INFLUENCE OF IRREGULARITIES IN HEIGHT AND DIFFERENT DESIGN CRITERIA
ON THE INELASTIC RESPONSE OF BUILDING MODELS

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ABSTRACT

Three-dimensional models of structures with different distributions of mass and stiffness in height and different fundamental vibration periods are analyzed. The overall behavior of the structure and the local behavior of the elements is studied when the structure is designed with global ductility factors of 2 and 4, and when over-strength levels due to the design of the sections is low, medium, and high. Global responses of the structure and maximum values of local rotation ductilities, maximum and accumulated, are not affected by the different over-strength levels considered. The number and the location of the sections that undergo inelastic behavior changes with to the over-strength level. The design level, measured through the global ductility, affects mainly the maximum values of the local rotation ductilities. The global responses of the structure are larger for the more irregular structures, and all the effects are more remarkable in the irregular structures. The maximum element forces become smaller as the structures become more irregular, and the largest maximum local rotation ductility values occur in the most irregular structures.

KEYWORDS

Nonlinear Analysis, Irregular Structures, Design Ductility, Earthquake Response, Design Over-strength

INTRODUCTION

The importance of the effects of irregularity in the height of buildings on their behavior has been studied recently. The different studies have had rather different objectives, all of them dealing with the many different aspects of this problem: response of a setback structure with flexible floor diaphragms (Jain and Sharma, 1988), verification of the provisions of the U.S. seismic codes for vertical irregularity (Dolce, 1988), verification of the provisions of the European codes for vertical irregularity (Pinto and Costa, 1993), validation of static analysis procedures for irregular structures (Hidalgo et al., 1993), and response study of a structure that suffered a large amount of damage in a large earthquake (Bonelli et al., 1993). In the present study, the influence that the irregularity of mass and stiffness distribution in the height of the buildings have on the elastic and the inelastic responses of the building is analyzed. Maximum values of the responses and their distributions over the height of structures with different fundamental periods and ductilities are studied. Another important topic where little is known corresponds to the factors influencing the distributions throughout the structure of local rotation ductility demand. In the design process, when the strength of a given element section is decided, the ductility that the section will need to provide when undergoing inelastic deformation, and the level of over-strength built into the structure are also determined.
either explicitly or implicitly. In current practice most of the time these decisions are only of qualitative type, given the procedures which at present are used to estimate the seismic responses of a structure, which normally are based on computing the response considering linear and elastic behavior, with a design spectrum that has been reduced considering the global energy dissipation capacity of the structure.

The level of the structural over-strength that the structure has is one of the important factors that influence its behavior when subjected to a severe earthquake. This over-strength level depends on the structural redundancy, the over-strength of the materials themselves, the load combinations considered, the participation of non-structural elements, and on the conservatism used in the design procedures (Fischinger et al., 1994; Uang and Bertero, 1991). Some of these factors are difficult to quantify and although it is always present in any actual design the level of structural over-strength can be taken advantage of only when it is adequately quantified within the design process (Uang, 1993). The evaluation of this factor can be done in an analytic manner considering the detailed inelastic behavior of the particular structure being analyzed, but this is a very difficult process. However, an approximation to this factor can be obtained through the compilation of a data base containing the largest possible number of case studies that would help to establish the parameters to develop an empirical approximation to the over-strength factor (Freeman, 1992). In this work the effect that the over-strength due to the design of the elements has on the behavior of a set of model structures is studied, thus providing some additional data for the proposed data base.

ANALYZED MODELS

To compare the behavior of building models characterized by different conditions of mass irregularity in plan and of mass and stiffness irregularity over the height of the resisting frames, and designed according to different criteria, several different models of five story buildings are studied. They are symmetric in the direction perpendicular to the earthquake action (U direction) and the different resisting frames behave essentially as flexural frames (Fig. 1). The nonlinear behavior is concentrated at the ends of the elements, and the force-deformation behavior is bi-linear, with a 10% of the initial stiffness after first yield. Stress and stiffness degradation are not considered. The three following configurations are studied: a) buildings with uniform distribution of mass and stiffness in height, b) buildings with a linear distribution of mass and stiffness in height, and c) buildings with a sharp decrease of mass and stiffness in a given story (setback).

Khan and Chopra (1976) demonstrated that for elastic behavior the analytical solution for buildings that have proportional characteristics of frames stiffnesses (called "compensable buildings") can be obtained by adequately combining the analytical solution of a single story model with three degrees of freedom (in plan floor displacements and rotation) and the analytical solution of the lateral displacements at the different stories of the corresponding plane frame model. As a consequence, the independence of the parameter called "irregularity over the height" and the ratio between torsional and lateral vibration frequencies for elastic behavior of this type of buildings can be demonstrated analytically. To test this independence when the behavior of the structure is inelastic and to observe the effect of these parameters on the different structural responses, different models are analyzed. Buildings characterized by different fundamental vibration periods $T_v$ (0.25, 0.75, 1.6, and 2.0 secs.), ratios of torsional to translational frequencies $\Omega$, and relative stiffness of the frames in the two orthogonal directions $K / K_{th}$, are studied (Table 1). The design level is represented by global ductilities $\mu = 1$ (elastic), 2, and 4. The models are subjected to the same set of eight records of the 1985 Central Chile earthquake, normalized to a maximum ground acceleration of 0.4g that has been used in previous studies (Cruz and Cominetti, 1990). The results considered are the average of the responses over the ensemble of records.

STRENGTH OF THE ELEMENTS

To define the design strength of each element $M_{\mu}$, a two step criterion has been considered. The first step is to determine the necessary strength as the strength required by the section if the behavior of the complete structure would remain elastic $M_{plastic}$, reduced by a factor $R_{\mu} = \sqrt{2\mu - 1}$, that is a function of the global design ductility $\mu$ (see Fig. 2). The second step is to recognize that in normal practice it is usual to assign
to some elements a strength level that is larger than the required, so as to obtain a simpler design. Normally this is done by providing the same strength to all those elements with similar forces, designing for the largest of each set. Because of the over-strength provided some of the sections do not yield, and therefore the distribution of the zones with nonlinear behavior is modified. To evaluate these effects, different designs are analyzed, some of them not very close to standard practice, but representative of extreme cases. These designs are specified in the following manner:

a) High over-strength level due to design: the strength assigned to all the beams corresponds to the greatest required throughout the structure (normally occurs in the second story beam). The strength assigned to all the columns is that corresponding to the greatest required throughout the structure (normally occurs at the base of the first story column).

b) Medium over-strength level due to design: the elements are grouped in sets with forces that do not vary in more than 20%, and are designed with the largest value among them. Column ends are designed for the largest force considering the end sections of the two columns at a given node.

c) Low over-strength level due to design: all sections are designed with their corresponding strength based on the maximum elastic forces computed. This implies that in a given node the strength at the column end in the story below it and that of the column end in the story above it are different.

RESULTS OBTAINED

Global Responses: Displacements and Rotations of the Diaphragms

The magnitude of the rotations and of the displacements of the floor diaphragms for structures with uniform distribution of mass and stiffness over the height is less than that obtained for structures with irregularity in height. The rotations of the floor diaphragms of the structures with linear distribution and with setback distribution are similar, especially when the behavior remains within the elastic range. The influence of the distributions of mass and stiffness over the height is more remarkable in flexible structures. In the structures with setback distribution the lateral displacements present sharp changes in the vicinity of the story where the discontinuity exists. In many cases this differences are larger when the behavior of the structure is inelastic.

The results obtained show that the two parameters that characterize the torsional behavior of a structure: the relative stiffness between the transverse frames and the longitudinal frames, $K_u/K_v$, and the ratio of torsional and translational frequencies, $\Omega = \omega_t/\omega_x$, do not modify the influence of the distributions over the height of mass and stiffness in the inelastic behavior of a building. For all practical purposes the global responses of the structure are not affected by the use of different design criteria.

Maximum Lateral Displacements of the Frames

The lateral displacements response of the individual frames depends primarily on the distance to the center of mass of the plan, because the frame displacements are a composition of the effects of the displacements and of the rotations of the diaphragm. As the structure irregularity increases, the frame displacements are larger and their distributions show more noticeable variations especially in the discontinuity zones. The frames lateral displacements are not significantly affected by the criteria used to design the element sections.

Requirements of Accumulated and Maximum Local Ductility of Rotation in the Beams

The structural over-strength level due to design of the elements produces an important effect in the location of the sections that yield and is of relative importance in the distribution of the inelastic maximum element forces. The over-strength level has almost no influence in the global responses of the structure and has
very little influence on the maximum values of accumulated and maximum local rotation ductility demand at the element ends, defined as the ratio of the accumulated and maximum plastic rotation and the rotation when the section yields respectively. In the variation of the maximum values of accumulated and maximum local rotation ductility in the beams, and in their distribution throughout the structure, the following parameters are involved:

a) Design ductility level $\mu$ and fundamental period $T_f$. These are the most important parameters affecting the maximum values of local ductility demand. The maximum values of local rotation ductility required by the beams are close to the global design ductility value considered ($\mu$) if the structure is rigid, and close to the strength reduction factor used ($R_{\mu}$) if the structure is flexible.

b) Mass and stiffness distribution over the height. As the change in mass and stiffness over the height becomes larger, the distribution of the beams local rotation ductility at the $V$ frames becomes less uniform. This effect is more notorious in the structures with larger $\mu$ values. The largest values are obtained in buildings with setback distribution and they are similar to the values of structures with linear distributions.

c) Structural over-strength due to design. This parameter has an important effect on the local rotation ductility distribution. The maximum ductility required by the beams of the upper stories is larger in the structures with low levels of over-strength due to design, and these values are significantly larger in the case of rigid structures. When structures have high levels of over-strength, the beams on the upper stories do not yield and the beams of the lower stories experience the largest ductility demands. The differences in the distribution over the height of the required ductilities are more pronounced in irregular structures when they are designed with different criteria. However in all the cases the maximum values of the local maximum rotation ductilities are quite similar; that is, a design with high over-strength does not produce significantly smaller values of maximum rotation ductilities with respect to those obtained designing without over-strength. In Fig. 3 the maximum ductilities required by beams designed with the three levels of over-strength are shown. The results for accumulated rotation ductility show similar behavior to that seen for maximum rotation ductility, but with different values.

Requirements of Accumulated and Maximum Local Ductility of Rotation in the Columns

In irregular buildings the columns require larger local maximum and accumulated rotation ductility. This effect is more notorious in the most rigid buildings. The variation however is not meaningful, the largest differences are about 40%, and occur only in a few cases, and in general they are not larger than 15%.

When all columns have been designed with the strength corresponding to the base of the first story column, the inelastic behavior is concentrated in this point, and the other sections remain in the elastic range. When a design with a medium over-strength level is used, the ductility demand is larger in the lower story columns, and decreases gradually in the upper stories. In the cases with low over-strength level the intermediate story sections have smaller ductility requirements, and the upper stories show the largest values, even larger than those at the lower stories sections. In spite of this important differences in the distribution over the height of the maximum and accumulated ductility requirements, the overall maximum values are not very different for the different design cases considered. Regular structures show smaller maximum and accumulated ductility values than irregular structures (the largest difference is about 35%). Only regular structures which are designed with a medium over-strength level show maximum ductility values that are larger than those in irregular structures. Because of the design criteria used, the actual values of the columns elastic strengths of irregular structures are similar for consecutive columns, while many of the column sections of the regular structures have strengths with differences greater than 20% and therefore are designed with their proportional strength. As a consequence, the regular structure has a smaller level of over-strength due to over-design. However, this does not modify the behavior of the accumulated ductilities, that are smaller in the regular structures than in the irregular structures. The accumulated ductilities for columns of structures designed with the three criteria are shown in Fig. 4.

All observations related to the local responses described correspond to the $V$ direction frame. The behavior
of the U direction frame responses is similar, but the results show a greater dispersion.

**Maximum Element Forces**

The magnitude of the maximum element forces computed from elastic and inelastic analysis depends strongly on the irregularity over the height of the structure. In structures with a uniform mass and stiffness distribution, the values are larger than those of irregular structures, and they decrease as the structure becomes more irregular. In the cases with low over-strength level due to design, the inelastic force distributions are similar to the elastic distributions. In structures having high over-strength level, the inelastic forces distribution is uniform.

**CONCLUSIONS**

The effect of the irregularity in height is important when the behavior of the structure remains in the elastic range as well as when it goes into the inelastic range. The responses that are most sensitive to this effect are the maximum lateral displacements of the diaphragm and the maximum values of local rotation ductility, both accumulated and maximum, at the element end sections, that tend to become larger as the structure becomes more irregular. On the other hand, the maximum element forces are not very sensitive to irregularities in height and tend to become smaller as it increases.

The use of different criteria to design the element sections, which determine different levels of over-strength, does not affect either the global responses of the structure or the maximum inelastic element forces but is very important in determining the location of the sections that undergo inelastic behavior. Nevertheless, the actual values of the maximum local rotation ductilities obtained are rather similar. When the elements design strengths are proportional to their computed maximum elastic forces, the ductility requirement distribution is uniform and fairly independent of the irregularity over the height. When the design has a high over-strength level, the largest ductility requirements are concentrated in a few locations, usually in the zones where the irregularities exist.

To assign different global ductility levels produces important changes in the maximum local ductilities, and normally the maximum values required for beams are close to the global ductility value when the structure is rigid, and close to the reduction factor when the structure is flexible. In the lower story columns of irregular buildings that do not have over-strength, the ductility requirements are up to 50% larger than the global ductility assigned to the building.

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**REFERENCES**


**Table 1. Summary of Analyzed Models Characteristics.**

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<th>Distribution</th>
<th>$\Omega_c$</th>
<th>$\Omega_{unc}$</th>
<th>$e/r$</th>
<th>$K_v/K_v$</th>
<th>Design Over-strength</th>
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<td>1.3</td>
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Note: $\Omega_c$ and $\Omega_{unc}$ are computed using frequencies of the vibration modes of the real coupled model and of the corresponding uncoupled model respectively.

**Fig. 1.** Description of the Analyzed Models.

**Fig. 2.** Force-Deformation Behavior Models.
High Level of Design Overstrength

Medium Level of Design Overstrength

Low Level of Design Overstrength

a) Uniform Distribution of K and M  b) Setback Distribution of K and M

Fig. 3. Maximum local rotation ductility in beams of the V frame (Structures designed with global ductility $\mu = 4$).
High Level of Design Overstrength

Medium Level of Design Overstrength

Low Level of Design Overstrength

a) Uniform Distribution of K and M  

b) Setback Distribution of K and M

Fig. 4. Accumulated local rotation ductility in columns of the V frame (Structures designed with global ductility $\mu=4$).