no out-of-plane beams.
2. Specimens with at least a minimal volume of transverse reinforcement. Joints constructed in the United States prior to 1970 typically have no transverse reinforcement; joint with these details were not included in the study.
3. Pseudo-static cyclic lateral loading was used to simulate an earthquake loading.
4. Availability of sufficient experimental data used for modeling and evaluation.

On the basis of criteria one through three, 14 experimental investigations comprising 80 individual test specimens were identified. However criteria four led to the selection of only 11 experimental investigations comprising 50 test specimens as appropriate for use in this study (Park and Ruitong [16], Durrani and Wight [17], Otani et al. [18], Blakeley et al. [19], Soleimani et al. [20], Meinheit and Jirsa [21], Noguchi and Kashawazaki [22], Oka and Shiohara [23], Kitayama et al. [24], Park and Milburn [25]). Of these tests, data from only four investigations (22 specimens) with parameters spanning the entire range of interest of new design (Table 1) are presented in this paper. These four experimental investigations were undertaken to identify the impact on joint response of concrete compressive strength, yield strength of reinforcement steel, percentage of beam and column longitudinal reinforcement, percentage of hoop reinforcement in the joint, column axial load, diameter of the beam longitudinal reinforcement bar, aspect ratio of the joint.

Table 1. Design input parameter variation of some selected specimens

Joint Specimen	Spcmn. No.	v_j/v_s	τ (MPa) top	τ (MPa) bot	φ (%)	f_c (MPa)
Park and Ruitong	PR1	0.41	2.89	2.89	1.15	45.86
	PR2	0.64	5.41	3.69	1.07	35.97
	PR3	0.46	2.89	2.89	0.60	36.17
	PR4	0.58	5.41	3.69	0.59	40.06
Durrani and Wight	DWX1	1.03	5.08	4.53	0.16	34.31
	DWX2	1.13	5.08	4.53	0.24	30.87
	DWX3	0.91	5.08	4.53	0.19	31.01
Noguchi and Kashiwazaki	NKOKJ1	1.25	7.77	7.77	0.14	69.94
	NKOKJ3	1.19	7.77	7.77	0.12	106.90
	NKOKJ4	1.20	7.77	7.77	0.14	69.94
	NKOKJ5	1.13	7.77	7.77	0.15	69.94
	NKOKJ6	1.20	7.77	7.77	0.16	53.45
Otani, Kobayashi and Aoyama	OKAJ1	0.80	4.24	4.24	0.07	25.70
	OKAJ2	0.83	4.24	4.24	0.14	24.03
	OKAJ3	0.90	4.24	4.24	0.29	24.03
	OKAJ4	0.74	4.24	4.24	0.07	25.70
	OKAJ5	0.70	4.24	4.24	0.07	28.74
	OKAS1	0.45	3.64	3.64	0.62	27.76
	OKAS2	0.48	3.64	3.64	0.58	27.76
	OKAS3	0.58	3.30	3.30	0.49	27.76
	OKAS4	0.56	4.92	4.92	0.53	25.11
	OKAS6	0.51	3.64	3.64	0.18	25.11

The results of these investigations indicate that joint response is determined primarily by the following parameters:

- Nominal joint shear demand capacity ratio, v_j / v_s, defined as the ratio of the maximum nominal joint shear demand to the nominal joint shear capacity. Demand and capacity are computed following the recommendations of ACI Committee 318R-02 [26]. The demand-capacity ratio is a function of the concrete compressive strength and joint volume.
- Bond stress demand, τ, defined as the average bond stress required within the joint to develop the tensile yield strength of the longitudinal beam reinforcement on one side. This demand is a function of the yield strength of the reinforcement steel and the diameter of the bar.
- Ratio of transverse steel capacity to the total shear demand of the joint, φ, defined as the ratio of the nominal strength of total transverse steel to the nominal joint shear force computed following the recommendations of ACI ACI Committee 318R-02 [26]. This ratio is a function of the yield strength of the transverse steel and the volume of transverse steel.
- The compressive strength of concrete, f_c.

The values of each of these parameters for the four experimental investigations presented here are listed in Table 1. The ranges of the above parameters included in the dataset are as follows:

- Nominal shear demand-capacity ratio: 0.3 to 1.5.
- Bond stress demand: 2.76 MPa to 15.86 MPa.
- Ratio of transverse steel capacity to the total shear demand of the joint: 0.02 to 1.0.
- Compressive strength of concrete: 24.13 MPa to 106.90 MPa.

MODEL SIMULATION

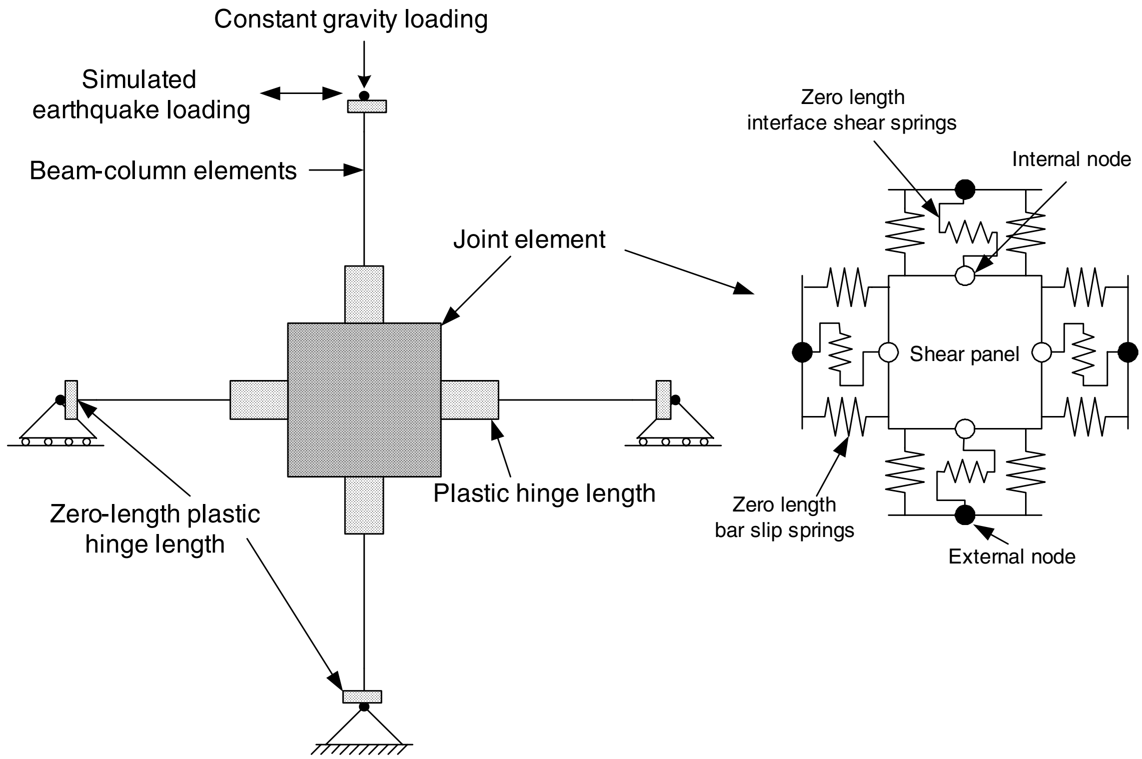
The joint model developed by Lowes and Altoontash [1] was implemented in OpenSees (http://opensees.berkeley.edu) to facilitate this study. OpenSees is an object-oriented open-source platform for finite element analysis developed as a part of the research effort of the Pacific Earthquake Engineering Research Center (http://peer.berkeley.edu). This platform includes beam-column element models and solution algorithms, thereby eliminating the need to develop these tools specifically for the current study.

The Basic model

Most of the experimental test specimens considered in this study were cruciforms, loaded as shown in Figure 1a. The typical model used to simulate the response of these specimens comprised the four-node joint element developed by Lowes and Altoontash [1] in combination with four classical force-based beam-column frame elements (Figure 1b).

Beam Column Elements

The force-based, lumped-plasticity beam-column element included in the OpenSees platform was used to simulate the response of the beams and columns that composed the laboratory test specimens. The lumped-plasticity model relies on a Gauss-Lobatto integration scheme in which two quadrature points are included within the user-specified plastic hinge length at each end of the element and a single quadrature point is included at mid-span of the element. A fiber-discretization of the member cross-section was used to simulate moment-curvature and axial load-deformation response within the plastic-hinge region. Concrete fiber response was defined using a modified Kent-Park concrete material model (Park et al. [27]) with degraded linear unloading/reloading stiffness as recommended by Karsan and Jirsa [28] to simulate compressive response and the assumption of zero tensile strength. The uniaxial response of reinforcing steel was simulated using a bilinear envelope and Menegetto-Pinto curves [29] to simulate unload-reload response. At the single mid-span quadrature point, the element was assumed to be elastic with an effective moment of inertia defined on the basis of the recommendations of ACI 318R-02 [26] Section 9.5.2.3.



(a) Typical Cruciform Model with boundary condition and loading (b) Joint Element

Figure 1. Experimental Sub-assemblage

Beam-Column Joint Element

The joint element proposed by Lowes and Altoontash is a super-element comprising 13 one-dimensional components that explicitly represent the three types of inelastic mechanisms that may determine the earthquake response of beam-column joints. Eight bar-slip springs (Figure 1b) simulate anchorage failure of beam and column longitudinal reinforcements embedded within the joint. A single shear panel (Figure 1b) simulates the shear failure of the joint core. Four interface shear springs (Figure 1b) simulates the shear transfer failure at the beam-joint and column-joint interfaces. The two-dimensional element has four external nodes, each with three degrees-of-freedom, and is appropriate for use with the beam-column elements described previously.

A hysteretic one-dimensional load-deformation model is used to define the material behavior of the 13 elements that comprise the joint super-element. A response (or backbone) envelope, an unload-reload path, and three damage rules that control the evaluation of the response paths define the one-dimensional material model. In the current implementation, the backbone envelope is defined to be multi-linear and the unload-reload path is tri-linear. Three damage rules define the evolution of the response envelope and the unload-reload paths as a function of load-deformation history. Damage is simulated through deterioration in the unloading stiffness (unloading stiffness degradation), deterioration in the strength developed in the vicinity of the maximum and minimum deformation demands (reloading stiffness degradation), and deterioration in strength achieved at previously unachieved deformation demands (strength degradation). Damage is defined as a function of peak displacement and cumulative energy dissipation.

The above one dimensional model, proposed by Lowes and Altoontash [1], includes recommendations for calibration of one-dimensional hysteretic models for bar-slip springs, shear-panel element and interface shear-springs that compose the super-element. For the bar-slip and shear panel, definition of backbone curves is based on the fundamental material and geometric properties of the specimen and the damage models are calibrated previously using the results of bond and reinforced concrete shear panel tests. The bar-slip spring calibration procedure employs the assumption of a uniform or piecewise-constant bond stress distribution within the joint; with bond capacity defined on the basis of experimental data. The shear-panel calibration procedure employs modified compressive field theory (MCFT) (Vecchio and Collins [30]) and the assumption of uniform shear within the joint. Interface shear-springs are assumed to be elastic and stiff.

MODIFICATION OF THE MODEL

The above model was employed to simulate the response of experimental test specimens as listed in Table 1. Upon review of the results of the investigation and re-evaluation of the model, the following improvements to the original model were proposed to improve simulation of response for joint designs considered in the study:

- *Height of tension-compression couple:* Frame member moments are transferred into the joint through tension-compression couples, where tension and compression forces are carried by the bar-slip springs. In the model proposed by Lowes and Altoontash [1] the distance between the bar-slip springs was defined to be equal to the depth of the beam and columns. However it was observed that the force transferred in the springs was less than that observed experimentally. Thus the distance between the bar-slip springs (height of the tension compression couple) was modified to be equal to the distance between tension reinforcing steel and centroid of the concrete compression zone in the frame member, at the member nominal flexural strength (positive and negative are averaged).
- *Post-peak response of bond-slip springs:* As proposed originally, the bar-slip springs predicted strength degradation once slip demand exceeded an empirically derived slip demand of 3 mm. However, in simulating the response of multiple specimens, it was found that this strength degradation resulted in failure of global convergence, even if arc-length continuation methods were employed. Thus, to mitigate this problem, the bond-slip constitutive law was modified so that strength degradation resulted only from cyclic loading. The result of this modification was that the bond-slip springs exhibited positive stiffness at all times, but strength deterioration upon reloading to a previously observed slip demand.
- *Anchorage length:* The joint *anchorage length* was used to determine the bond stress capacity within the joint. If the anchorage length required to develop the post-yield strength of the bar exceeds the *anchorage length*, then a reduced bond stress capacity is assumed. This results in a reduced post-yield stiffness (load vs. displacement) for the bar-slip springs. In the modified version of the model, the anchorage length for the beam (column) reinforcement was reduced from the column width (beam height) to the distance between the bar-slip springs, which better represents the observed response.

Comparison of Simulated (Modified Model) and Observed Response

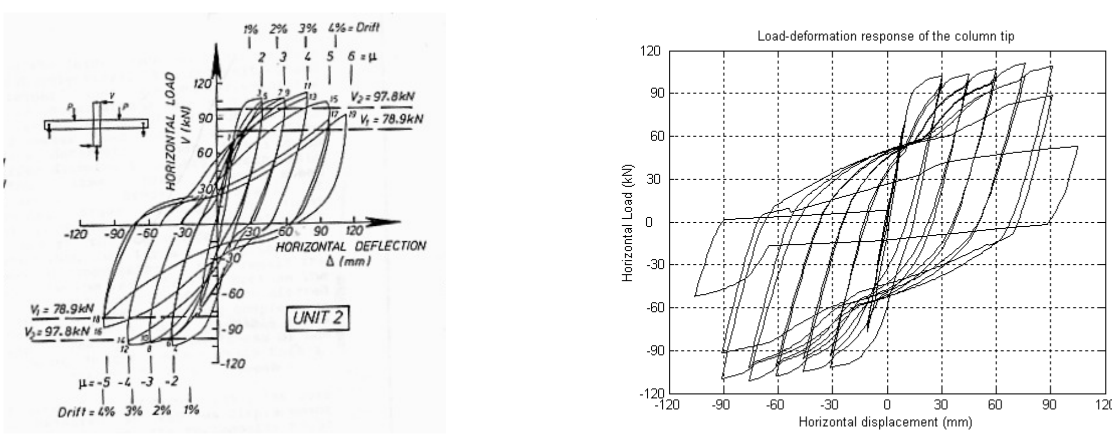
The modified model was evaluated by comparing simulated and observed failure mechanisms and load-displacement response for the specimens identified in Table 1. Figure 2, Figure 3 and Figure 4 shows simulated and observed data for a typical specimen, PR2 tested by Park and Ruitong [16], which exhibits anchorage failure. Table 2 and Table 3 list specific numerical response values that were used to evaluate the specimens. Joint failure was classified as due to bond if the model/test specimen achieved the nominal yield strength of the beams and exhibited some ductility and shear if it did not. Load-displacement response was characterized on the basis of

- Nominal, Maximum and Failure strength. Nominal flexural strength was defined as the minimum load corresponding to a beam developing nominal flexural strength (per ACI 318R-02). If beams did not develop nominal flexural strength, nominal strength was defined equal to the maximum strength. Failure strength is defined as the maximum strength observed during the first load-cycle for which peak strength during the cycle was less than 80% of the maximum strength.
- Secant stiffness values at 60% of nominal strength and unloading stiffness at maximum strength.
- Displacement values at nominal flexural, maximum and failure strength.

Observations from Analysis Results using the Modified Model

The data in Table 2 and Table 3 and Figure 2, Figure 3 and Figure 4 provide a basis for evaluation of the model. The following conclusions can be made regarding the model:

- The model accurately predicts the observed failure mechanism only for joints with at least moderate shear capacity ($\varphi > 0.15$). The response of the joint core in shear is modeled assuming 1) that shear stress is uniform and 2) that shear stress-strain response may be simulated using the MCFT. However experimental data suggest that in joints with low transverse steel volumes, shear is transferred primarily through a compression strut. This mechanism is stronger and stiffer than predicted by MCFT.
- The model accurately predicts the observed load-displacement histories, with the exception that the model 1) predicts faster strength loss than is observed in the laboratory and 2) under-predicts unloading stiffness at maximum strength. The first limitations of the model likely results from the calibration of the bar-slip damage rule to simulate the strength loss observed for local (i.e. short anchorage length) rather than average bond-slip tests. The second limitation of the model likely results from the use of RC shear-panel data, which are not wholly representative of joints, to calibrate stiffness damage rules for the shear-panel.



(a) Observed response (b) Simulated response

Figure 2. Load-Deformation Response of Park and Ruitong Specimen PR2

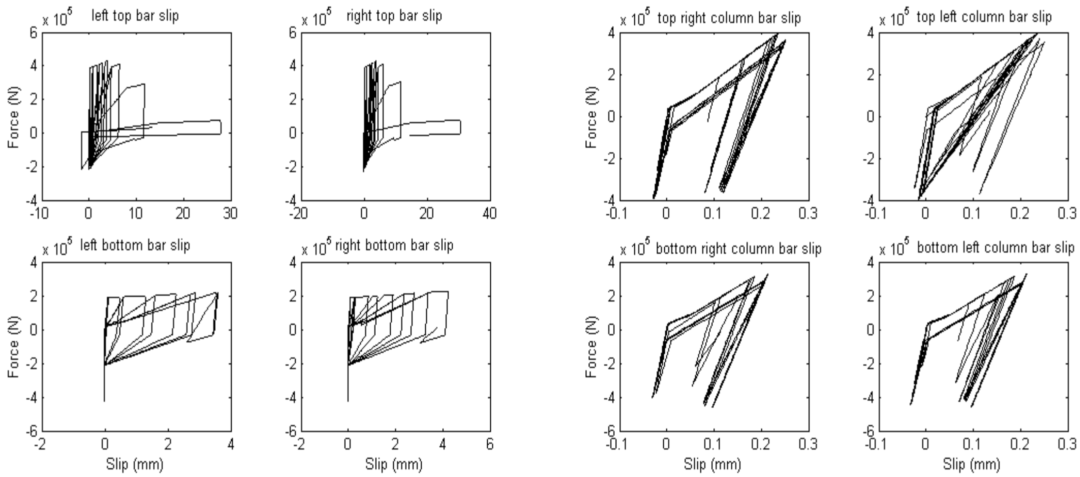


Figure 3. Bar-Slip Component Response of the Joint for Specimen PR2

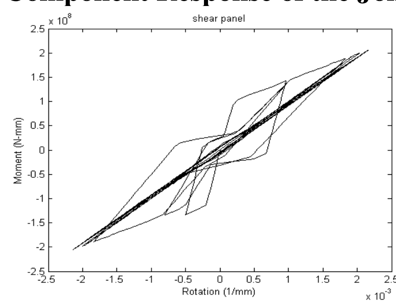


Figure 4. Shear Panel Component Response of the Joint for Specimen PR2

Table 2. Strength and Stiffness comparison between observed and simulated specimen results

Specimen	Initial Stiffness		Nominal Strength		Maximum Strength		Unloading stiffness @ Max. Strength		Failure Strength	
	Exp. (kN/mm)	Smlt./Exp.	Exp. (kN)	Smlt./Exp.	Exp. (kN)	Smlt./Exp.	Exp. (kN/mm)	Smlt./Exp.	Exp. (kN)	Smlt./Exp.
PR1	2.33	1.00	70.0	1.00	80.3	0.98	1.15	0.83	NA	NA
PR2	3.50	0.97	105.0	0.97	111.7	0.99	2.23	1.00	92.5	0.58
PR3	2.40	0.91	72.0	0.91	79.4	0.94	1.59	0.79	NA	NA
PR4	3.30	1.01	99.0	1.01	106.5	0.97	1.77	0.84	79.7	1.05
DWX1	4.89	0.94	186.8	0.94	191.2	0.94	6.26	0.74	171.2	0.85
DWX2	5.25	0.95	186.8	0.94	197.9	0.93	6.49	0.56	182.4	0.57
DWX3	4.57	0.89	151.2	0.89	151.2	0.90	3.96	0.75	137.9	0.65
NKOKJ1	8.00	1.02	250.0	0.51	250.0	0.51	13.89	0.46	200.0	0.51
NKOKJ3	8.33	0.90	295.0	0.54	295.0	0.48	29.50	0.32	250.0	0.46
NKOKJ4	7.33	1.22	245.0	0.61	245.0	0.59	14.41	0.30	200.0	0.35
NKOKJ5	8.00	1.04	250.0	0.52	250.0	0.51	12.50	0.30	150.0	0.30
NKOKJ6	6.83	1.90	220.0	0.89	220.0	0.56	11.00	0.20	150.0	0.20
OKAJ1	5.63	1.48	90.1	0.74	117.7	0.57	1.10	0.20	96.1	0.20
OKAJ2	3.68	1.90	117.6	0.95	127.4	0.88	1.02	0.20	97.5	0.25
OKAJ3	3.90	0.98	125.0	0.98	132.3	1.00	3.15	0.83	NA	NA
OKAJ4	3.67	0.68	117.7	0.68	117.7	0.68	4.71	0.78	68.6	0.20
OKAJ5	3.67	0.68	117.7	0.68	117.7	0.68	2.35	0.90	83.3	0.20
OKAS1	4.17	0.95	66.7	0.95	70.6	0.98	1.24	1.00	NA	NA
OKAS2	4.17	0.97	66.7	0.97	74.5	0.96	1.43	0.94	NA	NA
OKAS3	4.48	0.94	71.6	0.94	79.4	0.95	3.31	0.90	77.4	0.90
OKAS4	4.90	0.95	78.3	0.95	80.9	0.95	1.37	0.91	76.4	0.87
OKAS6	4.10	0.98	64.0	0.98	70.6	0.99	1.19	1.00	NA	NA

Table 3. Displacement comparison between observed and simulated specimen results

Specimen	Disp @ Nominal Str.		Disp. @ Max. Str.		Disp. @ Failure	
	Exp. (mm.)	Smlt./Exp.	Exp. (mm.)	Smlt./Exp.	Exp. (mm.)	Smlt./Exp.
PR1	30.0	1.0	105.0	1.00	NA	NA
PR2	30.0	1.0	75.0	1.00	105.0	1.00
PR3	30.0	1.0	90.0	1.00	NA	NA
PR4	30.0	1.0	90.0	1.00	105.0	1.00
DWX1	38.1	1.0	58.5	0.98	101.7	0.94
DWX2	35.6	1.0	66.2	1.07	116.9	0.92
DWX3	33.1	1.0	76.3	0.88	104.3	0.95
NKOKJ1	30.0	0.5	45.0	0.67	60.0	0.75
NKOKJ3	30.0	0.6	45.0	0.67	75.0	0.40
NKOKJ4	30.0	0.5	45.0	0.45	75.0	0.40
NKOKJ5	30.0	0.5	45.0	0.39	75.0	0.40
NKOKJ6	30.0	0.5	45.0	0.38	75.0	0.40
OKAJ1	16.0	0.5	64.0	0.25	96.0	0.33
OKAJ2	32.0	0.5	64.0	0.25	96.0	0.33
OKAJ3	32.0	1.0	64.0	1.00	NA	NA
OKAJ4	32.0	1.0	32.0	1.00	96.0	0.67
OKAJ5	32.0	1.0	64.0	0.50	96.0	0.67
OKAS1	16.0	1.0	64.0	1.00	NA	NA
OKAS2	16.0	1.0	64.0	1.00	NA	NA
OKAS3	16.0	1.0	32.0	1.00	64.0	1.00
OKAS4	16.0	1.0	32.0	1.00	64.0	1.00
OKAS6	16.0	1.0	64.0	1.00	NA	NA

CONCLUSIONS

The model proposed by Lowes and Altoontash [1] was evaluated using an experimental dataset that spanned the range of plausible joint geometries, design parameters and material properties for interior building joints with modern design details. On the basis of this evaluation, the model was modified to include 1) a revised height for the tension-compression couple formed by bond-slip springs at the perimeter of the joint, 2) revised damage rules for bar-slip material envelope, 3) a new definition of the bar anchorage length. These modifications result in more accurate prediction of observed response. Additionally, it was found that the model including the recommended modifications is appropriate for use in simulation of the response of joints with ratio of transverse steel capacity to the total shear demand of the joint greater than 0.15. These joint typically exhibit anchorage failure.

The results of this study suggest that improved accuracy could be achieved by 1) improved simulation of the bar-slip strength loss and 2) improved simulation of joint shear response for joints with ratio of transverse steel capacity to the total shear demand of the joint lower than 0.15.

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