WAVE PROPAGATION IN SOIL BY THEORETICAL AND EXPERIMENTAL METHODS

PH.Y. PHILIPOV, K.G. ISHTEV, P.S. DINEVA

Department of Automatics, Technical University, Sofia, Bulgaria
Fax: + 359 2 636 34 23
E-mail: RANDI@BGEARN.BITNET

ABSTRACT

Theoretical and experimental results for seismic wave propagation in multilayered soil region have been discussed. The layers are parallel and horizontal. The waves are SH - waves and "anti-plane" problem has been solved. It is satisfied condition for propagation of linear wave. Experimental records of seismic signal on the bed rock and on the free surface have been done. The theoretical obtained signals on the free surface use the experimental obtained signal records on the bed rock. Comparison of the dynamic characteristics on the free surface obtained by both: experimental and theoretical methods are shown.

KEY WORDS: SH-seismic anti-plane wave propagation problem, structural method approach, oriented graph and Mason formula

Introduction

The knowledge of real input seismic excitation is very important for aseismic design of structures. Seismic waves propagate from epicenter, coming through a geological region and come into the structure foundation. The wave field depends on the geometrical and mechanical properties of a given geological field. Such physical phenomena as refraction, reflection, diffraction, interference, damping, attenuation, geometrical and physical dispersion of seismic waves occurs.

The main aim of this paper is to present theoretical results for seismic wave propagation solution on the base the structural method. The results obtained are compared with experimental ones and the experimental method is described.
Theoretical statement of the problem

The assumptions for formulation of the wave propagation problem in soil are the following:

Soil layers are parallel and horizontally.
The incident waves on the bed rock are given by experimental records.
The “anti-plane” problem is considered, i.e. displacement has only one component \(u_z\), perpendicular to the plane \((x,y)\), where wave propagates.

The only traction component \(P_z\), which is not zero is:
\[
P_z = \sigma_{zx} n_x + \sigma_{zy} n_y \neq 0
\]
here:
\[
\sigma_{zx} = \mu \frac{\partial u_z}{\partial x},
\sigma_{zy} = \mu \frac{\partial u_z}{\partial y}
\]
\[
P_z = \left( \mu \frac{\partial u_z}{\partial x} n_x + \mu \frac{\partial u_z}{\partial y} n_y \right) = \mu \frac{\partial u_z}{\partial n}
\]

The soil is pure elastic field

The following boundary-value problem for transient wave propagation process is stated:
\[
\rho \frac{\partial^2 u_z}{\partial t^2} - \mu \Delta u_z = 0
\]
where:
\[
\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}; \rho - \text{density}; \mu - \text{shear module};
\]
\[
\frac{\partial u_z}{\partial n} = 0 \text{ on the free surface}
\]
\[
u_z^i(x, y, t) = \nu_z^{i+1}(x, y, t) \text{ on the boundaries between layers}
\]
\[
(\sigma_y n)_{z}^{-i} = (\sigma_y n)_{z}^{i+1}
\]
\[
u_z(x, y, t) = \nu_B(x, y, t) \text{ on the bed rock boundary}
\]
At infinity the Sommerfeld’s radiation condition is satisfied.

Structural method for solution of the above formulated problem.

Block diagram presentation.
Block-diagram models can be applied as a visual and effective approach for solving a wave propagation problems in multilayered media. The boundary conditions and the fundamental solutions of the problems forms the structural image of the problem in the view of an block-diagram.

In the case of horizontal and parallel layers, when the incident waves are perpendicular to boundaries a block diagram is given in Fig.1. Here arrows indicate wave propagation direction. The term \( 1+\beta_i \) presents the refraction coefficient of the signal \( U_{i-1} \) passing from \( i-1 \) to the \( i \)th layer. The block \(-\beta_i\) presents the refraction coefficient of the signal \( V_i \) on the boundary between \( i \)th and \((i-1)\)th layer. The unit denotes by \( \otimes \) express summation of the both signals. The result of summation is the signal \( U_i^+ \). The block \( W_i \) represents the layer transfer function which gives the connections of Laplace images of signals \( U_i^+ \) and \( U_i^- \). It express change of wave passing through \( i \)th layer. The units \( 1+\beta_{i-1} \) and \( \beta_{i+1} \) present the refraction and reflection coefficients on the boundary between \( i \)th and \( i+1 \)th layer.

**Transfer function of multilayered system.**

For the aims of earthquake engineering it is very important the knowledge of transfer function between signals of rock bed foundation \( X_b(s) \) and surface layer \( X_s(s) \):

\[
W(s) = \frac{X_s(s)}{X_b(s)} \quad (5)
\]

In order to obtain this transfer function the block diagram can be transformed by rules widely used in control theory. For complex systems (great number of layers) the block diagram reduction become laborious. In these cases Mason formula [1] from signal flow graphs theory can be used. In the common case of multilayered soil systems the transfer function, basing on the Mason formula is:

\[
W_{i,j} = \frac{\left( \sum_{d=1}^{r} W_d \right) \prod_{l=1}^{p} (1+W_l)^*}{\prod_{l=1}^{p} (1+W_l)^*} \quad (6)
\]

where: \( \sum_{d=1}^{r} W_d \) is the sum of transfer functions of all direct ways from point \( j \) to point \( i \); \( W_i \) is the product of layer transfer functions and refraction coefficients of blocks which take part in \( l \)th closed contour of the block.
Fig. 1. Soil block diagram
The block diagram in fig.1. has only one direct way \((r=1)\) from \(X_b(s)\) to \(X_s(s)\) with transfer function \(W_d = 2 \prod_{i=1}^{m} (1 + \beta_i) W_i\), where \(m\) is the number of layers. The transfer function of each closed contour consist at least one block from this direct way and they will not take part in the numerator of formula (6). Then the relation between bad rock foundation and surface layer is simplified and can be given by:

\[
W(s) = \frac{2 \prod_{i=1}^{m} (1 + \beta_i) W_i}{\left[ \prod_{i=1}^{p} (1 + W_i) \right]^*}
\]  

(7)

here \(p=(m+1)m/2\) is the number of all closed contours. The denominator of expression (7) is function only of second power of layer transfers functions of each layer, because \(W_i\) attains in each contour twice - when the waves propagate from bottom to surface and back.

The transfer function can be used for obtaining the signal on free surface at given bed rock signal. For the purpose Fourier integral transformation can be used. Then the formula (5) can be written as:

\[
X_s(\mathrm{j}\omega) = W(\mathrm{j}\omega) X_b(\mathrm{j}\omega)
\]

(8)

where \(W(\mathrm{j}\omega)\) is the frequency response function, \(X_b(\mathrm{j}\omega)\) and \(X_s(\mathrm{j}\omega)\) are the complex Fourier spectra of the bed rock and surface signals. Using back Fourier transformation from obtained by formula (8) complex spectra \(X_s(\mathrm{j}\omega)\) the realization of the surface signal \(X_s(t)\) can be calculated.

**Experimental methods and comparison with theoretical results.**

The experimental situation is shown on Fig.2. Recorder D1 is placed at 24 m depth and it records the signal at the bed rock. Experimentally obtained bed rock signal is shown in Fig.3. Using formula (8) we obtain the theoretical signal on the free surface, using proposed here structural method. Both signals: theoretical and experimental (recorder D2 is placed on the surface) are shown in Fig.4. The good coincidence between signals obtained by experiment and theoretical models proposed here can be seen. The normalized velocity spectra and Fourier spectral density are calculated for the signals shown on Fig.4.. The differences between both
signals (experimental and theoretical) can be explained by the fact that the theoretical model does not account for the nonelastic behavior of the soil and the damping of the seismic signals in the soil region.

Conclusion remarks

The comparison between experimental results and numerical results on the base of proposed structural method for seismic wave propagation in multilayered soil media shows:

- excellent coincidence of results in the case when velocity response spectra of signals on the free surface are compared.
- good coincidence of results in the case when Fourier spectral density curves by experimental method and structural method, which leads to analytical solution of the problem, are compared.

This work was sponsored by the grant N Е - 517/95 with the National Foundation of Science of Bulgaria.

References.

1. Jon Van de Vegte, 1994,
   Feedback Control Systems, Prentice - Hall.
D2 Surface recorder

<table>
<thead>
<tr>
<th>N</th>
<th>H</th>
<th>ρ</th>
<th>V_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.5</td>
<td>1900</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>1800</td>
<td>270</td>
</tr>
<tr>
<td>3</td>
<td>6.0</td>
<td>2200</td>
<td>420</td>
</tr>
<tr>
<td>4</td>
<td>10.0</td>
<td>2000</td>
<td>580</td>
</tr>
</tbody>
</table>

D1 Bed rock recorder

Fig.2. Experimental situation.

Fig.3. Experimental bed rock signal.

Fig.4. Experimental and theoretical surface signals.

Fig.5. Normalized velocity response spectrums of the experimental and theoretical surface signals.

Fig.6. Fourier spectrums of the experimental and theoretical surface signals.