A SIMPLE METHOD FOR THE SEISMIC ANALYSIS OF PILES AND ITS COMPARISON WITH THE RESULTS OF CENTRIFUGE TESTS

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SUMMARY

One of the main factors which affects the seismic behaviour of piles is the soil yielding which is ignored in available purely elastic methods for dynamic analysis of pile foundations. In this paper a relatively simple time domain methodology for the seismic analysis of pile foundations is developed in which, the pile is modeled as a beam and the surrounding soil is assumed to have an elasto-plastic behaviour. The elastic behaviour is modeled by taking advantage of the Mindlin fundamental solution with appropriate extensions to account for the soil radiation damping. The plastic behaviour is modeled via an iterative procedure, which ensures that nowhere along the pile-soil interface will the soil pressure exceed its ultimate lateral value. At the end of this paper, the developed method is used to estimate the response of a model single pile and a 2x2 pile group tested in centrifuge tests. The tests have been carried out in the California Institute of Technology and have been well documented. It is demonstrated that, despite its simplicity, the developed methodology gives reasonable results for the single pile and correctly estimates the maximum value of the pile moment and shear in the pile group. However, the depth of influence of the pile group cap-mass is underestimated.

INTRODUCTION

In recent years simple dynamic linear Winkler methods for the seismic analysis of pile foundations have become available and popular amongst designers. These methods do not need a lot of computer coding and can be easily used via simple spreadsheet calculations. The attractiveness of these methods may make designers complacent about their limitations, the most significant of which is the soil yielding which is often inevitable in any lateral pile loading. It is therefore important that the effect of soil yielding be taken into account in the dynamic model. [Parmelee et al., 1964] and [Penzien, 1970], who developed the earliest organised methods for dynamic analysis of pile foundations, employed a nonlinear discrete model. Penzien used the Mindlin solution and determined the nonlinear spring constants of a Winkler model. The pile inertial effects were modelled by lumped masses and the radiation damping was accounted for by viscous dampers. This methodology was later extended by Matlock, Reese, Prakash and their co-workers and resulted in the development of the so-called p-y curves. [Kagawa, 1980] and [Kagawa and Kraft, 1980 & 1981] studied soil-pile-structure interaction via a finite element method which could represent a nonlinear soil response and examined the effect of pile, soil and loading conditions on the p-y curves. They also developed a simple Winkler model for seismic analysis of single piles in which p-y curves are used. [Norris, 1994] presented an “equivalent linear subgrade modulus profile approach” in which the modulus obtained from p-y curves is replaced by an equivalent linear modulus profile. [El Naggar and Novak, 1996, 1995] developed a nonlinear Winkler model for dynamic lateral analysis of pile response. In this model the soil has been divided into two parts; the first part is an inner nonlinear field, and the second part is a linear far field which accounts for wave propagation away from the pile. This method is, in essence, similar to a Winkler method developed by Nogami and his co-workers for nonlinear analysis of pile foundations. [Nogami and Konagai, 1988 & 1987] developed a time domain Winkler model which for flexural response consists of three Voigt models of springs and dashpots connected in series. The Finite Element Method, a versatile tool in dealing with nonlinear soil behaviour is, in general, computationally very expensive for seismic analysis of pile...
foundations. Finn and [Wu, 1996] developed a quasi-3D method which permits dynamic nonlinear effective stress analysis of pile groups in layered soils. In this method the boundary conditions associated with a full 3D analysis have been relaxed through simplified assumptions for the generation of waves at the soil-pile interface.

The available simplified nonlinear methods for dynamic analysis of pile foundations lack the simplicity of equivalent linear methods and generally do not help drawing general conclusions about the seismic behaviour of pile foundations. In this paper a simplified method is presented which is an extension of a static methodology developed by [Poulos, 1973 & 1982] for piles subjected to lateral soil movement.

**METHODOLOGY**

The pile is modelled as a beam and the soil as an elastic material whose stress strain relationship can be modelled by the Mindlin equation and the effect of radiation damping is taken into account separately. As long as the soil is elastic, the displacement of the pile and soil at their interface are equal and compatibility condition may be enforced between them. This can be done by discretizing the pile into elements (Fig. 1) and requiring that at the centers of these elements the soil and pile displacements be equal. This will lead to the following equation which along with equations obtained from boundary conditions provide a solution to the problem [Tabesh, 1997], [Tabesh & Poulos, 1997]:

\[
\begin{bmatrix}
D + \frac{\text{II}}{E_p I_p} \cdot E_s \cdot \delta^4 \end{bmatrix} \{\Delta u\} + \frac{[M]\delta^4}{E_p I_p} \{\Delta \ddot{u}\} + \\
\frac{[C]\delta^4}{E_p I_p} \{\Delta u\} - \{\Delta \ddot{u}_e\} = \frac{[\text{II}].E_s \cdot \delta^4}{E_p I_p} \{\Delta u_e\}
\]

(1)

where [II] is [I]-1, the inverted soil-displacement-factor matrix elements of which are obtained from Mindlin equation, \{\Delta u\} and \{\Delta u_e\} are the incremental displacement at soil pile interface and the incremental external soil movement respectively. D is the matrix of finite difference coefficients used to model the pile, Ep, Ip, Es, and \delta are pile modulus, Pile cross-section moment of inertia, soil modulus and element length respectively. [M] and [C] are mass and damping matrices and a dot superscript means differentiation with respect to time.

![Fig. 1: Pile elements in the lateral seismic analysis of single piles](image)

The Mindlin hypothesis, of course, does not automatically satisfy the condition of soil radiation damping, and this should be accounted for separately. Several formulations for the radiation damping are available in the literature which give fairly close results. In this paper the value of 5psVsd presented by [Kaynia, 1988] for the amount of damping per unit length of the pile is used in which ps is soil mass density, Vs is soil shear wave
velocity and \( d \) is pile diameter. This value was obtained by matching the head response of a single pile modelled by the Finite Element and Winkler models.

The vector of external soil movement \( \{u_e\} \) in equation (1) may be obtained from free-field ground analysis. By assuming that the earthquake consists of vertically incident SH waves the site response can be obtained using the concept of wave propagation in a layered medium as used in the development of the well-known SHAKE program [Schnabel et al, 1972]. However, as the presented method is in time domain a nonlinear methodology used in the development of the ERLS program [Poulos, 1991] has been employed here. In this method the soil layers are modelled as a multi degree of freedom system of mass-spring-dashpots, in which the spring and dashpot coefficients are strain dependent and nonlinear. This system is excited at the bottom level by the earthquake record and the response is calculated.

The assumption that the soil and pile will have the same displacement during the earthquake and imposition of displacement compatibility between the soil and the pile is not correct when soil yielding occurs. A solution scheme can be introduced in which the value of the pressure at the soil-pile interface is monitored at each element and at every time step. As long as this value is less than ultimate lateral soil pressure, \( P_y \), the compatibility equation is replaced by the condition that the pressure at that element is equal to \( P_y \); this violates the equilibrium of forces applied to the pile, therefore, the pressure at all pile elements is recalculated and it is ensured by iteration that at no element does the pressure exceed \( P_y \). The amount of radiation damping after soil yield is not known, but several tests have shown that it is far less than the value obtained by the elastic assumption [Nogami, 1987], [Chako, 1995]. In the absence of any well-developed guidelines, and based on the results of such tests, in this analysis the radiation damping is ignored wherever soil yields; therefore, for the soil elements that have remained elastic, the radiation damping is taken into account based on the Kaynia formula but is ignored at the elements that have undergone yielding. The ultimate lateral pressure of soil \( (P_y) \) can be obtained based on [Broms', 1964-a & 1964-b] recommendations or the formulae presented by the American Petroleum Institute [API, 1991]. In this paper Broms' values are used.

The methodology explained in this sections has been incorporated in a FORTRAN code named SEPAP (Seismic Elasto-Plastic Analysis of Piles) and verified by extensive comparisons with other methods, laboratory tests and field measurements. The details of these comparisons may be found elsewhere [Tabesh, 1997], [Tabesh and Poulos, 1997].

### COMPARISON WITH CENTRIFUGE TESTING

**Table 1: Details of the model pile and pile head masses**

<table>
<thead>
<tr>
<th>Item</th>
<th>Dimensions (mm)</th>
<th>Weight (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pile Head Mass</td>
<td>Diam.=43.7, Height=23.1</td>
<td>2.356</td>
</tr>
<tr>
<td>Pile Head Insert</td>
<td>Diam.=9.3, Height9.5</td>
<td>0.016</td>
</tr>
<tr>
<td>Pile Head Clamp</td>
<td>Area=19.0 X 19.0, Height=5.08</td>
<td>0.044</td>
</tr>
<tr>
<td>Conical Pile Tip</td>
<td>Diam.=9.6, Height=10.9</td>
<td>0.014</td>
</tr>
<tr>
<td>Weight of Steel Tube (including strain gauges, glue, and lead wires from base of pile head mass to tip of pile)</td>
<td>Diam.=9.52, Length=209.5</td>
<td>0.114</td>
</tr>
<tr>
<td>Steel Tube (only)</td>
<td>Diam.=9.52, Length=209.5</td>
<td>0.109</td>
</tr>
</tbody>
</table>

In this section the results of a comparison between the measurements in a centrifuge testing and the calculations based on the methodology explained in this paper is presented. The tests were performed by [Finn and Gohl, 1987] both on a single pile and a 2x2 pile group. The details of the tests along with their instrumentation and results have been reported extensively by [Finn and Gohl, 1987] and are not repeated here due to space limitation. The details of the model pile and pile head masses for the single pile test, however, are shown in Table 1 for quick reference.
The prototype pile of Table 1 was analysed by SEPAP. The head boundary condition was assumed to be free, but the eccentricity of the mass was taken into account. The tip was also assumed to be free. The soil layer was divided into 24 sub-layers with a depth of 0.5 metres each. The shear modulus at each layer was obtained from an equation presented by [Hardin and Black, 1968] as proposed by Finn and Gohl. The maximum acceleration of the input base motion is 0.158 g. The soil ultimate lateral pressure was estimated by [Broms’, 1964-b] equation, \( P_y = N_p P_p \), in which \( P_p \) is passive pressure and \( N_p \), based on a suggestion by [Poulos, 1992], was taken to be 3.

Fig. 2-a compares the computed and measured surface acceleration-time histories. They are very similar, but there are certain relatively high frequency components present in the computed response, that are not seen in the measured one. In order to see this better, the absolute value of the Fourier amplitude of the two time histories are compared in Fig. 2(b). As can be seen, the computed response contains high amplitudes for periods between 0.2 to 0.4 seconds. At other frequencies the two responses follow each other rather closely, with the computed response underestimating certain peaks.

The reason for these differences is probably the fact that in the analysis by SEPAP, the soil profile has been modeled as a layered medium. It is also likely that the measures taken during the tests to minimize the effect of lateral boundaries of the soil container have not removed all the intervening waves generated by the boundary. It is also noted that in SEPAP the free-field motions are obtained based on the assumption that the layers are horizontally infinite and no account is taken of the waves reflected by lateral boundaries.

One of the outstanding results of the single pile test was the unequivocal illustration of the interaction between the pile and its cap-mass. The recorded acceleration at the pile head was found to be significantly different from the surface acceleration. Many of the high frequency components present in the free-field record were damped out in the pile head record. This can be seen in Figs. 2 (c,d). The latter figure shows that the free field surface acceleration has 2 main peaks, one around a period of 0.37 sec. and the other around a period of 1 sec. The pile head acceleration, however, has one main peak around a period of 1 sec. The amplitudes around this period have clearly been amplified compared to the free field surface, but at the same time the amplitudes around the period of 0.37 sec have been substantially reduced compared to the free-field surface response.

The same phenomenon is quite remarkably seen in the computed response, showing that the interaction between the pile-soil system and the pile cap mass is captured in the analysis by SEPAP adequately. Fig. 2(e) shows the absolute value of the Fourier amplitude of the measured and computed pile head acceleration-time histories. It can be seen that the computed and measured responses are very close, except for a small period range around 1 sec, where the computed amplitude is smaller than the measured value.

The time histories of the measured and computed pile-head acceleration records are compared in Fig. 2(f). The two records follow each other rather closely, but the measured response has slightly higher peaks and troughs at certain times.

The computed and measured moment-time histories at the location of strain gauge No. 1 are compared in Fig. 3(a) which shows a trend very similar to the one observed for the pile head acceleration. The absolute value of the Fourier amplitude of the time histories is depicted in Fig. 3(b) which illustrates a remarkable agreement between the measured and computed values, except at a short range around the period 1 sec. in which the measured amplitude is much higher than the computed one.
In order to compare the measured and computed moments along the pile, the envelopes of the positive and negative moment at the location of the strain gauges 1 to 7 are depicted in Fig. 3 (c). This figure shows that while the measured and computed maximum moments are close, the computed values are smaller beyond a depth of 3.5 metres. This means that the part of the pile length influenced by the cap-mass during the test has been larger than the one obtained in the SEPAP analysis.

In the course of the analysis it was noted that if the elasto-plastic analysis used to compute the response was replaced by an elastic analysis, the response would have been much larger than the measured one. This can be
seen in Fig. 3(d) which compares the measured and computed pile head acceleration-time histories. The maximum pile head acceleration obtained from the elastic analysis is equal to 0.36g compared to the measured 0.18g. Similar differences exist in the computed and measured moments.

![Test 12 - Single pile - The computed and measured moment time histories at SG1](image1)

![Test 12 - Single pile - Moment at SG1](image2)

(a) The computed and measured moment-time histories at the location of strain gauge No. 1 (SG1)

(b) The absolute value of the Fourier amplitude of the measured and computed moment-time histories at SG1.

![Test 12 - Single pile - The envelope of the positive and negative moment](image3)

![Test 12 - Single pile - Pile head acceleration, elastic analysis](image4)

(c) The envelope of the positive and negative computed and measured moment at strain gauges 1-7.

(d) The computed and measured pile head acceleration-time history, using elastic analysis.

Fig. 3: The comparison of the computed and measured response in Test No. 12

**COMPARISON WITH THE 2X2 PILE GROUP TEST**

Although the method explained in this paper and incorporated in the SEPAP program has been developed for single piles it is instructive to see how well this method can estimate the maximum internal response of a pile in a pile group. One of the piles in the 2x2 pile group test carried by [Finn and Gohl, 1987] was considered and it was assumed that the pile is alone carrying one forth of the pile group cap-mass. The details of the pile group test may be found in [Finn and Gohl, 1987].

In order to compare the results of the test with the measured response the pile head boundary condition needs to be determined. Two extreme cases i.e. a free and a fixed head boundary condition were considered. Fig. 4 shows the envelope of the measured and calculated response for both boundary conditions. As can be seen the maximum values of the moment obtained in the SEPAP analyses are close to the measured values, however, neither of the two analyses have correctly estimated the location of maximum moment along the pile. In the fixed head analysis the maximum moment occurs at the head level and in the free head analysis it occurs at a depth of 2 meters bellow the head. In the measured response the maximum value of the moment occurs around the 4 meter depth. This shows that the depth of influence of the cap-mass in the response of pile groups is far greater than the one that may be obtained in a single pile analysis.
CONCLUSIONS

In this paper a simple time domain methodology for the seismic analysis of single piles was presented. The result of a comparison between the measurements in a number of centrifuge tests and the computations using the developed method was also presented. It was shown that the calculated response is close to the measurements for the case of single piles. The maximum value of the pile moment obtained in a test on a 2x2 pile group was closely estimated by the developed method, but the location of maximum moment along the pile was far deeper in the measured response.

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REFERENCES


