SIMULATION OF ENERGY INFLUX AND EFFLUX THROUGH SOIL STRUCTURE INTERFACE

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SUMMARY

A new method to simulate soil-structure interaction effects in shaking table tests has been presented recently by the authors. In the method, analog circuits or digital signal processors are used to produce soil-foundation interaction motions in real time. Their expressions of soil-structure interaction motions are based on published rigorous formulations of impulse response functions of foundations resting on or embedded in homogeneous or layered soils of semi-infinite extents. This paper introduces in its first half the method for simulating soil-structure interaction effects in shaking table tests and some pieces of contrivance for better control of shaking tables. The latter half then describes a simple example of soil-structure interaction simulations using the present method.

INTRODUCTION

Such devastating events as Sounth-Hyogo Earthquake of 1995 seem to have stimulated a sharp rise in demand for huge shaking tables that allow models weighing for example more than thousand tons to be tested. Shaking tables are usually so driven by servo-hydraulic actuators that they follow closely input seismic motions. However, a shaking table, when heavily loaded with a structure model to be tested, interacts with the model, and this interaction often causes the table’s motion to deviate from the intended time history. Recent advances in signal-processing technology have certainly enhanced controllability of shaking tables to a great extent (Horiuchi, et al., 1995, Stoten, 1998), and yet, the motions of a table are often required to be adjusted, through iterative trials, to the intended base motions by modifying the input time histories. Generally, the larger a table is, the more difficult it is for the table to be controlled at will.

A large table with improved performance is certainly a necessity in many earthquake-related researches. However, faithful reproduction of free-field ground motions on the table may not necessarily be adequate, because actual structures interact with their foundations and the surrounding soils in real earthquakes, causing the ground motions at the structures’ bases to deviate from the free-field ground motions. This dynamic interaction is a phenomenon associated with the influx and efflux of energy which is generated by the earthquake excitation and transmitted through the soil-structure interface. It is noted that the difference between the influx and efflux is exactly the energy stored up within a structure, and thus, is closely related to the extent of damage to the structure. If this interaction effects are rationally simulated in shaking table tests, one will obtain necessary pieces of information for interpreting failure processes of prototype structures in terms of energy.

Konagai and Nogami (1998-1998b) have recently developed a method to produce soil-structure interaction effects in a shaking table test on a structure model, without using physical ground model. In their method, soil-structure interaction effects are simulated by adding appropriate soil-structure interaction motions to the free-
field ground motions at the shaking table. Their expressions of soil-structure interaction motions are based on published rigorous formulations of flexibility functions and/or impulse response functions of foundations resting on or embedded in homogeneous or layered soils of semi-infinite extents. In general, radiation damping will cause the total damping of a soil-structure system to be greater than that of the structure itself. Thus, incorporation of soil-structure interaction effects in a shaking table test will lead to reducing the demands on the capacity of the shake table, and the structure model may not necessarily be shaken too forcibly. However, real-time adjustment of the shaking table’s motion is definitely a prerequisite for the present method, and one can not do it through iterative trials.

This paper introduces in its first half simple descriptions of foundation stiffness parameters in terms of a limited number of frequency-independent parameters: the descriptions allowing soil-structure interaction motions to be simulated on a shaking table. The latter half then presents simple examples of soil-structure interaction simulations using the present method are also given in this chapter.

**PRESENT METHOD**

Figure 1 shows a schematic view of the set-up in a shaking table test for earthquake excitation, in which a superstructure model is placed directly on the table without a physical ground model. Soil-structure interaction effects are simulated by adding appropriate soil-structure interaction motions to the free-field ground motions at the shaking table. In the simulation, first, the transducers at the base of the foundation pick up the signals of the base forces, $p_x$ and $p_y$, in sway and rocking motions, respectively. These two amplified signals are then applied to the circuits $H_{xx}$, $H_{xy}$, $H_{xy}$, and $H_{yy}$ to produce the outputs corresponding to the soil-structure interaction motions, $\tilde{u}_x (=\tilde{u}_x + \tilde{u}_y)$ and $\tilde{u}_y (=\tilde{u}_x + \tilde{u}_y)$. The output signals are added to the free-field motion signals, $u_x$ and $u_y$, to produce the signals of foundation motions, $u_x + \tilde{u}_x$ and $u_y + \tilde{u}_y$. The signals are translated into the shaking table motions by the shaking table controller.

The above method requires a device that generates the signals corresponding to the soil-structure interaction motions, and a digital signal processor (DSP), which is capable of producing a variety of transfer functions, is used as this device. The transfer functions to be realized on the DSP are designed from the analytical expressions for stiffness or flexibility functions of the foundations. Konagai (1999) has shown that $n_p$ piles closely grouped together beneath a superstructure can be viewed as a single equivalent upright beam whose stiffness matrix is described with two stiffness parameters, $EI_p$ and $EI_G$. The parameter, $EI_p$, is identical to $n_p \times EI_{pp}$, where $EI_{pp}$ is the bending stiffness of an individual pile, whereas, $EI_G$ is evaluated following the same procedure as that used for the evaluation of bending stiffness of a reinforced concrete beam. Namely, $EI_G$ is assumed to be equal to the sum of the Young’s-modulus-weighted products of all the elementary areas times their distances squared from the centroid of the cross-section $A_G$ (Figure 2). Careful examination of deflections of grouped

![Figure 1: Simulation of soil-structure interaction on a shaking table](image-url)
piles reveals that most piles are indeed flexible in practice in the sense that they do not deform over their entire lengths. Instead, pile deflections become negligible below their active lengths, $L_a$; the active pile length is given as a function of $EI_p$ and the shear modulus of the surrounding soil, $\mu$, as:

$$L_a = aL_0 = a\sqrt{\frac{EI_p}{\mu}}$$

(1)

The parameter $a$ in the above equation differs in different soil profiles. Konagai, Ahsan and Maruyama (1999) carefully examined the approximate solutions of pile cap stiffness, $S_{xx}$, in sway motion by using the upright single beam analogy, and have shown that they are closely approximated by the following expression:

$$S_{xx} = k_x + ia\cdot c_x - a^2 m_x$$

(2a)

where, $a = \frac{\omega \cdot R_0}{v_x}$

(2b)

with $\omega$ = circular frequency, $R_0$ = the radius of the equivalent upright beam, and is assumed to be identical to the radius of a circle with the same area as the cross-section, $A_G$, that includes all the grouped piles enclosed by the broken line in Figure 2, and for a homogeneous soil,

$$k_x = \mu\left(2\pi R_0 + \frac{\pi}{2} L_0\right), \ c_x = \mu \cdot 2\pi L_0 \quad \text{and} \quad m_x = \mu \frac{\pi L_0}{4}$$

(2c)-(2d)

Equation (2a) clearly shows that the performance of a soil-pile group system is approximated by that of a simple-damped oscillator with a spring $K$ ($=k_x$), a damper $C$ ($= R_0 c_x / v_x$) and a mass $M$ ($= R_0^2 m_x / v_x^2$). The similar expressions have been utilized for rigid spread foundations (Meek and Wolf) resting on semi-infinite half spaces. Needless to say, not all stiffness and/or flexibility expressions are of the same simple form. Reviewing these expressions, however, Konagai (1999) has found that the flexibility expressions in the time domain are closely approximated by summing up exponential and/or exponentially decaying sine and cosine functions of time $t$, the functions being easily produced by a DSP.

**Figure 2:** Assumptions for evaluation of equivalent single beam

**CONTROL OF SHAKING TABLE**

It is noted that the system illustrated in Figure 1 is realized on condition that a shaking table loses no time in producing faithfully its input motion. The motion produced by the shaking table, however, is not exactly identical to the intended time history because the ratio of output-to-input amplitude of the shaking table system
does not remain the same over the desired frequency range. The performance of the system’s transfer function is also affected by the presence of models on the shaking table; this fact may cause the motion of the table to further deviate from the intended time history. A controller with the transfer function $T$ normally performs like a low pass filter, and experiments on the table are conducted below its cut-off frequency. Below this frequency yet, there remains a time delay $\Delta t$ between the produced motion and the input signal. The effect of the time delay, described in the frequency domain as $T \approx e^{-i\omega \Delta t}$, could be canceled by multiplying the flexibility function $H$ by $T^{-1}$. Assuming that the performance of a soil-foundation system is approximated by that of a simple-damped oscillator with a spring $K$, a damper $C$ and a mass $M$ (Figure 3), the flexibility function $H_{ss}$ is expressed as:

$$H_{ss} = \frac{1}{K - \omega^2 M + i\omega C} \quad (3)$$

Thus, the cancellation of the time-delay effects is made by

$$H_{ss} T^{-1} = \frac{e^{i\omega \Delta t}}{K - \omega^2 (M - \Delta M) + i\omega (C - \Delta C)} \quad (4a)$$

For smaller values of $\omega \Delta t$, equation (4a) is rewritten as:

$$H_{ss} T^{-1} = \frac{1}{K - \omega^2 (M - \Delta M) + i\omega (C - \Delta C)} \quad (4b)$$

where, $\Delta M = C \cdot \Delta t$ and $\Delta C = K \cdot \Delta t$ \quad (4c) and (4d)

Equation (4b) shows that the equivalent mass and the viscous damping coefficient are reduced by $C\Delta t$ and $K \cdot \Delta t$, respectively. The reduced mass $M - \Delta M$ and the damping coefficient $C - \Delta C$ must be positive, calling for:

$$\frac{\Delta M}{M} = 4\pi^2 \frac{\Delta t}{t_0} < 1 \quad \text{and} \quad \frac{\Delta C}{C} = \frac{\Delta t}{t_c} < 1 \quad (5a) \quad (5b)$$

with $t_c = C / K$ and $t_0 = 2\pi \sqrt{M / K}$ \quad (6a) and (6b)

The above conditions (equations (5a) and (5b)) are usually satisfied in reality for many cases of soil-structure interaction, because radiation of waves from a foundation leads the motion of the structure to be noticeably damped.

It is, however, necessary for the time delay to be minimized when equations (5a) and (5b) are not satisfied. Adaptive and/or robust control of a shaking table will cause the time delay to be reduced to some possible extent (Igarashi, 1999 and Horiuchi, 1994). Some attempts that the authors are trying out will be addressed in later publications.

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**Figure 3:** Equivalent spring-damper system supporting a rigid spread foundation

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In order to provide a proper perspective on the usefulness of the present method, a simple example of simulations of soil-structure interaction effects was conducted (Konagai, et al. 1998). A rigid cylindrical block is assumed to be put on a rigid and circular spread foundation resting on a semi-infinite half medium of soil (Figure 4a). The dimensions of both the prototype block and foundation are listed in Table 1, whereas Table 2 shows the parameters for the soil medium. Poisson’s ration of the soil was set at 0.5 on the assumption that the ground is an alluvial soft soil deposit that is totally saturated with water. According to the approach by Meek and Wolf (1992a-1993b), the soil supporting a circular spread foundation is idealized for each degree of freedom as a truncated semi-infinite cone (Figure 4b) with its own apex height $z_0$. The apex ratio $z_0 / r_0$, or the opening angle of the cone is determined by equating the static stiffness coefficient of the disk on the semi-infinite soil half-space to that of the corresponding cone: whereas the wave propagating through the cone with the velocity $v$ dominates the stiffness within the considerably high frequency range. For a translational cone, the velocity $v$ is found identical to the shear wave velocity $v_s$, and the stiffness of the foundation for sway motion, $S_{ax}$, is obtained as:

$$S_{ax} = K + i\omega C - \omega^2 M$$

(7a)

where, $K = \frac{Dv_s^2 \cdot \pi r_0^2}{z_0}$, $C = \rho_s \cdot v_s \cdot \pi r_0^2 \cdot \frac{z_0}{r_0} = \frac{\pi}{2} (2 - \nu)$ with $\nu$ = Poisson’s ratio and $M$ = mass of the foundation.

(7b)-(7e)

Equation (7a) is noticed to be identical to the unit-impulse response function of a simple spring-damper system. A model of the soil-structure system is then prepared by reducing the parameters, $M$, $K$, and $C$ to the uniform scale of 1 to 100. Since the ratio of these parameters is kept unchanged, the time scale is not changed at all.

**Table 1:** Dimensions of block and foundation

<table>
<thead>
<tr>
<th>(a) block</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>7.1×10^5 kg</td>
<td>7 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) spread foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
</tr>
</tbody>
</table>

**Figure 4:** Rigid block put on a rigid mat foundation resting on a semi-infinite soil medium
Table 2: Mechanical properties of soil

<table>
<thead>
<tr>
<th>Density</th>
<th>Shear wave velocity</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6x10^3 kg/m^3</td>
<td>100 m/s</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure 5 shows the model put on a shaking table. The steel block in the middle is the model of the rigid block, and the shaking table itself virtually represents the motion of the mat foundation on the semi-infinite soil half-space. The block is put not directly on the shaking table but on a flat steel plate supported by four stiff upright legs with strain gages pasted on them. These gages pick up the base shear force from the block. An impulse as shown in Figure 6 is given to the shaking table as an input motion \( u_x \). The test was also conducted for the above block model put on the rigid base. Figure 7 shows time histories of both the displacement of the shaking table and the distance that the block has slipped. Dotted lines in this figure show the motions without the interaction effect being taken into account, whereas thick lines show the motions affected by the soil-structure (foundation-block) interaction. Incorporation of the soil-structure interaction leads to slight increase in the duration of the base motion and drastic decrease of the distance that the block has slipped. The mass of the block is the direct cause of the increase in the duration of the base motion, and the decrease of the sliding distance is closely linked with the increase of the energy that has dissipated as outwardly propagating waves into the virtually spreading soil medium. The present method allows both influx \( E_{\text{input}} \) and efflux \( E_{\text{dissipated}} \) of energy through the foundation to be measured in real time. These two kinds of energy are respectively:

\[
E_{\text{input}} = \int_0^t p \cdot \dot{u}_x \cdot dt \quad \text{and} \quad E_{\text{dissipated}} = \int_0^t -p \cdot \ddot{u}_x \cdot dt \quad (8a), \quad (8b)
\]

The energy, \( E_{\text{consumed}} \), used up within the model on the shaking table is then obtained as:

\[
E_{\text{consumed}} = E_{\text{input}} - E_{\text{dissipated}} \quad (8c)
\]

Figure 8a shows the variations of these energies with time where the interaction effects are ignored, and thus, the cumulative loss of energy through friction ends up to be the same amount as the energy influx. On the other hand, Figure 8b, in which soil-structure interaction effects are incorporated, shows that a part of influx energy dissipates away and just the remainder is used up through friction.

Figure 5: Block model on shaking table
Figure 6: Displacement of shaking table and distance that block has slipped

Figure 7: Influx, efflux and consumption of energy

CONCLUSIONS

A new method for a model experiment on a shaking table has been presented. The present method allows soil-structure interaction to be simulated. In the present method, soil-structure interaction effects are simulated by adding appropriate soil-structure interaction motions to the free-field ground motions at the shaking table. This method requires a device that generates the signals corresponding to the soil-structure interaction motions, and a digital signal processor (DSP), which is capable of producing a variety of transfer functions, is used as this device.

Expressions of soil-structure interaction motions are based on published rigorous formulations of stiffness or flexibility functions of foundations resting on or embedded in homogeneous or layered soils of semi-infinite extents. The idealization of grouped piles as a single equivalent upright beam and the concept of the active pile length have facilitated the derivation of a simple expression of pile-cap stiffness in terms of frequency-independent mass, damping and stiffness parameters. The similar expressions have been utilized for rigid spread foundations (Meek and Wolf) on semi-infinite half spaces. Needless to say, not all stiffness and/or flexibility expressions are of the same simple form. Reviewing these expressions, however, it has been found that the flexibility expressions in the time domain are closely approximated by summing up exponential and/or exponentially decaying sine and cosine functions of time $t$, the functions being easily produced by a DSP.
In general, the transfer function of a shaking table system permits the transmission of signals with frequencies below a certain cutoff value with little attenuation. But even below this limit of frequency, a signal is delayed a certain time. This delay leads to the virtual increases in both mass and damping constant of a soil-foundation system. This error must be compensated by modifying the signals corresponding to soil-structure interaction motions.

In order to provide a proper perspective on the usefulness of the present method, a simple experiment was conducted, in which a steel block was put on a shaking table that virtually represents the sway motion of a rigid circular mat foundation on a semi-infinite half space of soil. An impulsive displacement was then given to the shaking table as an input free-field motion, and both the displacement of shaking table and the distance that the block slipped were measured. Incorporation of the soil-structure interaction led to slight increase in the duration of the base motion and noticeable decrease of the distance that the block slipped. The present method allows both influx and efflux of energy through the foundation to be measured in real time, and offered clear physical interpretation to the observed response of the block.

REFERENCES


