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INFLUENCE OF STRAIN HARDENING ON STABILITY OF STEEL SYSTEMS DURING SEISMIC RESPONSE

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ABSTRACT

The influence of cyclic strain hardening on the dynamic instability of steel systems subjected to seismic excitation is examined. Results indicate that positive post-yielding slopes combined with strength gains due to cycling play a significant role in the stabilization of steel systems. An expression to estimate the critical intensity of a strain-hardening system from the predictions obtained with an elasto-plastic analysis is proposed.

KEYWORDS

Strain Hardening, Critical Intensity, Hysteresis, Stability Coefficient, Post-yielding Range, Instability.

INTRODUCTION

The presence of positive post-yielding slopes in the restoring curve of steel systems combined with strength gains due to cycling lead to safety margins against dynamic instability which are larger than those predicted by conventional elasto-plastic models. A study presented here demonstrates that reasonable predictions of the instability threshold considering strain hardening can be obtained by means of elasto-plastic analyses, provided that some of the system's parameters are manipulated appropriately.

INFLUENCE OF STRAIN HARDENING

The well known equation of motion of a SDOF system subjected to horizontal ground motion can be written, in incremental form, as;

$$\Delta\ddot{u} + 2\omega_o\xi_o\Delta\dot{u} + (K_t - \theta\omega_o^2)\Delta u = -\Delta\ddot{x}_h \tag{1}$$

where the symbol Δ is used to indicate increment, and the dot superscript symbolizes derivative with respect to time. ω_o = natural frequency of the system, ξ_o = critical damping ratio, θ = stability coefficient, which represents the reduction on the system's first-order tangent stiffness (K_t) due to P-δ effect, u = horizontal displacement, and $\ddot{x}_h$ = horizontal ground acceleration.

In elasto-plastic systems, the reduction in the first-order tangent stiffness due to P-δ effect is reflected in a negative effective slope after the yielding strength is reached, as shown in Fig. 1. The post-yielding slope passes from a zero value in absence of gravity load, to a constant value of $-\theta\omega_o^2$ when gravity load is present.

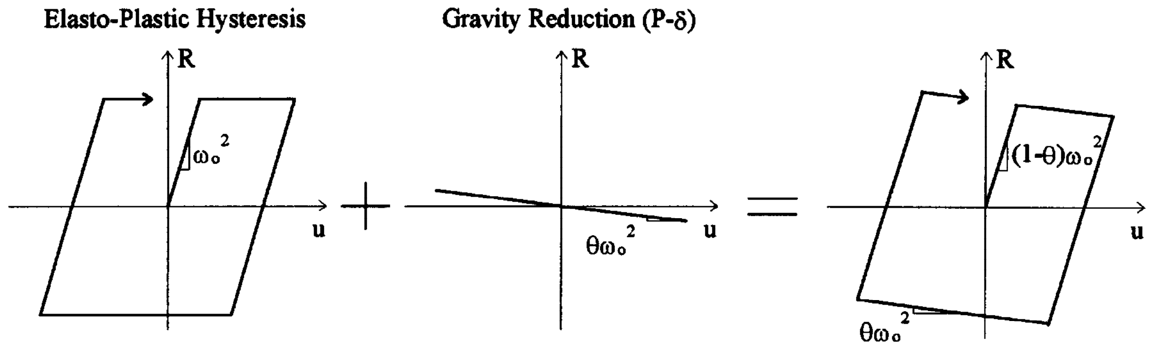

Fig. 1. Gravity reduction in an elasto-plastic system.

In systems with strain hardening, however, the situation is not as simple, since the slope varies continuously as inelasticity develops. For a given displacement amplitude, such slope depends mainly on the number of cycles previously experienced by the system. Fig. 2 shows the first-order hysteresis of a 100 inch long (A-36 Steel) cantilever beam, when cycled back and forth within a maximum amplitude of four times the yielding displacement ($\mu = 4$).

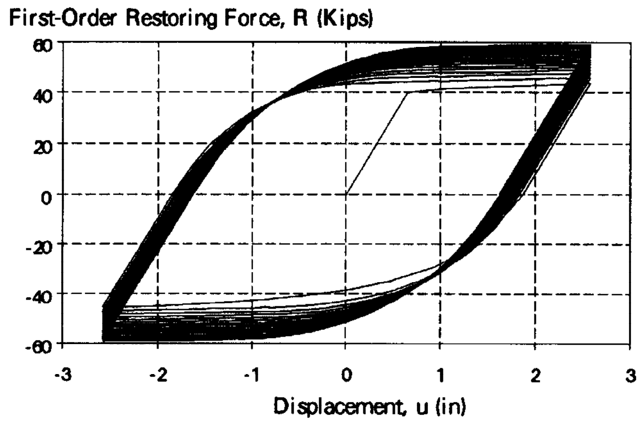

Fig. 2. Post-yielding slope and strength variation per cycle.

It can be observed that, associated with a strength gain, there is a progressive drop in the post-yielding slope with every cycle. This situation is illustrated in a more convenient form in the two plots of Fig. 3, in which the slope and strength variation registered at the maximum amplitude (normalized to the elastic slope and the yielding strength, respectively) are shown as a function of the number of cycles.

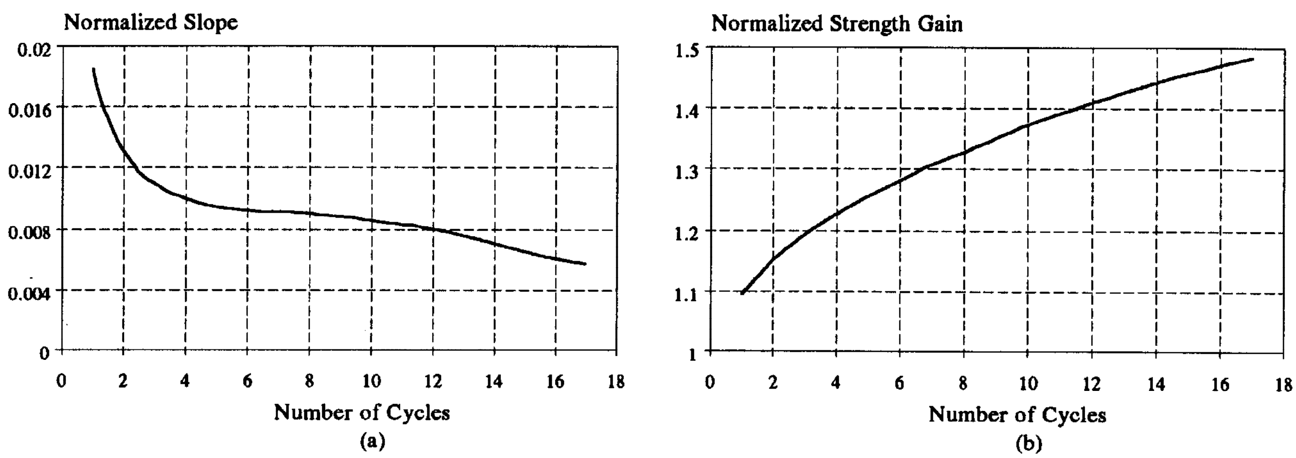

Fig. 3. Normalized slope and strength gain at $\mu = 4$ as a function of the number of cycles.

Since the gravity induced reduction takes place in this case from positive post-yielding slopes, the negative slopes that result when the P-δ effect is considered are necessarily smaller (in absolute terms) than the $-\theta\omega_o^2$

value associated with the elasto-plastic system. It is evident that such slopes vary constantly and depend on the amount of inelasticity previously developed by the system.

Depending on the relative value of the stability coefficient and the post-yielding slopes, the number of cycles needed to attain negative effective slopes (a necessary although not sufficient condition for instability) may vary from one system to another. The study of instability of steel systems then requires the development of a computer algorithm to solve the equation of motion given by Eq.(1) incorporating rules to consider the strain hardening effect. A series of hysteretic rules that reproduce such a effect together with a computer program to integrate the SDOF equation of motion are presented elsewhere (Paniagua, 1995).

CRITICAL INTENSITY OF A STRAIN-HARDENING SYSTEM

A parameter which is adequate for establishing the safety margin against an instability failure of a SDOF system subjected to a ground motion excitation is provided by the critical intensity, I_c. This intensity corresponds to the ground motion scale factor, not necessarily larger than one, above which the response displays instability. In order to examine quantitatively the influence of strain hardening in the critical intensity, let us consider once again the cantilever beam. An elasto-plastic first-order idealization, based on the plastic moment ($M_p = ZF_y$) of the section at the base, is compared in Fig. 4 with a piece-wise linear representation of the restoring force obtained using a fiber discretization.

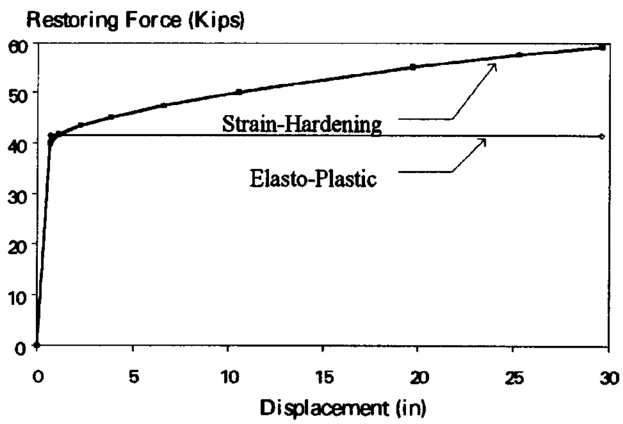

Fig. 4. Elasto-plastic (EP) and strain-hardening (SH) first-order idealizations.

For illustration, consider strain-hardening (SH) and elasto-plastic (EP) SDOF models, based on the two restoring curves presented in Fig. 4. With a damping ratio $\xi_o = 0.02$ and a period $T_o = 0.8$ sec, both systems are analyzed under three different ground motions, namely El Centro 1940 (ELC), Pacoima Dam 1971 (PAC), and Imperial Valley College 1979 (IVC). Values of the critical intensity associated with two stability coefficients (namely $\theta = 0.03$ and 0.05) are obtained with both, the strain-hardening and the elasto-plastic models. The results are compared in Table 1.

Table 1. Comparison of critical intensities with and without strain hardening.

RECORD	$\theta = 0.03$			$\theta = 0.05$		
	$I_{c(EP)}$	$I_{c(SH)}$	$I_{c(SH)}/I_{c(EP)}$	$I_{c(EP)}$	$I_{c(SH)}$	$I_{c(SH)}/I_{c(EP)}$
ELC	1.047	1.551	1.48	0.708	0.865	1.22
PAC	0.545	0.737	1.35	0.387	0.460	1.19
IVC	0.596	0.732	1.23	0.470	0.534	1.14

The results in Table 1 illustrate the fact that due to the presence of positive post-yielding slopes combined with a cyclic strength gain, the critical intensity is always higher when strain hardening is accounted for. It is evident, however, that the ratio between both critical intensities varies widely, from a minimum of 1.14 for $\theta =$

0.05 to a maximum of 1.48 for $\theta = 0.03$. These results suggest that the safety margin against instability due to strain hardening decreases when the stability coefficient is raised. Although some variability with the ground motion can be observed, the ratio between the critical intensities for a prescribed stability coefficient are about the same order. From a practical perspective, it appears that an attempt to link the critical intensity ratio with the stability coefficient could be made, independently on the ground motion characteristics.

INFLUENCE OF THE POST-YIELDING SLOPES

Because of the presence of positive post-yielding slopes, which oppose the slope reduction due to P-δ effect, it seems feasible to treat the strain hardening effect as a reduction on the stability coefficient. In this regard, it is interesting to determine the value of a modified stability coefficient θ^* such that, when used in the elasto-plastic idealization, leads to the larger critical intensity obtained with the strain-hardening model. With this objective, a numerical study for two stability coefficients and three ground motions was conducted on the elasto-plastic idealization of the cantilever beam. The results are presented in Table 2.

Table 2. Modified stability coefficients for the elasto-plastic system.

RECORD	$\theta = 0.03$		$\theta = 0.05$	
	θ^*	$\Delta\theta$	θ^*	$\Delta\theta$
ELC	0.0183	0.0117	0.0370	0.0130
PAC	0.0200	0.0100	0.0376	0.0124
IVC	0.0201	0.0099	0.0379	0.0121

In addition to the modified stability coefficient θ^*, the difference between this value and the original (defined as $\Delta\theta = \theta - \theta^*$) is presented in Table 2. It is worth noting that, for a given stability coefficient, the value of the modified coefficient does not seem to vary much from one ground motion to another. Moreover, it can be observed, from inspection of Fig. 3(a), that the number of cycles at which the reduction in the first-order slope of the strain-hardening system equals the $\Delta\theta$ values reported in Table 2 is approximately four for $\theta = 0.03$, and two for $\theta = 0.05$.

Since the values of the post-yielding slopes depend on the degree of inelasticity developed, which can be conveniently expressed in terms of the number of cycles experienced by the system, an approach for predicting the critical intensity of strain-hardening systems, using elasto-plastic models, consists in establishing a link between the stability coefficient and the number of cycles needed to induce collapse. Provided that such a number can be reasonably estimated, the plot in Fig. 3(a) could be used to determine the $\Delta\theta$ value by which the stability coefficient has to be reduced in order to approximate the effect of strain hardening using an elasto-plastic system.

Effective Stability Coefficient

Although the results summarized in Table 2 are restricted to the case of a particular cantilever beam, the approach outlined identifies some of the parameters which are relevant to the study of the effect of strain hardening on instability predictions. The objective now is to generalize the previous results to consider, in principle, any SDOF system. The first step in this direction is to establish the shape of a curve, similar to the one presented in Fig. 3(a), to determine the value of the post-yielding slope at a given amplitude as a function of the number of cycles. In this regard, the variation of the post-yielding slopes of seven SDOF systems with different first-order restoring characteristics, including the cantilever beam previously presented, is studied. The progressive decay of the post-yielding slope at an arbitrary amplitude of four times the yielding displacement ($\mu = 4$) is plotted in terms of the number of cycles in Fig. 5. The slope results are normalized to the monotonic post-yielding slope at $\mu = 4$, referred from now on as $\rho_o\omega_o^2$.

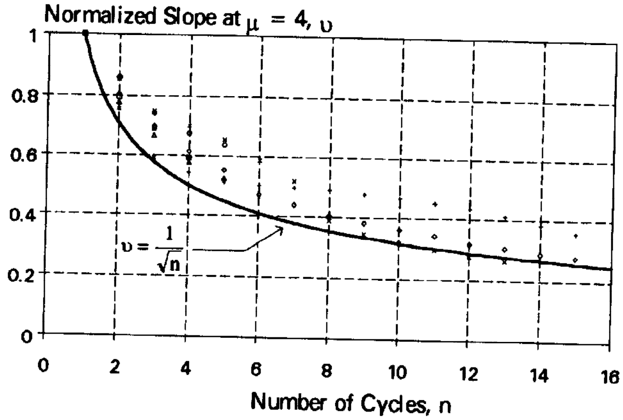

Fig. 5. Decay of the post-yielding slope as a function of the number of cycles.

It is evident from Fig. 5 that, even though the decay of the slope with every cycle varies from one SDOF to another, all the systems considered show a similar tendency. A smooth-varying curve, which represents the low-bound of all the results is presented in Fig. 5 as well. This curve is considered to predict post-yielding slope values which are conservative, from the instability perspective, and can be used to estimate the value of the minimum first-order post-yielding slope that can be expected to result after a given number of inelastic cycles has been experienced. A simple equation of such curve, obtained from a fitting process, is given by;

$$\upsilon = \frac{1}{\sqrt{n}} \tag{2}$$

where υ = fraction of the first-order post-yielding slope at $\mu = 4$, and n = number of cycles of constant amplitude.

Provided that the number of cycles, as indicative of the inelasticity undergone by the system before collapsing, can be estimated reasonably the possibility of obtaining instability predictions based on elasto-plastic analyses seems feasible. Under this perspective, the presence of positive post-yielding slopes can be treated as a reduction of the stability coefficient in the elasto-plastic system, and the effective stability coefficient θ^* may be defined as;

$$\theta^* = \theta - \upsilon \rho_o \tag{3}$$

where θ = stability coefficient of the original strain-hardening system, υ is given by Eq.(2), and ρ_o = slope of the first-order monotonic curve at $\mu = 4$, normalized to the elastic slope ω_o^2.

Equivalent Number of Cycles. The number of cycles is a measure of the energy dissipated by the system through hysteretic behavior. An estimate of such energy is provided by the area enclosed in the force-displacement hysteresis, which in turn depends on the ground motion characteristics, the first-order restoring skeleton, and the stability coefficient. An approach to determine the equivalent number of cycles that a system with a prescribed stability coefficient can undergo before collapsing when subjected to a particular ground motion is proposed by Paniagua (1995). A study on the equivalent number of cycles, performed with seven different strain-hardening SDOF systems subjected to various ground motion records leads to the following observations:

1) For systems with reasonable values of the stability coefficient (i.e. typical gravity loads encountered in common structural systems), the equivalent number of cycles is, in most of the cases, between two and four.

2) For a given SDOF system, with a prescribed stability coefficient, the equivalent number of cycles does not vary much from one ground motion to another.

3) For a given SDOF system subjected to a prescribed ground motion, the equivalent number of cycles depends on the value of the stability coefficient. The larger the stability coefficient, the smaller the equivalent number of cycles.

4) Even though the scatter of the results is considerable, the best correlation between the equivalent number of cycles and the system's parameters is achieved in terms of the difference (θ - ρ_o). The plot presented in Fig. 6 contains the data obtained in the analysis of the SDOF systems and illustrates this point.

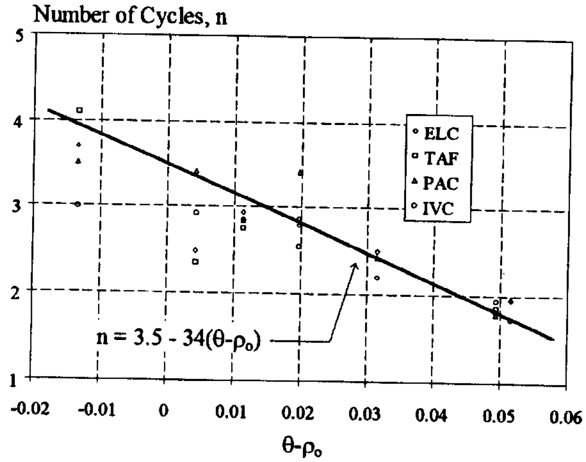

Fig. 6. Variation of the number of cycles.

Recognizing the limitations imposed by the few data available, the straight line shown in Fig. 6 is judged to be an adequate representation of the variation of the equivalent number of cycles in terms of the difference (θ - ρ_o). The equation of such a straight line is;

$$n = 3.5 - 34(\theta - \rho_o) \tag{4}$$

Since the reduction in the post-yielding slope increases with the equivalent number of cycles (see Fig. 5), Eq.(4) was selected so relatively conservative estimates of the number of inelastic cycles produced by the ground motion on the verge of collapse are obtained. With the exception of one data value around (θ - ρ_o) = 0.02, Eq.(4) provides with an upper bound approximation of the equivalent number of cycles.

INSTABILITY PREDICTIONS BASED ON EQUIVALENT ELASTO-PLASTIC (EEP) ANALYSES

It has been illustrated that reasonable predictions of the instability threshold of a SDOF system taking into account the strain hardening effect can be obtained by means of a conventional elasto-plastic analysis, provided that some of the system's parameters are manipulated appropriately. The procedure for determining the critical intensity of a strain-hardening (SH) SDOF system using an equivalent elasto-plastic (EEP) system is described with detail by Paniagua (1995). This procedure involves obtaining the elastic slope ω_o^2 and the post-yielding slope $\rho_o\omega_o^2$ at $\mu = 4$ from the strain-hardening system's monotonic first-order restoring characteristics, determining the number of cycles n with Eq.(4) and the fraction υ of the first-order post-yielding slope with Eq.(2), and calculating the stability coefficient θ^* of the equivalent elasto-plastic system using Eq.(3). The critical intensity I_c for a particular ground motion can be determined by means of a series of dynamic analyses using the conventional elasto-plastic hysteresis.

Following the previous procedure, for a damping ratio $\xi_o = 0.05$, the critical intensity of two SDOF systems is obtained for four ground motions (the ground motion Taft 1952 (TAF) is included as well). The results are compared in Table 3 with the critical intensities of the strain-hardening systems and with the predictions obtained using conventional elasto-plastic (EP) idealizations. The ratios between the strain-hardening and the equivalent elasto-plastic predictions ($I_{c(SH)}/I_{c(EEP)}$), and between the strain-hardening and the conventional elasto-plastic predictions ($I_{c(SH)}/I_{c(EP)}$) are presented in Table 3 as well. Values of such ratios near to 1.0 indicate a close correlation between the predictions and the *actual* values.

Table 3. Critical intensities and critical intensity ratios.

RECORD	SDOF 1					SDOF 2				
	$I_{c(SH)}$	$I_{c(EEP)}$	$I_{c(EP)}$	$I_{c(SH)}/I_{c(EEP)}$	$I_{c(SH)}/I_{c(EP)}$	$I_{c(SH)}$	$I_{c(EEP)}$	$I_{c(EP)}$	$I_{c(SH)}/I_{c(EEP)}$	$I_{c(SH)}/I_{c(EP)}$
ELC	5.60	5.67	3.84	0.99	1.46	3.22	3.09	2.61	1.04	1.23
TAF	9.74	9.57	6.10	1.02	1.60	6.72	6.39	5.79	1.05	1.16
PAC	3.07	2.70	1.53	1.14	2.01	1.99	1.72	1.39	1.16	1.43
IVC	2.55	2.01	1.22	1.27	2.09	1.60	1.17	0.91	1.37	1.76
	AVERAGE			1.11	1.80	AVERAGE			1.16	1.40

The average correlation index for the equivalent elasto-plastic predictions is 1.13 whereas for the conventional elasto-plastic predictions an average value of 1.60 is obtained. These results indicate the feasibility of the proposed approach for estimating I_c using an equivalent elasto-plastic model based on the characteristics of the strain-hardening system, and illustrate that the improvement of its predictive capability respect to the conventional elasto-plastic system is substantial.

INFLUENCE OF THE CYCLIC STRENGTH GAIN

In addition to the progressive decay of the post-yielding slope, which has been already addressed, another feature exhibited by systems with strain hardening is the strength gain due to cycling. Although the gain respect to the yielding strength is important (as illustrated in Fig. 3(b)), the study on the effect of the post-yielding slopes reveals that the critical intensity seems to depend mainly on the latter exclusively. It appears, therefore, that the cyclic strength gain is not fundamental in the value of the critical intensity of a strain-hardening system. This situation can be explained if one realizes that the equivalent number of cycles needed to induce collapse is relatively small, being in most cases between two and four. According to Fig. 3(b), the strength gain for such range varies approximately between 1.15 and 1.25. It should be noted, however, that these values represent an estimate of the maximum strength gain that can be expected during the response. Since the average effective gain is likely to be smaller, the influence of the cyclic overstrength on the critical intensity is not necessarily of the same order, and for practical purposes can be disregarded.

GAIN IN CRITICAL INTENSITY DUE TO STRAIN HARDENING

Statistical results derived from the study of twenty four ground motions (Bernal, 1992) indicate that, with the peak ground velocity and the duration of the ground motion held constant, the critical intensity for elasto-plastic systems is inversely proportional to the stability coefficient to the power of 0.75. Since the critical intensity of a strain-hardening (SH) system can be estimated adequately using an equivalent elasto-plastic (EEP) model, the ratio between the *actual* critical intensity of the strain-hardening system ($I_{c(SH)}$) and the prediction based on a conventional elasto-plastic system ($I_{c(EP)}$) which neglects the strain hardening effect can be obtained. Such a ratio can be expressed in terms of the original and the effective stability coefficients as;

$$\frac{I_{c(SH)}}{I_{c(EP)}} = \left[\frac{\theta}{\theta^*}\right]^{0.75} \qquad (5)$$

Combining Eq.(4), Eq.(2), and Eq.(3), and substituting the result into Eq.(5), one can get;

$$\frac{I_{c(SH)}}{I_{c(EP)}} = \left(\frac{1+\dfrac{\eta}{\rho_o}}{1+\dfrac{\eta}{\rho_o}-\dfrac{1}{\sqrt{3.5-34\eta}}}\right)^{0.75} \qquad \text{where } \eta = \theta - \rho_o \qquad (6)$$

Plots of Eq.(6), obtained for different values of ρ_o, are presented in Fig. 7. It is evident that the increase in the safety margin against collapse due to strain hardening varies widely. The plots indicate that, for a prescribed value of the ρ_o coefficient, the stabilizing effectiveness of strain hardening is considerably bigger when small values of the stability coefficient are considered, what confirms the results presented in Table 1.

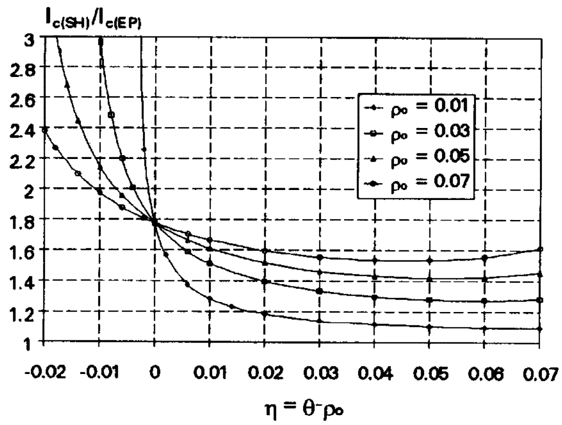

Fig. 7. Gain in critical intensity due to strain hardening.

It is interesting to note that, independently on the parameter ρ_o, for values of η equal to zero (i.e. the gravity induced reduction equals the post-yielding slope at the prescribed ductility) the gain in critical intensity due to strain hardening is approximately equal to 1.80. Needless to say that positive values of η correspond to cases in which the gravity induced reduction is larger than the post-yielding slope, then leading to a smaller effectiveness of the strain hardening phenomenon.

As illustrated in Fig. 7, for cases of practical interest (i.e. gravity loads encountered in typical systems) the critical intensity values that account for strain hardening are at least 1.10 times bigger than predictions in which its influence is neglected. For either very small stability coefficients, or large values of the post-yielding slope (as represented by ρ_o), the critical intensity with strain hardening can be as big as two to three times the elasto-plastic prediction. It is worth pointing out, however, that these cases are of limited interest since they correspond to systems in which unstable behavior due to P-δ effect does not represent a real threat.

CONCLUDING REMARKS

The study of steel systems subjected to ground motion excitation indicates that the presence of positive post-yielding slopes, combined with strength gains due to cycling, lead to values of the critical intensity which are larger than those predicted by conventional elasto-plastic models. Results show that reasonable instability predictions can be obtained by using appropriate effective reduced stability coefficients in the elasto-plastic models. A procedure is proposed to obtain such a stability coefficient as a function of known characteristics of the strain-hardening system. This procedure is shown to produce adequate predictions of the instability threshold. Finally, statistically derived results obtained on the stabilizing effectiveness of strain hardening show that the safety margin against collapse varies widely, being bigger for systems with small stability coefficients. For cases of practical interest, the critical intensity values when strain hardening is accounted for vary from 1.1 to 2.0 times the predictions obtained with elasto-plastic models.

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