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ANALYSIS OF WAVE PROPAGATION IN AN OCEANIC STRUCTURE USING A NORMAL MODE SUPERPOSITION METHOD

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ABSTRACT

The effects of solid-fluid interaction on wave propagation in a deep oceanic structure was investigated in this paper. Numerical calculations were performed by a normal mode superposition method. According to the numerical results, effects of solid-fluid interaction had an influence on propagation of Rayleigh waves.

KEYWORDS

Discrete wavenumber method; finite element method; normal mode superposition; solid-fluid interaction.

INTRODUCTION

It is important to examine the effects of fluid layer on wave propagation in an oceanic structure. Previously, some reserch has already been carried out regarding the wave propagation in an oceanic structure. For example, Biot (1952) examined the sub oceanic Rayleigh wave mode using the analytical approach, and Tan (1989) formulated finite element method for a layered solid-fluid medium. In this paper, wave propagation in a deep oceanic structure is investigated to examine the effects of solid-fluid interaction. A normal mode superposition method (Touhei, 1994 and 1995) is introduced here to examine the solid-fluid interaction effects in the Rayleigh wave modes.

THEORY AND METHOD

Governing Equations for the System

The wave motion in an isotropic elastic space is expressed as

$$(\lambda + \mu)\nabla\nabla \cdot \boldsymbol{u} + \mu\nabla^2\boldsymbol{u} - \rho_s\partial_t^2\boldsymbol{u} = -\boldsymbol{f} \qquad (1)$$

where $\boldsymbol{u}$ is displacement, λ and μ Lamé's constants, ρ_s the mass density of solid, ∇ the gradient operator, ∂ the partial differential operator whose subscript denotes the derivative argument and $\boldsymbol{f}$

the force density in solid. Assume that fluid is compressible and doesn't have viscosity. The governing equation for fluid is given by

$$\nabla^2 p - \frac{1}{c^2}\partial_t^2 p = -g \tag{2}$$

where p denotes fluid pressure, c the sound velocity of fluid and g the force density of fluid.

The continuity of the displacement at solid-fluid interface is represented using Euler's equation,

$$\boldsymbol{n} \cdot \nabla p = -\rho_f \partial_t^2 \boldsymbol{n} \cdot \boldsymbol{u} \tag{3}$$

where ρ_f is the mass density of fluid and $\boldsymbol{n}$ the outward normal unit vector from the solid surface. The equilibrium of the traction is as follows

$$\boldsymbol{\sigma} = -\boldsymbol{n}p \tag{4}$$

where $\boldsymbol{\sigma}$ denotes the traction vector of solid.

Green's Function in the Time Domain

Following Olson *et al.* (1984), solid displacement $\boldsymbol{u}$ and fluid pressure p can be represented as follows

$$\boldsymbol{u}(\boldsymbol{x}, t) = \sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}\left[U_{zk_n}^m(z,t)\boldsymbol{R}_{k_n}^m(r,\phi) + U_{rk_n}^m(z,t)\boldsymbol{S}_{k_n}^m(r,\phi) + U_{\phi k_n}^m(z,t)\boldsymbol{T}_{k_n}^m(r,\phi)\right] \tag{5}$$

$$p(\boldsymbol{x}, t) = \sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty} P_{k_n}^m(z,t)Y_{k_n}^m(r,\phi) \tag{6}$$

where m is azimuthal order number, r, ϕ, z the cylindrical coordinate notation, k_n the discrete wavenumber and $\boldsymbol{R}_{k_n}^m(r,\phi), \boldsymbol{S}_{k_n}^m(r,\phi), \boldsymbol{T}_{k_n}^m(r,\phi)$ the surface vector harmonics whose components are given by

$$\boldsymbol{R}_{k_n}^m(r,\phi) = Y_{k_n}^m(r,\phi)\boldsymbol{e}_z \tag{7}$$

$$\boldsymbol{S}_{k_n}^m(r,\phi) = \frac{1}{k_n}\partial_r Y_{k_n}^m(r,\phi)\boldsymbol{e}_r + \frac{1}{k_n r}\partial_\phi Y_{k_n}^m(r,\phi)\boldsymbol{e}_\phi \tag{8}$$

$$\boldsymbol{T}_{k_n}^m(r,\phi) = \frac{1}{k_n r}\partial_\phi Y_{k_n}^m(r,\phi)\boldsymbol{e}_r - \frac{1}{k_n}\partial_r Y_{k_n}^m(r,\phi)\boldsymbol{e}_\phi \tag{9}$$

$$Y_{k_n}^m(r,\phi) = J_m(k_n r)e^{im\phi} \tag{10}$$

where $\boldsymbol{e}_z, \boldsymbol{e}_r, \boldsymbol{e}_\phi$ are the base vectors in cylindrical coordinate system. Note that discrete wavenumber k_n satisfies the following zero equation,

$$J_m(k_n R) = 0 \tag{11}$$

where R is positive real number which is chosen large enough so that effects of image source can be removed in the time window.

Complete solution for Eqs. (1) to (4) can be expressed using a normal mode superposotion method, which is as follows,

$$\left\{\begin{array}{c} u(r,\phi,t) \\ p(r,\phi,t) \end{array}\right\} = \sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}[C_{k_n}^m(r,\phi)]\int_0^t [V_{k_n}^m][\Phi_{k_n}^m(t-\tau)][V_{k_n}^m]^{-1}[M]^{-1}\left\{\begin{array}{c} F_{k_n}^m(\tau) \\ G_{k_n}^m(\tau) \end{array}\right\} d\tau \tag{12}$$

where $C_{k_n}^m$ is the matrix whose components are the surface vector harmonics, M the mass matrix of the system, $V_{k_n}^m$ the modal matrix of the system, $F_{k_n}^m$ and $G_{k_n}^m$ the force density in solid and fluid corresponding to the discrete wavenumber domain and $\Phi_{k_n}^m(t)$ the orthogonal matrix whose components are

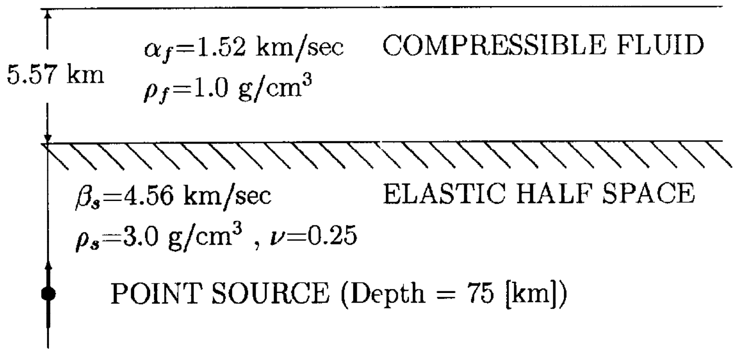


Fig. 1 A deep oceanic structure around Solomon Islands (Ewing *et al.*, 1957)

Table 1. The thickness of the finite element of Solomon Islands oceanic structure

Depth [km]	Thickness of the element [km]
0.0 ~ 5.57	0.2228
-15.0 ~ 0.0	1.0
-75.0 ~ -15.0	3.0
-100.0 ~ -75.0	5.0
-200.0 ~ -100.0	10.0
-1000.0 ~ -200.0	32.0

$$\Phi_{k_n}^m(t) = \text{diag.}\left[\varphi_{k_n(1)}^m(t), \cdots, \varphi_{k_n(N)}^m(t)\right] \tag{13}$$

$$\varphi_{k_n(i)}^m(t) = \frac{\sin\sqrt{\lambda_{k_n(i)}^m}\,t}{\sqrt{\lambda_{k_n(i)}^m}}\theta(t) \tag{14}$$

where $\lambda_{k_n(i)}^m$ is eigenvalue and $\theta(t)$ a unit step function.

NUMERICAL CALCULATIONS

A deep oceanic structure shown in Fig. 1 is analyzed to investigate the effects of solid-fluid interaction. The oceanic structure shown in Fig. 1 is around Solmon Islands (Ewing *et al.*, 1957). In the analyzed model, focal depth is 75 km, thickness of fluid 5.57 km, mass density of fluid 1.0 g/cm^3 and sound velocity 1.52 km/sec. In addition, mass density of solid is 3.0 g/cm^3, S-wave velocity 4.56 km/sec and Poisson's ratio 0.25. A rigid basement is imposed deep enough from the solid-fluid interface, which is set at the depth of 1000 km to remove the effects of the reflected waves from rigid boundary in the time window. Thickness of each element is shown in Table. 1. In the Table, the origin of the depth is set at the solid-fluid interface.

Dispersion curves of the normal modes for the oceanic structure are shown in Fig. 2. In the figure, the ordinate denotes the normalized phase velocity in which the unit phase velocity denotes the S-wave velocity of solid. It is found that most of the dispersion curves approach the P- and S-wave velocity and undispersive property of the body waves is constructed by the sum of the dispersive normal modes. Some of the normal modes are located under the S-wave velocity and flatten out near the sound wave velocity. The dispersion curves are compared with those from period equations of Biot (1952). The present dispersion curves are in almost complete agreement with solutions from the period eauation.

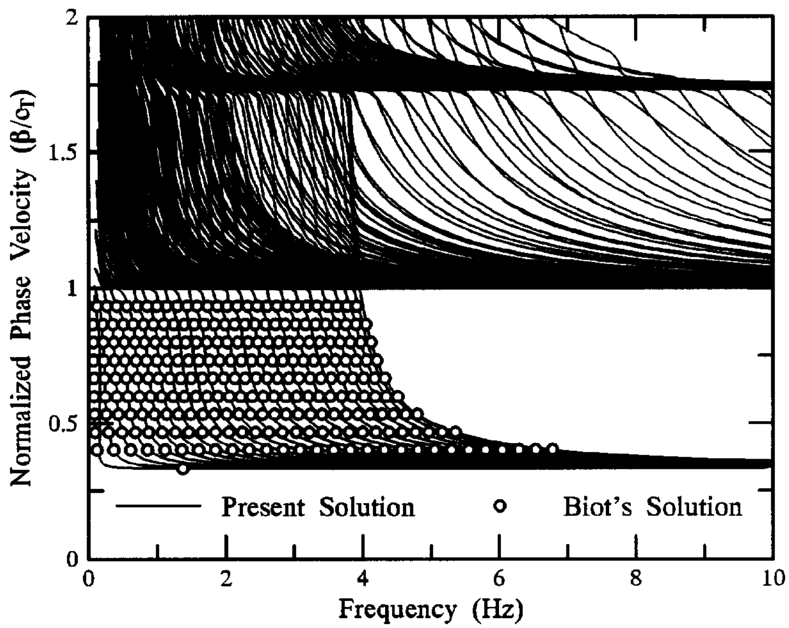


Fig. 2 Dispersion Curves of the Normal Modes

Figure 3 shows the modal shapes for fundamental and first higher Rayleigh wave mode. Modal shapes for the fluid pressure and solid displacement are normalized to have the fluid pressure have a unity amplitude. The amplitude of solid displacement is found to be quite small when compared with that of the fluid pressure, which shows the effects of solid-fluid interaction is very small.

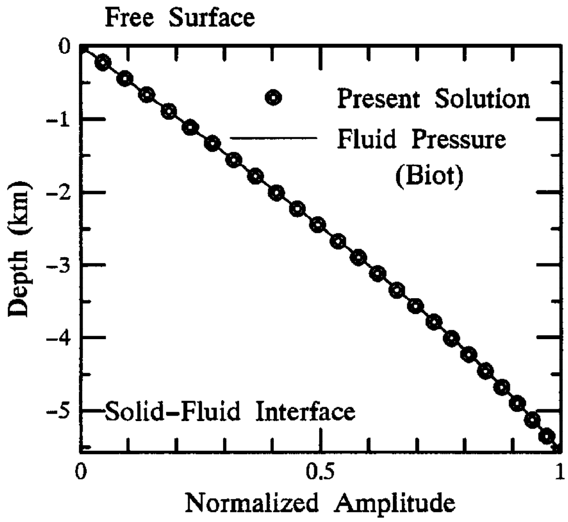
Time histories of solid displacement and fluid pressure at the solid-fluid interface are shown in Fig. 4. In the figure, the observation point is set at the epicentral distance of 375 km. The time axis in Fig. 4 is normalized so that the arrival of S-wave is unity, while solid displacement and fluid pressure is normalized using the following equations,

$$W = -\frac{\pi^2 \mu L}{3F} w \tag{15}$$

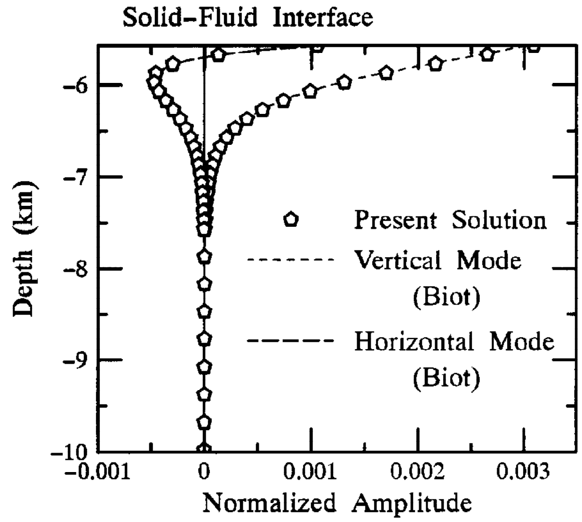
$$U = -\frac{\pi^2 \mu L}{3F} u \tag{16}$$

$$P = \frac{\pi^2 L^2}{3F} p \tag{17}$$

where W, U denote normalized vertical and horizontal solid displacement, P the normalized fluid pressure. In addition, w, u, p actual displacement and pressure, F actual magnitude of point source, L the distance between source and observed point and μ the shear modulus. In Fig. 4, sloid line indicates the present solution and broken line the exact solution of Pekeris *et al.* (1957), which is the solution of Lamb's problem in an elastic half space. It is found that solid-fluid interaction influences the arrival of Rayleigh waves, but does not affect the arrival of body waves. The real time lag of the Rayleigh wave between these solutions is about 4.5 sec.

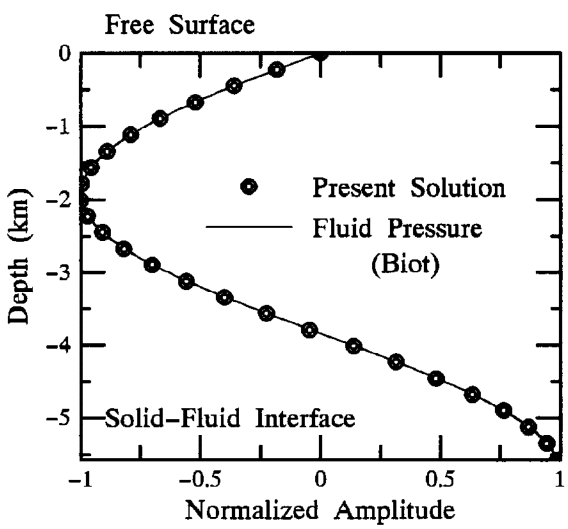


Fluid Pressure

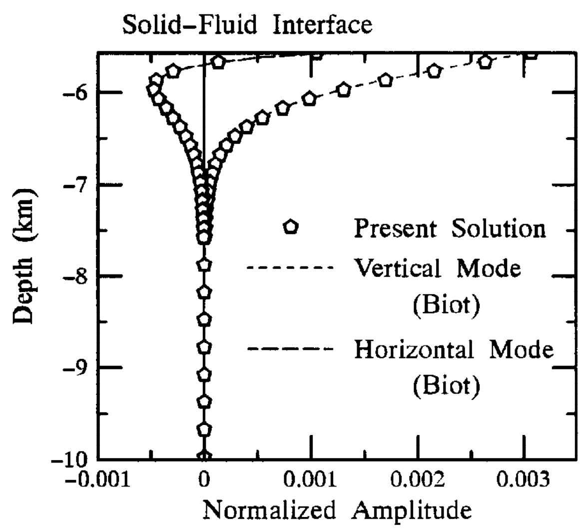


Solid Displacement

Fundamental Mode (1.02 Hz)



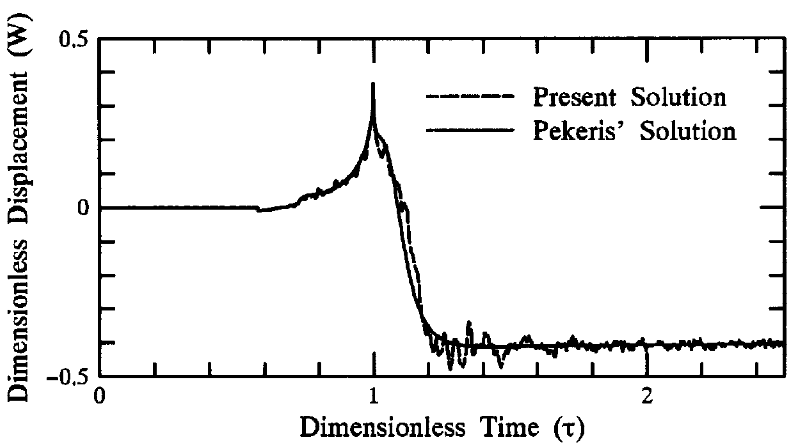
Fluid Pressure



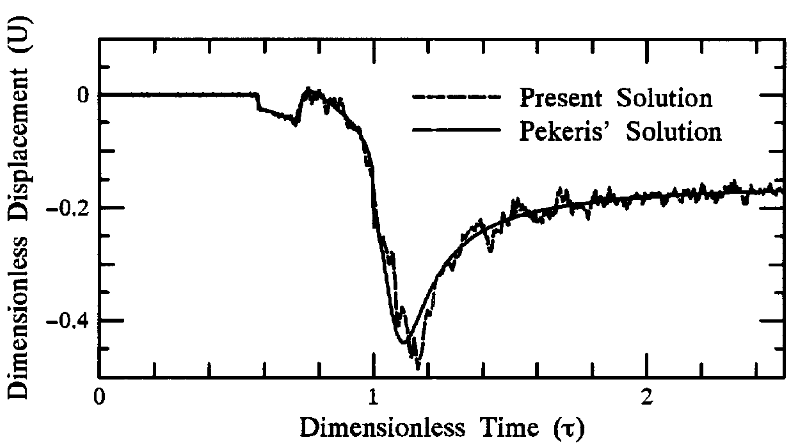
Solid Displacement

1 st Higher Mode (1.04 Hz)

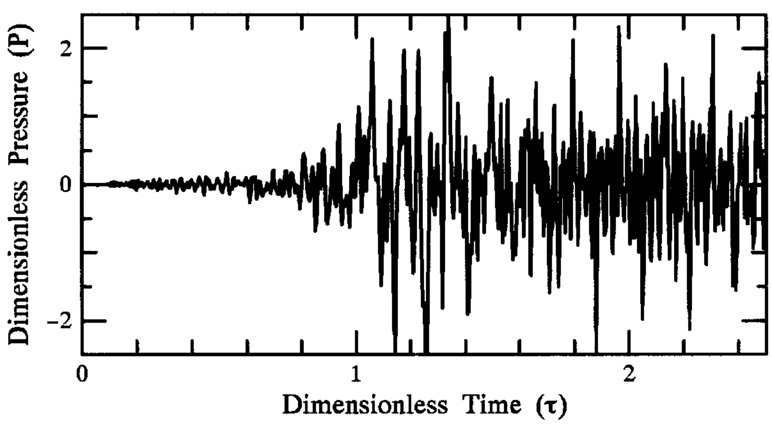
Fig. 3 Modal Shapes for Solid-Fluid Interaction System



Vertical Component



Horizontal Component



Fluid Pressure

Fig. 4 Time History ($\Delta = 375$ km)

CONCLUSION

The effects of solid-fluid interaction in a deep oceanic structure were investigated in this paper. According to the modal shapes of the solid-fluid system, the effects of solid-fluid interaction were small. It was found from the time histories of solid displacement that the apearance of the Rayleigh wave was slightly delayed due to the solid-fluid interaction.

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