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SHEAR VIBRATION OF 3-D INHOMOGENEOUS EARTH DAMS IN TRIANGULAR CANYONS

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ABSTRACT

In this paper, by means of the shear wedge analysis, an elastic model is developed for evaluating the dynamic characteristics, namely natural frequencies and modes of transversal vibration of inhomogeneous earth dams in triangular canyons, where the inhomogeneity of the dam materials is taken into account by assuming a specific variation of the stiffness properties along the depth. By the use of the method of separation of variables and Bubnov-Galerkin approach, an approximate eigenvalue solution is given for the fundamental natural frequency of transversal vibration of the dams, and some calculation formulas for transversal earthquake response of the dams are obtained. At the end, a computed example is presented.

KEYWORDS

Transversal vibration; inhomogeneity; triangular canyon; fundamental natural frequency; earthquake response.

INTRODUCTION

In the majority of earth dams shaken by severe earthquakes, two primary types of damage have occurred (Ambraseys, 1960): longitudinal cracks at the top of the dams and transversal cracks sometimes accompanied by crest settlement. The longitudinal cracks appear to have been caused primarily by the horizontal component of the earthquake motion in the upstream-downstream direction, that is, the direction perpendicular to the axis of a dam. In contrast, transversal cracks of an earth dam can result from longitudinal dynamic strains induced by earthquake motion in the longitudinal direction (Seed et al., 1978). A rigorous analytical solution has been developed for the transversal linear shear vibration of inhomogeneous earth dams in rectangular canyons (Oner, 1984), and other two rigorous analytical solutions have been presented for the lateral linear shear response of homogeneous embankment dams in semi-cylindrical canyons (Dakoulas and Gazetas, 1986) and in semi-elliptical canyons (Dakoulas and Hsu, 1993) respectively. This paper develops an approximate method for evaluating the transversal earthquake response of inhomogeneous earth dams in triangular canyons.

DIFFERENTIAL EQUATION OF TRANSVERSAL VIBRATION

In view of the fact that earth dams are large three dimensional structures constructed from inelastic and inhomogeneous materials, the determination of their dynamic characteristics such as the natural frequencies and modes of vibration is extremely difficult. As a result, the following simplifying assumptions are made in order to derive the governing equation of earthquake motion: (1) The dam is represented by an elastic wedge with symmetrical triangular section in a rigid symmetrical triangular canyon (see Fig. 1). (2) The dam is modelled by a non-uniform elastic material that has uniform mass density and variable stiffness along the depth. The continuous variation of soil stiffness may be represented by a simplified function relationship, which the shear modulus increases as the (l/m)th power of the depth (Abdel-Ghaffar et al., 1981):

$$G(y) = G_0\left(\frac{y}{H}\right)^{1/m} \tag{1}$$

where G_0 is the shear modulus of the dam material at the base, H the height of the dam and l/m is constant (l/m = 0, 1/3, 2/5, 1/2 or 1 etc.). (3) The dam materials are linearly elastic. (4) The direction of ground motion is horizontal and perpendicular to the dam axis. (5) The interaction between the dam and the water in reservoir is negligible.

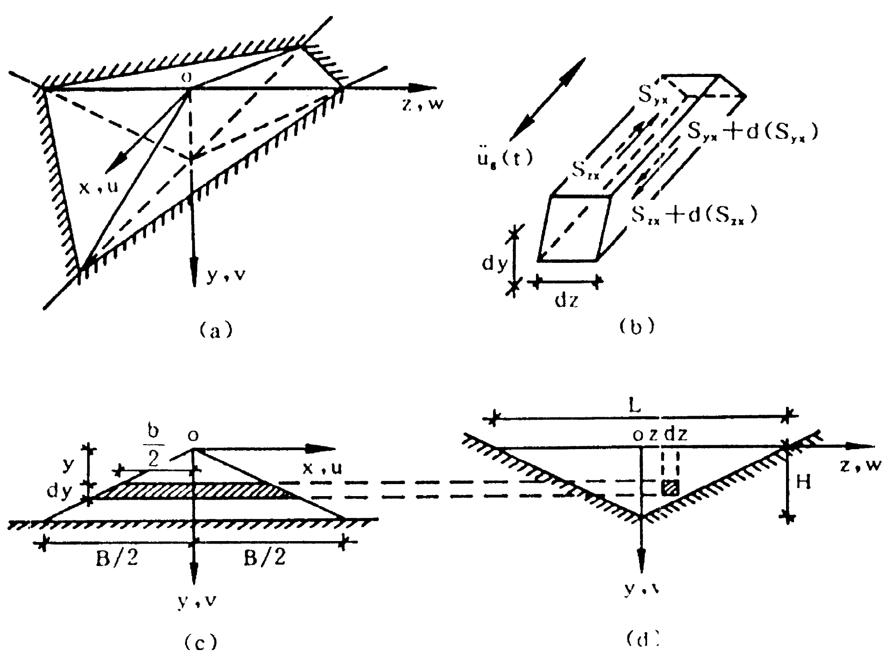

Fig. 1. Analytical model of earth dam in triangular conyon for shear wedge analysis

Figure 1 shows a dam, an axial slice of the dam, a maximum longitudinal section and a maximum transversal section. Forces acting on an element in the x-direction, as shown in Fig. 1 (b), are

(1) Inertial force:

$$F_i = \rho \frac{1}{2}\left[\frac{B}{H}y + \frac{B}{H}(y + dy)\right]dydz \frac{\partial^2 u}{\partial t^2} \approx \rho \frac{B}{H}y \frac{\partial^2 u}{\partial t^2}dydz$$

(2) Shear force on horizontal face:

$$S_{yx} = \tau_{yx} \frac{B}{H}ydz = \frac{B}{H}yG(y) \frac{\partial u}{\partial y}dz$$

(3) Shear force on vertical face:

$$S_{zx} = \tau_{zx} \frac{1}{2}\left[\frac{B}{H}y + \frac{B}{H}(y + dy)\right]dy \approx \frac{B}{H}yG(y) \frac{\partial u}{\partial z}dy$$

where t is time, ρ the density of the dam material, B the maximum width at the base, $u(y,z;t)$, τ_{yx} and τ_{zx} are the vibrational displacements and the shear stresses on horizontal and vertical face at depth y in the x-direction respectively.

For the equilibrium of an element [Fig. 1 (b)], the following equation is obtained:

$$F_i = \frac{\partial}{\partial y}(S_{yx})dy + \frac{\partial}{\partial z}(S_{zx})dz \tag{2}$$

Substituting the forces into eq. (2), the equation of motion governing free lateral vibration of the dam is obtained:

$$\frac{\partial^2 u}{\partial t^2} = \frac{v_{S0}^2}{H^{1/m}}\left(1 + \frac{1}{m}\right)y^{\frac{1}{m}-1} \frac{\partial u}{\partial y} + \frac{v_{S0}^2}{H^{1/m}}y^{\frac{1}{m}} \frac{\partial^2 u}{\partial y^2} + \frac{v_{S0}^2}{H^{1/m}}y^{\frac{1}{m}} \frac{\partial^2 u}{\partial z^2} \tag{3}$$

where $v_{S0}=(G_0/\rho)^{1/2}$ is the shear wave velocity at the base of dam material.

The boundary conditions are

$$\begin{cases} u = 0 & \text{at} \quad y = H \pm Kz \\ \frac{\partial u}{\partial y} = 0 & \text{at} \quad y = 0 \\ \frac{\partial u}{\partial z} = 0 & \text{at} \quad z = 0 \end{cases} \tag{4}$$

where $K = 2H/L$, and L is the length of the dam crest.

SOLUTION FOR FIRST NATURAL FREQUENCY

By the method of separation of variables $[u(y,z;t)=\Phi(y,z)T(t)]$, the following equations are obtained for the time and space variables:

$$T'' + \omega^2 T = 0 \tag{5}$$

$$\left(1 + \frac{1}{m}\right)y^{\frac{1}{m}-1} \frac{\partial \Phi}{\partial y} + y^{\frac{1}{m}} \frac{\partial^2 \Phi}{\partial y^2} + y^{\frac{1}{m}} \frac{\partial^2 \Phi}{\partial z^2} + \frac{\omega^2 H^{1/m}}{v_{S0}^2}\Phi = 0 \tag{6}$$

where ω is the natural frequency. The solution of eq. (5) is

$$T(t) = A_1\cos\omega t + A_2\sin\omega t \tag{7}$$

in which A_1 and A_2 are arbitrary constants.

Since the boundary conditions given by eq. (4) must be satisfied at all times, the following boundary conditions can be imposed on the function $\Phi(y,z)$:

$$\begin{cases} \Phi = 0 & \text{at} \quad y = H \pm Kz \\ \dfrac{\partial \Phi}{\partial y} = 0 & \text{at} \quad y = 0 \\ \dfrac{\partial \Phi}{\partial z} = 0 & \text{at} \quad z = 0 \end{cases} \tag{8}$$

The closed form solution of eq. (6) is difficult to be obtained. However, an approximate eigenvalue solution of eq. (6) can easily be obtained, which is a rather accurate value for the first natural frequency of vibration of the dam system.

According to the Bubnov-Galerkin method, if a function $\Phi(y,z)$ which satisfies the boundary conditions given by eq. (8) can be found, the following integral

$$\int_0^H \int_{-\frac{1}{K}(H-y)}^{\frac{1}{K}(H-y)} \left[\left(1 + \frac{1}{m}\right) y^{\frac{1}{m}-1} \frac{\partial \Phi}{\partial y} + y^{\frac{1}{m}} \frac{\partial^2 \Phi}{\partial y^2} + y^{\frac{1}{m}} \frac{\partial^2 \Phi}{\partial z^2} + \frac{\omega^2 H^{1/m}}{v_{S0}^2} \Phi \right] \Phi \, dy dz = 0 \tag{9}$$

yields an algebraic equation from which the frequency of the system can be determined.

It can easily be proved that the following function

$$\Phi(y,z) = \frac{1}{H^4}(y + H + Kz)(y + H - Kz)(y - H + Kz)(y - H - Kz) \tag{10}$$

can satisfy eq. (8). Substituting eq. (10) into eq. (9) and performing the integration, the following algebraic equation about the natural frequency is obtained:

$$\frac{H^2}{225 v_{S0}^2}\omega^2 - \left[\frac{4}{105}(2 + K^2)\frac{m}{1+m} + \frac{2}{15}(3 + K^2)\frac{m}{1+3m} - \frac{2}{3}(2 + K^2)\frac{m}{1+4m} + \frac{2}{3}(1 + K^2)\frac{m}{1+5m} + \frac{2}{15}(3 - K^2)\frac{m}{1+6m} - \frac{2}{105}(11 + 2K^2)\frac{m}{1+8m}\right] = 0 \tag{11}$$

Solving eq. (11), the formula for the natural frequency of transversal vibration of earth dam in triangular canyon is given:

$$\omega = \frac{15 v_{S0}}{H}\left[\frac{4}{105}(2 + K^2)\frac{m}{1+m} + \frac{2}{15}(3 + K^2)\frac{m}{1+3m} - \frac{2}{3}(2 + K^2)\frac{m}{1+4m} + \frac{2}{3}(1 + K^2)\frac{m}{1+5m} + \frac{2}{15}(3 - K^2)\frac{m}{1+6m} - \frac{2}{105}(11 + 2K^2)\frac{m}{1+8m}\right]^{\frac{1}{2}} \tag{12}$$

The frequency given by eq. (12) is the first natural frequency ω_1, and the function by eq. (10) is corresponded to the first mode shape Φ_1.

TRANSVERSAL EARTHQUAKE RESPONSE OF DAM

It is easily proved that the equation governing transversal vibration of the dam with damping under earthquake can be written as:

$$\frac{\partial^2 u}{\partial t^2} + \frac{c}{\rho}\frac{\partial u}{\partial t} - \frac{v_{S0}^2}{H^{1/m}}\left[\left(1 + \frac{1}{m}\right) y^{\frac{1}{m}-1}\frac{\partial u}{\partial y} + y^{\frac{1}{m}}\frac{\partial^2 u}{\partial y^2} + y^{\frac{1}{m}}\frac{\partial^2 u}{\partial z^2}\right] = -\ddot{u}_g(t) \tag{13}$$

where $\ddot{u}_g(t)$ is the acceleration of canyon in the x-direction and c is the coefficient of damping.

By the method of separation of variables and based on the orthogonality of mode shape, the following two equations for the first mode shape are obtained:

$$T_1'' + 2\lambda_1\omega_1 T_1' + \omega_1^2 T_1 = -\eta_1 \ddot{u}_g(t) \tag{14}$$

$$\left(1+\frac{1}{m}\right)y^{\frac{1}{m}-1}\frac{\partial\Phi_1}{\partial y} + y^{\frac{1}{m}}\frac{\partial^2\Phi_1}{\partial y^2} + y^{\frac{1}{m}}\frac{\partial^2\Phi_1}{\partial z^2} + \frac{\omega_1^2 H^{1/m}}{v_{S0}^2}\Phi_1 = 0 \tag{15}$$

where ω_1 is the first natural frequency given by eq. (12), λ_1 is the damping ratio of first mode, $\lambda_1 = c/2\rho\omega_1$, and η_1 is the mode participation coefficient:

$$\eta_1 = \frac{\int_0^H \int_{-\frac{1}{K}(H-y)}^{\frac{1}{K}(H-y)} \Phi_1 y\,dy\,dz}{\int_0^H \int_{-\frac{1}{K}(H-y)}^{\frac{1}{K}(H-y)} \Phi_1^2 y\,dy\,dz} \tag{16}$$

After substituting the Φ_1 from eq. (10) into eq. (16) and performing the integeration, $\eta_1 = 1.856$ is obtained. Thus, the solution of eq. (14) is obtained:

$$T_1(t) = -\frac{\eta_1}{\omega_1}\int_0^t \ddot{u}_g(\tau)e^{-\lambda_1\omega_1(t-\tau)}\sin\omega_1'(t-\tau)d\tau \tag{17}$$

where $\omega_1' = \omega_1\sqrt{1-\lambda_1^2}$, the Duhamal integeral in eq. (17) may be calculated by numerical integeration method.

Because the higher modes have little effect on earthquake response of the dam, only a few lower modes (1~3 orders) are adopted for practical requirement. So the transversal earthquake response of the dam in triangular canyon can be approximately written as follows:

$$u \approx \Phi_1 T_1,\quad \dot{u} \approx \Phi_1\dot{T}_1,\quad \ddot{u} \approx \Phi_1\ddot{T}_1,\quad \tau_{yx} \approx G\Phi_{1y}'T_1,\quad \tau_{zx} \approx G\Phi_{1z}'T_1 \tag{18}$$

where Φ_1, Φ_{1y}' and Φ_{1z}' can be determined by eq. (11) and its derivatives, T_1, $\dot{T}_1$ and $\ddot{T}_1$ can be obtained by eq. (17) and its derivtives.

In engineering it is most interesting in the maximum response of the dam, so the following formulas of maximum response are useful for earthquake-resistant design of the earth dam:

$$u_{max} \approx |\eta_1\Phi_1|S_d,\quad \dot{u}_{max} \approx |\eta_1\Phi_1|S_v,\quad \ddot{u}_{max} \approx |\eta_1\Phi_1|S_a,$$
$$\tau_{yx,max} \approx G|\eta_1\Phi_{1y}'|S_d,\quad \tau_{zx,max} \approx G|\eta_1\Phi_{1z}'|S_d \tag{19}$$

where S_d, S_v and S_a are displacement response spectrum, velocity response spectrum and acceleration response spectrum respectively.

For most earthquake motions, there is not obvious difference between the quasi-velocity response spectrum S_{pv} and velocity response spectrum S_v. So it is acceptable that the two response spectra are equal, then we get

$$\omega_1 S_d = S_{pv} = \frac{1}{\omega_1}S_a \approx S_v \tag{20}$$

Therefore, the earthquake response computation of earth dam may be performed only by use of the velocity response spectrum of earthquake motion. For example, figure 2 shows the velocity response spectrum of the NE80° component of earthquake motion in Olympia, Washington, America, in April 13, 1949 (Housner, 1959).

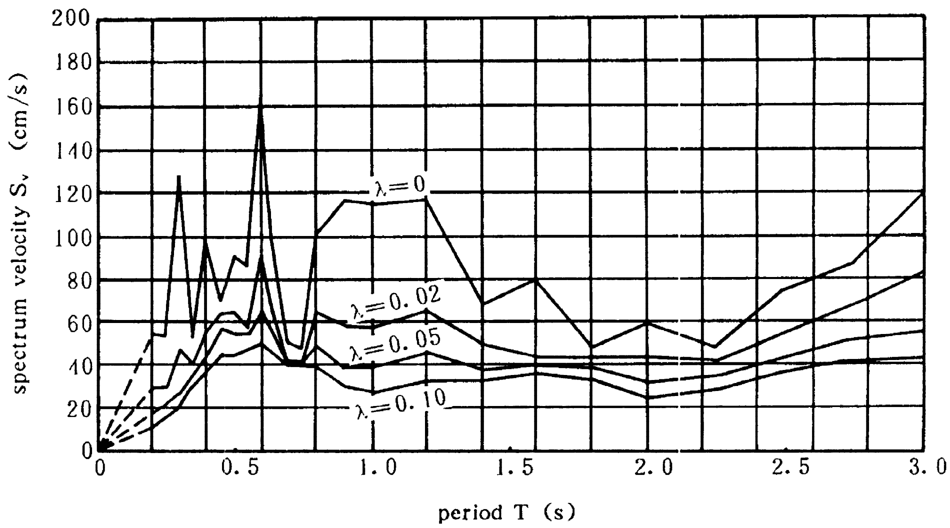

Fig. 2. The velocity response spectrum of the NE80° component of earthquake motion in Olympia, Washington, America, in April 13, 1949 (Housner, 1959).

COMPUTED EXAMPLE

Suppose a symmetrical earth dam in a symmetrical triangular canyon is subjected to a transversal earthquake motion (Olympia, Washington, April 13, 1949). The length of the dam crest is L=200 m, and the maximum height of the dam is H=50 m. The property of the dam material are ρ=2000 kg/m^3, G_0=80 MPa and λ_1=0.05. Determine the various maximum response on the centrial and L/4 sections of the dam.

Substituting the aforesaid known parameters into eq. (12), the first natural frequency ω_1 and first natural period T_{D1} are obtained. Then, according to T_{D1} and λ_1, the maximum response spectra can be determined from the response spectrum curves (shown in Fig. 2), and substituting these response spectra (S_d, S_v and S_a) and the shear modulus G into eq. (19), the maximum response by the earthquake are obtained. For the different (l/m)th powers, the different values of the first natural frequency ω_1, first natural period T_{D1}, displacement response spectrum S_d, velocity response spectrum S_v, acceleration response spectrum S_a, and the maximum value of the crest displacement u_{max}, crest velocity $\dot{u}_{max}$, crest acceleration $\ddot{u}_{max}$, and crest shear stresses on horizontal face $\tau_{yx,max}$ and on vertical face $\tau_{zx,max}$ are shown in Table 1, where ① is corresponded to the central section and ② to the L/4 section. The maximum response on central and L/4 sections for three (l/m)th powers (l/m = 0, 1/3, 1/2) are shown in Fig. 3.

Table 1 The maximum value of maximum response on central and L/4 sections for five different (l/m)th powers (l/m = 0, 1/3, 2/5, 1/2, 1)

$\frac{1}{m}$	ω_1 rad/s	T_{D1} (s)	S_d (cm)	S_v (cm/s)	S_a (cm/s²)	u_{max} (cm)	$\dot{u}_{max}$ (cm/s)	$\ddot{u}_{max}$ (cm/s²)	$\tau_{yx,max}$ (kPa)	$\tau_{zx,max}$ (kPa)
0	14.14	0.44	3.80	53.8	760.8	① 7.06	99.9	1112.3	173.5	0.0
						② 3.97	56.2	794.4	226.0	113.0
$\frac{1}{3}$	11.38	0.55	4.80	54.6	621.5	① 8.90	101.4	1153.6	184.6	0.0
						② 5.01	57.0	648.9	226.2	113.1
$\frac{2}{5}$	10.95	0.57	5.28	57.8	632.9	① 9.80	107.3	1174.8	196.3	0.0
						② 5.51	60.4	660.8	237.6	118.8
$\frac{1}{2}$	10.35	0.61	6.10	63.2	654.3	① 11.33	117.3	1214.6	216.6	0.0
						② 6.37	66.0	683.2	256.4	128.2
1	8.08	0.78	5.61	45.3	365.9	① 10.41	84.1	679.2	166.5	0.0
						② 5.86	47.3	382.0	166.6	83.3

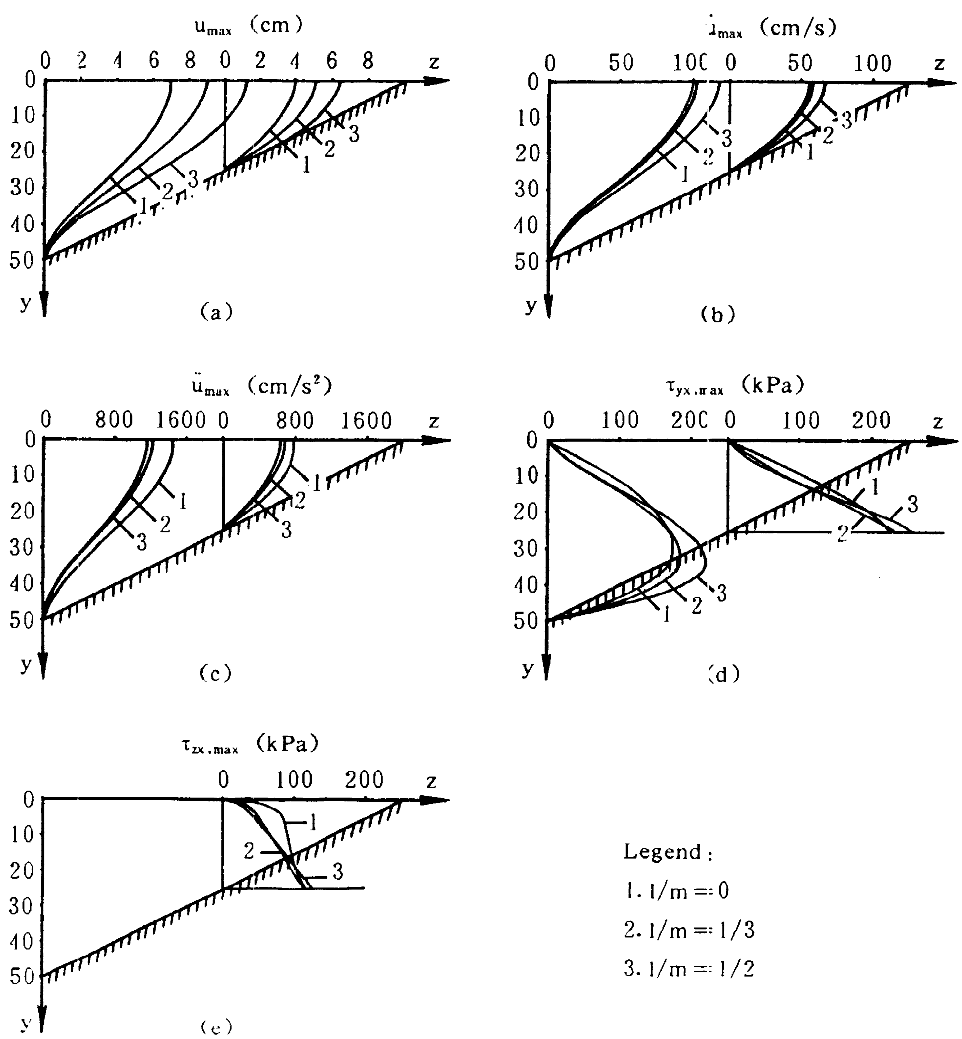

Fig. 3. The maximum response on central and L/4 sections for three different (l/m)th powers (l/m = 0, 1/3, 1/2)

CONCLUSION

The approximate analytical formulas developed in this paper are simple and they can be used for the analysis of transversal vibration of inhomogeneous earth dams in triangular conyons under earthquake motion, by use of corresponding simplified method, such as response spectrum technique. The detailed example has shown that the analytical model presented here will provide information of practical as well as academic significance.

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