

EVALUATION OF CODE TYPE REDUCTION FACTORS FOR STRUCTURES WITH IRREGULARITIES IN HEIGHT AND DESIGNED UNDER DIFFERENT CRITERIA

S. COMINETTI and E.F. CRUZ

Dept. Ingeniería Estructural y Geotécnica, P. Universidad Católica de Chile. Casilla 306, Correo 22, Santiago, Chile.

ABSTRACT

An explicit form of code type response modification factors that allow to compute an estimate of the maximum inelastic response of structural models characterized by a wide range of parameters is presented. The inelastic response for a given structure is obtained using these factors and the maximum elastic response obtained through the classic methods of dynamic analysis. The most important parameters influencing the response are identified and their effects provide the basis for the approximation. The average errors obtained using the approximation do not exceed 30% and the "mean plus one standard deviation" values are usually below 50%. Given the uncertainty of many of the parameters that define the real behavior of a structure to be subjected to a real earthquake, these errors are considered adequate for the purpose of understanding the probable inelastic behavior of the structure, and thus should allow the designer to make better design decisions.

KEYWORDS

Nonlinear Analysis, Response Modification Factors, Reduction Factors, Earthquake Response, Irregular Structures, Design Ductility.

INTRODUCTION

In current practice, to estimate the maximum response of a building considering only elastic behavior is rather simple, given the available computational tools at the practicing engineering level. However, to compute the response when the building is required to behave in the inelastic range still requires an important investment both of time and money that normally the designers can not afford. Furthermore, it must be considered that to carry out an analysis of this type a greater number of parameters are necessary, and that not all of them are simple to compute or even estimate.

In actual practice, an inconsistency exists between the analyses that are carried out, that consider elastic behavior of the structure, and the process used for the design of the sections, which is one that considers the possibility of inelastic behavior of the elements. In the design process the maximum element forces that the different sections of a structure should withstand are determined, but the inelastic deformations that accompany them remain unknown. There have been efforts towards estimating the inelastic response of a structure based on simplified models of it that consider the most important parameters that characterize its inelastic behavior (Bruneau, 1990). However this approach has several drawbacks, the most important

being the difficulty to represent in a simple model a complex system that includes a great diversity of parameters. Acknowledging this difficulty and realizing that the study of an elastic system through relatively complex models is already available for the designer, the idea of using the elastic analysis results with the tools used in a design office to estimate the inelastic behavior of the structure becomes feasible.

In previous studies of an assorted range of structures (Cruz and Cominetti, 1994), it has been observed that the trends in the relationship that normally are obtained between the maximum response computed from elastic analysis and those computed from inelastic analysis of a given structure are very stable. Thus it seems reasonable to generalize them and to use them in estimating the maximum inelastic response. Explicit formulations for the empirical response modification factors and for the displacements amplification factor used in American codes have been presented (Rojahn, 1988; Uang, 1991). These results are consistent with those obtained here in the sense that these factors depend on the existing over-strength and on the global ductility factor μ considered in the design. In the following paragraphs a set of simple expressions are presented to estimate the inelastic maximum response of a structure based on the response obtained from an elastic dynamic analysis, with errors smaller than 30 percent.

STRUCTURAL MODELS, EXCITATION AND RESPONSES

Basic Model, Parameters, Excitation, and Design Strength

Simple three-dimensional models of buildings, with properties typical of reinforced concrete structures are analyzed. The resisting structure consists of simple frames of a single bay and five stories (Fig. 1). Models with a single frame in the direction of the earthquake action (V direction), are considered to represent structures with low level of over-strength due to structural redundancy. Models with three frame in the direction of earthquake action are considered to represent systems with greater levels of over-strength. The resisting frames in the direction perpendicular to earthquake action (U direction) have equal characteristics to those of the V direction frames, with stiffness properties that are proportional. All the models used are characterized by having a beam to column stiffness ratio $\rho = 0.125$ in all the floors, that represents systems with typical rigid frame flexural behavior. The material behavior curve for all the sections is bilinear, with a post-yield stiffness of only 10 percent of the initial stiffness. The models consider the axial deformation of the columns of each frame. However, in the three-dimensional model used, the compatibility of the vertical deformations of the corner columns is not considered. The beams are considered to be axially rigid. The beams and the columns are designed neglecting the interaction of the axial forces with the flexural moments.

The study includes models with different fundamental periods (translational), $T_v = 0.25$, 0.75, 1.60, and 2.0 seconds, designed with global ductility factors $\mu = 1$, 2, 4, 10, and 30. The structures also have different ratios of torsional to translational frequencies, defined as $\Omega = \omega_{\theta}/\omega_{v}$ (the frequencies ω_{θ} and ω_{v} correspond to the coupled real model). The ratio Ω ranges from $\Omega = 0.75$, that corresponds to a structure with low torsional stiffness, to values of $\Omega = 1.0$ and 3.0 related to structures with greater torsional stiffness. The relative eccentricity varies between values e/r = 0 and e/r = 1.3. The floor plan aspect ratio has been varied, using values of a/b = 1.0, 0.50, and 0.33. To study the effect on the responses of the distribution over the height of the mass and of the stiffness model structures with distributions that are uniform, linear, and with a sharp decrease at the third floor (to 30 percent) are considered. The ratios between the stiffness of the frames in the U and V directions has been varied between 1, 2, 3, and 10. The values of the parameters in the actual models that were analyzed are summarized in Table 1.

The earthquake actions considered correspond to the same 8 ground acceleration records of the Central Chile earthquake of March 3, 1985, normalized to a maximum acceleration of 0.4 g used in previous studies (Cruz and Cominetti, 1990). For the building models with a single frame in the V direction a single component earthquake is used, aligned with that direction. For the building models with more than one frame in the V direction two types of earthquake loading is considered: a) single component earthquake in V direction, and b) two component earthquake, in which the component with the largest ground acceleration is used in the V direction, and the other corresponding component is used in the U direction (both scaled,

but maintaining the ratio of the original maximum ground accelerations).

For the models with a single frame in the V direction, all the frames are designed with the forces obtained from an elastic analysis of the structure subjected to the earthquake action in the V direction. This represents structures with low level of over-strength provided by the transverse frames in terms of their own strength as well as due to the existence of a single frame in the longitudinal direction (low redundancy). In the case of the model structures with three frames in the V direction, the different frames are designed using the provisions of the Chilean earthquake code NCh433Of.93 (INN, 1993), i.e. the design of the frames in each direction (U and V) is done independently considering the structure subjected to the earthquake action in the corresponding direction. These models represent structures with larger amounts of over-strength due to the greater design forces used for the transverse frames, as well as due to the existence of several frames in the direction of the earthquake action, allowing for redistribution of forces after initial yielding.

The strength at the different element sections is determined reducing the maximum moment obtained from an elastic analysis (average over the set of earthquakes) by a factor $R_{\mu} = \sqrt{2\mu}-1$. The design strength of the sections is defined according to three different criteria, that represent the level of over-strength of each design. For a large level of over-strength all the beams and columns are designed with the same strength value (the greatest moment among all elements). For a low level of over-strength the elements are designed using standard practice criteria: elements are assigned the same strength if their computed maximum moments (elastic analysis) do not differ by more than 20%. A design with no over-strength is obtained by designing each section with its corresponding maximum moment.

Response Modification Coefficients

Maximum of Local Rotation Ductility in Beams and in Columns. To evaluate the plastic rotation level at the elements when undergoing inelastic behavior, the local ductility of maximum rotation at each element ends are computed. It is defined as the ratio between the maximum plastic rotation $\theta_{\text{pl.max}}$ and the rotation that the section has when it reaches yielding θ_y , i.e. $\mu_{\theta} = \theta_{\text{pl.max}} / \theta_y$. The larger variations of the average maximum values of the local rotation ductility in the ends of the beams can be blamed on the design criteria used, based on using a reduction factor R_u and the elastic forces computed for the elements. Flexible structures usually experience local rotation ductilities in the beams and in the lower end of the first story columns that are smaller than those of moderately rigid structures, thus showing a dependency on the fundamental period of the structure. For rigid structures the maximum values of the local rotation ductility are similar to the global design ductility value used, while for the more flexible structures the values are similar to the reduction factor used for the design forces R_{μ} . Based on these observations the response modification coefficient for estimating local ductilities C_{μ} is defined as $C_{\mu} = \mu_1 / \mu$ for small T_1 , and as $C_{\mu} = \mu_{l} / R_{\mu}$ for medium and large T_{l} where μ_{l} is the local ductility value to be estimated, μ is the global design ductility value, and R_u is the reduction factor used for the design. In general, the maximum values of the local ductilities occur in the beams of the second story and in the base of the columns of the first story. Although values for the accumulated ductility were computed, it is not possible to obtain a consistent approximation for them because they show very large dispersion.

Maximum Lateral Displacements of the Diaphragm and of the Frames. The study of the response of a wide range of torsionally stiff structures, $\Omega > 1.0$, shows that low period structures experience maximum inelastic displacements that are larger than the corresponding maximum displacements computed from elastic analysis. For more flexible structures the inelastic displacements are smaller and their actual values are similar to the corresponding elastic maximum displacements. As the design forces reduction factor R_{μ} increases, the ratio between the elastic displacements and the corresponding inelastic displacements does not vary in proportional form, and it becomes smaller. For torsionally stiff structures it is convenient to define the ratio $C_{dr} = \delta_{max.el} / R_{\mu} \delta_{max.inel}$. For structures that are torsionally flexible, $\Omega < 1.0$, the behavior observed for the inelastic displacements of the story levels is exactly the opposite of that described for the torsionally stiff structures, i.e. for low period structures they are similar and only slightly larger. The

inelastic rotations of the story diaphragms are smaller than the corresponding elastic rotations in all the structures. Therefore, the corresponding displacement ratio for torsionally flexible structures can be defined as: $C_{df} = \delta_{max.inel} / R_{\mu} \delta_{max.el}$. The behavior of both C_{dr} and C_{df} is similar as the parameters that affect the inelastic behavior are varied.

ANALYSIS OF THE RESPONSE MODIFICATION COEFFICIENTS

Maximum Local Rotation Ductility in Beams and in Columns

The results of the evaluation of the ratios $C_{\mu b}$ and $C_{\mu c}$ (corresponding to beams and columns respectively) for the different model structures considered can be represented by curves that are essentially linear with values close to 1.0 and slowly decreasing as the fundamental period increases (Fig. 2). The analysis of the values obtained, allows to identify the influence of the different parameters, as follows:

- a) Ratio of Torsional and Translational Frequencies, Ω : in structures with low torsional stiffness the local ductilities increase in average in about 20% with respect to structures that have large torsional stiffness. This effect can be approximately represented by a coefficient $A_{\Omega\theta}$ equal to 1.0 for torsionally stiff structures and to 1.2 for torsionally flexible structures.
- b) Relative Stiffness of Longitudinal and Transverse Frames: as the relative stiffness of the frames in the U direction with respect to that of the frames in the V direction increases the actual values as well as the slopes of the lines decrease by different amounts, all changes being within a range of about 10%. The overall influence of these changes is not significant and will be neglected in the approximation.
- c) Level of Design Over-strength in the Elements: the maximum values of the local ductilities in the sections are dependent on the level of design over-strength assigned to the elements of the structure (Cominetti and Cruz, 1995a). To design with different levels and distributions of over-strength generates changes in the distribution of the maximum local ductilities in the height of the structure. When designing with low levels of over-strength the distribution is uniform in the height, while in the case of designing with high levels of over-strength the beams in the upper stories do no reach inelastic behavior. The maximum ductility values tend to increase when a regular structure is designed with high over-strength, and tend to decrease in the case of an irregular structure. The maximum ductility at the base of the column of the first story as a rule increases as the level of over-strength decreases and therefore C_{uc} is amplified.
- d) Irregularity of Mass and Stiffness in the Height: as a structure becomes more irregular, the values of the maximum local ductilities in the beams and in the base of the first story columns become larger (Cominetti and Cruz, 1995b), so that the corresponding values in the curves become larger.
- e) Design Level: an increase in the global design ductility has a moderate effect in the curves for $C_{\mu b}$ for structures with low levels of design over-strength. The effect is larger in the curves for $C_{\mu c}$ independent of the level of design over-strength.
- f) Level of Over-strength due to Structural Redundancy: as the over-strength due to structural redundancy increases the maximum values of the local ductilities in beams and in columns decrease, especially in the case of structures designed with large values of global design ductility.
- g) Transverse Component of the Excitation: the addition of the transverse component of the earthquake action does not generate important variations in the values of maximum local ductilities in the elements of the V direction frames. On the other hand the elements of the frames in the U direction, that tend to remain elastic for single component earthquake action, show inelastic behavior with local rotation ductility levels of the order of 75% of those developed in the elements of the V direction frames.

The relative eccentricity e/r and the aspect ratio of the plan a/b do not affect significantly the behavior of these curves.

The curves obtained for C_d are very stable. For structures with low levels of over-strength due to structural redundancy the values change rapidly for low period structures, and are practically constant for more flexible structures. For structures with larger levels of over-strength due to redundancy, the curves present a common slope for the complete range of periods considered. For different design force reduction factors R_{μ} the curves are moved in parallel form towards lower values, with decreasing differences as R_{μ} increases. This behavior is shown schematically in Fig. 3. As a rule, the variation of the different parameters does not significantly affect the values of this coefficient, except for the design forces reduction factor R_{μ} and the response quantity that is analyzed. The curve for C_d corresponding to the lateral displacement of the individual frames has the same shape of the curve for the displacements of the diaphragm, with slightly smaller values. The values depend on the design level assigned to the structure (through the reduction factor R_{μ}) and on the distribution of mass and of stiffness over the height.

PROPOSED APPROXIMATION

Estimate of Maximum Local Rotation Ductility

Recognizing that rather large uncertainties exist in the determination of the parameters that control the behavior of a real structure when subjected to a large earthquake and the rather large dispersion observed in the responses of the model studied, in the approximation being presented only the most important factors affecting the response have been considered. Nevertheless, the resulting curves, which are rather simple and have only a few parameters still provide a reasonable estimate of the inelastic response of the structure. The approximate expressions obtained for the ratios $C_{\mu b}$ and $C_{\mu c}$ defined before are:

$$C_{\mu b} = \left[1.0 A_{b1} - \frac{0.05}{A_{b1}} \right] A_{b2} A_{\Omega \theta}$$
 and $C_{\mu c} = \left[1.0 A_{c1} - \frac{0.08}{A_{c1}} \right] A_{c2} A_{\Omega \theta}$ (1)

The values of the constants A_{b1} , A_{b2} , A_{c1} , and A_{c2} are shown in Tables 2 and 3.

Estimate of Maximum Inelastic Displacements

The curves for the displacement ratios C_{dr} and C_{df} defined previously show a very stable shape, and basically vary according to the level of the design represented by the reduction factor R_{μ} , to the response being considered, and to the torsional stiffness of the structure. These effects are included in the approximation through the constants A_{l} , A_{d} , and $A_{\Omega d}$ (values are given in Tables 4, 5 and 6) resulting in the expression:

$$C_{dr} = \left[\left(0.6 + 0.2 T_{v} \right) A_{I} - \left[\frac{1}{R_{\mu=2}} - \frac{1}{R_{\mu}} \right] A_{d} \right] A_{\Omega d}$$
 (2)

where T_v is the period of the natural vibration mode with the largest equivalent translational mass (Chopra, 1981), that for structures with low torsional stiffness is not usually the same as T_1 , the fundamental vibration period. The approximation for C_{df} has the same expression with the constant A_1 always equal to 1.0. For structures with a low level of over-strength due to structural redundancy the values of C_{dr} and C_{df} should not exceed the corresponding value at $T_1 = 0.75$ secs.

The approximation for C_{dr} can be used for the displacements of the diaphragm and for the lateral displacements of the individual frames in torsionally stiff structures, and also for the lateral displacements of the individual frames in torsionally flexible structures. The approximation for C_{df} is adequate for the displacement of the diaphragm of torsionally flexible structures.

EVALUATION OF ERRORS IN THE ESTIMATE OF INELASTIC RESPONSE

The values of the maximum local rotation ductilities in beams and columns are computed using the simplified curve proposed and they are compared with the values obtained by averaging the corresponding values computed from nonlinear analysis of the structure subjected to the set of eight earthquake records mentioned earlier. Considering this average response as the "exact" value, the errors in the local rotation ductility computed with the approximate curve are in most cases within 30%, and normally do not exceed 20%. The standard deviation of the errors is normally below 35% for the beams and 45% for the columns. The average of the absolute values of the errors is about 12% for both beams and columns, and the average of the "mean plus one standard deviation" values of the errors is about 40%. These results suggest that the quality of the approximation obtained can be considered acceptable, at least from a practical stand point.

The errors in the estimates of the maximum inelastic displacements are quite low, of the order of 10% to 15%. There are a few cases where larger errors are obtained, but even those do not exceed 30%. The standard deviation of these errors varies normally between 10% and 20%, with a few cases in which they can reach up to about 35%. The "mean plus one standard deviation" value of the errors rarely exceeds 50% and has an average of about 30%.

CONCLUSIONS

The study has included a large set of model structures characterized by an wide range of parameters. A comparison of the responses obtained from elastic analysis and from nonlinear analysis has allowed to identify relationships between maximum responses in both cases that show trends which are regular through a wide range of variation of the parameters, thus allowing to generate approximate expressions for estimating the maximum values of some of the responses in the inelastic range based on the results of a conventional elastic dynamic analysis.

The parameters of the structure that have the most important effects in these expressions are the global design ductility, the fundamental period, the torsional stiffness, and the level of over-strength both at element level and in a global sense.

The response quantities that can be estimated are the maximum rotation ductility in the ends of the beams and the base of the first story columns, and the maximum displacement of the diaphragms and of the individual frames. These estimates are obtained with an error level that is considered acceptable for the purpose of a preliminary analysis of the inelastic response of real structures that should provide valuable information for design decisions.

Since only model structures have been considered, further research is needed to validate this approximation with real buildings.

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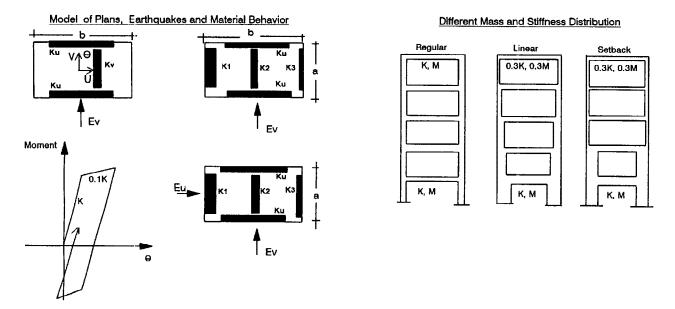


Fig. 1. Schematic representation of analyzed models.

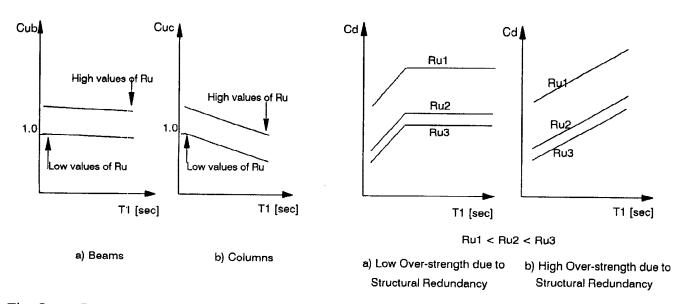


Fig. 2. Response modification coefficients for ductility in beams and columns.

Fig. 3. Response modification coefficients for displacements.

Table 1. Summary of the Analyzed Models.

a) Low Over-strength due to Structural Redundancy Tv = 0.25, 0.75, 1.6, 2.0 sec.

a/b	Ω	Ku/Kv	e/r	Distribution	Design	Global
					Overstrength	Ductility
0.5	3	1	1	Regular	High	1 2 4 10 30
0.5	3	1	1	Regular	Medium	1 2 4 10 30
0.5	3	1	1	Regular	Low	1 2 4 10 30
0.5	3	1	1	Linear	High	124
0.5	3	1	1	Linear	Medium	124
0.5	3	1	1	Linear	Low	124
0.5	3	1	1	Setback	High	124
0.5	3	1	1	Setback	Medium	124
0.5	3	1	1	Setback	Low	124
0.5	3	3	1.3	Regular	High	124
0.5	3	3	1.3	Linear	High	124
0.5	3	3	1.3	Setback	High	124
0.5	1.5	3	0.34	Regular	High	124
0.5	1.5	3	0.34	Linear	High	124
0.5	1.5	3	0.34	Setback	High	124
0.5	3	2	1.2	Regular	High	124
0.5	2	1	0.62	Regular	High	124
0.5	2	2	0.78	Regular	High	124
0.5	2	3	0.77	Regular	High	124
0.5	1.5	2	0.4	Regular	High	124
0.5	0.75	1	0.16	Regular	High	124
1	3	1	1.23	Regular	High	124
0.33	3	1	8.0	Regular	High	124
1	3	2	1.23	Regular	High	124
0.33	3	2	1	Regular	High	124
1	3	3	1.23	Regular	High	124
0.33	3	3	1.1	Regular	High	124
0.33	2	3	0.66	Regular	High	124
0.33	2	2	0.6	Regular	High	124

b) High Over-strength due to Structural Redundancy $\label{eq:tv} Tv = 0.25,\, 0.75,\, 2.0 \; \text{sec}.$

0.5	1	1	0	Regular	High	1 2 10
0.5	1	1	0.52	Regular	High	1 2 10
0.5	1.1	1	1.03	Regular	High	1 2 10
0.5	1.2	2	0	Regular	High	1 2 10
0.5	1.3	2	0.52	Regular	High	1 2 10
0.5	1.5	2	1.03	Regular	High	1 2 10
0.5	2.3	10	0	Regular	High	1 2 10
0.5	2.5	10	0.52	Regular	High	1 2 10
0.5	2.6	10	1.03	Regular	High	1 2 10

Table 2. A_{b1} and A_{c1} Constants.

Redundancy Overstrength	Design Overstrength	Global Ductility	Ab1	Ac1
Low	High	Low	1.0	1.0
	•	High	1.0	1.75
	Low	Low	1.0	1.0
		High	1.25	1.25
High	High	Low	1.0	1.0
		High	1.0	1.25
	Low	Low	1.0	1.0
		High	0.7	0.7

Table 3. A_{b2} and A_{c2} Constants.

Response	K, M Distribution	Design Overstrength	Ab2	Ac2
	Regular	High	1.0	1.0
V Frames		Low	1.0	1.25
	Irregular	High	1.0	1.0
		Low	1.25	1.25
	Regular	High	0.75	0.75
U Frames		Low	0.75	0.94
bidir. eq.	Irregular	High	0.75	0.75
		Low	0.94	0.94

Table 4. A_d Constant.

Redundancy	Response	Global	Ad
Overstrength		Ductility	
High and Low	Diaphragm Displacement	Low	1.0
	and V Frames Displaceme	High	1.25
High	Diaphragm Rotation	Low	1.0
	and U Frames Displaceme	High	0.5
Low	Diaphragm Rotation	Low	1.0
	and U Frames Displaceme	High	1.25

Table 5. A₁ Constant.

Response	Redundancy	Global	Α
	Overstrength	Ductility	
V Frame Displacements	Low	Low	0.8
		High	0.7
	High	Low	1.0
		High	1.0
U Frames Displacements	High	Low	1.0
		High	1.0
U Frames Displacements	High	Low	0.8
(bidir. earthquake)		High	0.6

Table 6. $A_{\Omega d}$ Constant.

Response	Frequency Ratio	AΩd
Diaphragm Displacement	> 1.0	1.0
	< 1.0	2/3
Diaphragm Rotation	> 1.0	1.0
and U Frames Displacement	< 1.0	1/3
V Frames Displacements	> 1.0	1.0
	< 1.0	2/3