CORRELATION BETWEEN THE EXPERIMENTAL AND ANALYTICAL RESULTS ON SEISMIC STRENGTHENING OF BYZANTINE CHURCHES

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ABSTRACT

Experimental and analytical investigations were performed to verify an original methodology that was developed for the repair and seismic strengthening of Byzantine churches. To investigate the nonlinear dynamic response of the structure, an original trilinear hysteretic model was developed on the basis of the experimental results obtained from seismic shaking table testing of a church model. This model allowed successful modeling and description of the behaviour of the church structures in all the characteristic phases: elastic range (no cracks), nonlinear range (occurrence of cracks), sliding range and range of heavily damage resulting in failure.

KEYWORDS

Byzantine churches, repair, seismic strengthening, analytical modeling, nonlinear dynamic response

INTRODUCTION

The investigations performed within the framework of the long-term scientific-research project "Seismic Strengthening, Conservation and Restoration of Churches Dating from the Byzantine period (9th - 14th century) in Macedonia" realized jointly by the Institute of Earthquake Engineering and Engineering Seismology - IZIIS - Skopje, the Republic Institute for Protection of Cultural Monuments RZZSK - Skopje and the "Getty Conservation Institute" - GCI from Marina del Rey, California, USA, represent unique, specific and multifaceted investigations in the field of protection of cultural heritage.

The main objective of these investigations was development of a methodology for repair and seismic strengthening of historic monuments in general, particularly churches dating from the Byzantine period located in the territory of the Republic of Macedonia. The methodology anticipated strengthening of the principal structural system of the churches in the process of their conservation, restoration and post-earthquake repair, for the purpose of obtaining a seismically resistant structure by minimal interventions.

This paper provides a brief presentation of the results from the last phase of the mentioned project that involved analytical modeling of the nonlinear dynamic response of these structures and its correlation with the experimentally obtained results.
BRIEF REVIEW OF PREVIOUS INVESTIGATIONS

The church of St Nikita as a prototype representative of Byzantine churches in Macedonia was selected on the basis of reconnaissance of 54 Byzantine churches located in Macedonia, and investigation into their main characteristics and structural systems, their typology, and global classification. In order to get an insight into the existing state of the selected church, detailed experimental, laboratory and field investigations were carried out as a prerequisite for definition of a seismic strengthening methodology. These investigations involved definition of the physical, mechanical, and chemical characteristics of the construction materials, definition of the seismic parameters of the terrain, definition of the main dynamic characteristics of the structure, and analysis of its seismic stability.

In accordance with the main principles of conservation and protection of historic monuments and taking full account of the traditional ways of construction, the specific characteristics of the church and the most recent requirements for seismic stability, an original methodology for strengthening of the structure of St. Nikita church was proposed. The methodology consisted of the following:

- Incorporation of horizontal steel ties at locations formerly occupied by existing timber belts (over the openings, at the base of the vaults, the base of the tambour and the dome) in order to assure structural integrity;
- Incorporation of vertical steel ties at the ends of the walls and around the openings for the purpose of increasing the bending resistance and the deformability capacity.

To verify experimentally the proposed methodology for repair and seismic strengthening, a model of the St. Nikita church (to a scale of 1 : 2.75, Fig. 1) was constructed in the Dynamic Testing Laboratory of IZIIS and tested on the seismic shaking table.

![Prototype and Model](image)

**Fig. 1.** Comparison between the prototype and the model of St. Nikita church

After seismic shaking table testing of the model by using selected types and intensities of earthquakes, the damaged model was repaired and strengthened in the originally proposed way and was again subjected to experimental tests. Comparison between the experimentally obtained results for the original and the strengthened model demonstrated the effectiveness of the method for increasing the seismic resistance of these structures. The method is easily applied and satisfies the basic principle in conservation and protection of historic monuments which is "minimal intervention - maximal protection".
Selection of an Appropriate Mathematical Model

The objective of the investigations in the subsequent phase of the project was the analytical modeling of the nonlinear dynamic response of the church model structure. To analyze the vibrational characteristics of a single structure, a simplified mathematical model was developed that reflects its mechanical characteristics in a most appropriate way.

The Byzantine churches studied in this project represent single-storey structures with a structural system consisting mainly of massive facade walls and two rows of symmetrically placed columns supporting the vaulted area (principal structure) and the dome through a system of pendentives and a tambour (top structure). The characteristic of single dome Byzantine churches is that they mostly suffer damage to the top structure during earthquakes. This is indicated by the fact that no original fresco painting could be found in the dome area of any church in the territory of Macedonia.

The experimental testing of the church model confirmed the vulnerability of the tambour and the dome. As a result of the high amplification of the dynamic response at the top level, the first damage occurred exactly on the dome and the upper part of the tambour, followed by loss of material from the arches and the cornices. This can be explained by the considerably lower rigidity of the top structure, which behaves as a separate structure, and its dynamic response depends upon the vibration of the principal structure.

The church model structure can be represented by a simplified mathematical model as a system with two masses, i.e., two degrees-of-freedom, concentrated at the level of the main vaults (level 1) and the dome (the top) (Fig. 2). However, in accordance with the very character of dynamic behaviour, in analyzing the dynamic response, the problem is reduced to two separate single-degree-of-freedom systems where the time history of acceleration, obtained analytically as a response of the principal structure, is used as an input excitation for the top structure.

![Diagram](image)

Fig. 2. Mathematical modeling of the church-model structure.

Selection of an Appropriate Hysteretic Diagram

The analysis of the experimentally obtained dynamic response of the church model made clear that the hysteretic model to be used for the performance of the nonlinear dynamic analysis will have to allow modeling of stiffness degradation in the post-elastic range of behaviour and the effect of shear forces. Hence and for these reasons, the trilinear hysteretic model of stiffness deterioration and pinching of the loop, i.e., the IZIIS model, Fig. 3, (originally proposed by Prof. Dr. P. Gavrilovic) was used. The main characteristics of this model are the following:
The model includes the post-ultimate range, U-M, which defines the stiffness modification even after reaching deformation, Δu, which is considered ultimate deformation in other models. This enables analysis of elements that have suffered considerable damage but have still not experienced failure;

- Considered in the model is the M-F range which enables modeling of states close to failure whereby an increase in deformation takes place without an increase of force, whereas the stiffness is practically zero;

- The model includes point S which enables consideration of stiffness variation under reversible load, i.e., modeling of the sliding effects and the effects of loss of adhesion between constitutive materials.

This sophisticated hysteretic model provides a much more realistic insight into the behaviour of the structures, i.e., their capability of absorbing hysteretic energy. Due to its complexity, there are no unique analytical expressions for definition of stiffness after points U and S which are usually either evaluated or read from experimentally obtained diagrams.

The P-δ diagrams obtained for the principal and the top structure during the seismic shaking table tests were used in the analyses presented in this paper. Figure 4 shows the procedure of gradual reading of the envelope points on the hysteretic diagram used in the analysis, from the experimentally recorded diagrams. The point of the first cracks and the stiffness after the occurrence of the first crack are obtained from the diagrams recorded under lower input accelerations (Fig. 4a), whereas the diagrams obtained under higher input accelerations (Fig. 4b) define the state of large nonlinear deformations (points M and S). The final experimental tests done under maximal input accelerations give information on the states close to failure.

Taking into account the defined laws on stiffness variation, a computer programme for performance of nonlinear dynamic analysis by applying the IZIIS-model was developed.

**Fig. 4. Definition of input parameters for the IZIIS model**
CORRELATION BETWEEN THE EXPERIMENTALLY AND THE ANALYTICALLY OBTAINED DYNAMIC RESPONSE OF THE CHURCH MODEL

Analysis of the Nonlinear Dynamic Response of the Church-Model Structure

The dynamic response of the church-model structure was obtained analytically by application of the IZIIS hysteretic model. Acceleration records applied during seismic shaking table tests were used as an input excitation for the analysis of the principal structure, whereas the acceleration obtained analytically as the response of the principal structure was used for the top structure (Fig. 2). By applying the previously mentioned computer programme, the dynamic response at both levels was obtained in the form of time histories of acceleration, force and relative displacement, frequency content of acceleration and force-displacement relationship. All the necessary input parameters for the IZIIS hysteretic model (stiffness, damping, displacements, etc.) were defined by identification of the analytical and the recorded response. Such an identification was performed for all the experimental tests of the original and the strengthened model for the purpose of defining the characteristic phases of behaviour of the church model structure. Comparison of the dynamic response obtained analytically and experimentally in all the characteristic stages of behaviour of the church model structure pointed to a high level of correlation.

Results of the Dynamic Response Analysis of the Church-Model Structure

A comparative presentation of the analytically and experimentally obtained dynamic response during one of the tests performed for the strengthened church model is given in Fig. 5 and Fig. 6. Selected in this case was one of the final tests performed by using the El Centro earthquake with an input acceleration of $a_{\text{max}} = 0.52g$ in order to present modeling of states of deep non-linearity. It is evident that there is a phase agreement of time histories of acceleration and displacement obtained for both the principal (level 1) and the top structure. The comparison of the maximal values points to a very high level of correlation between the analytically and the experimentally obtained results. Similarly, comparison of the frequency contents of analytically and experimentally obtained accelerations points to agreement of the main dynamic characteristics (predominant period, frequency, stiffness).

![Graphs](image_url)

**Fig. 5** Comparison between the analytical and experimental response of the church model (acceleration and displacement time histories)
Fig. 6 Comparison between the analytical and experimental response of the church model (frequency content and force-displacement relationship)

On the basis of the results from the nonlinear dynamic analyses performed for the original and the strengthened model by application of the IZIIS hysteretic model, the bearing (expressed via the shear base coefficient, Cbs) and deformability (expressed through the ductility) characteristics are the following:

### Table 1. Bearing characteristics of the church-model obtained by nonlinear dynamic analysis

<table>
<thead>
<tr>
<th></th>
<th>Cbs at the occurrence of the first crack</th>
<th>Ductility ($\mu=D_{\text{max}}/D_{\text{s}}$) for different input acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>level 1</td>
<td>top</td>
</tr>
<tr>
<td><strong>Original model</strong></td>
<td>0.20</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Strengthened model</strong></td>
<td>0.29</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The above results could be interpreted as follows:

- At input accelerations of $a_{\text{max}} = 0.20$ g (corresponding to an earthquake return period of 100 years), the structure of the original model will suffer considerable nonlinear deformations, whereas the strengthened model will remain completely non-damaged ($\mu < 1.0$).
- At input accelerations of $a_{\text{max}} = 0.40$ g (corresponding to a return period of 200 years), failure of certain parts of the top structure as well as nonlinear damage to the bearing walls will take place in the original model. However, the strengthened model will suffer the first heavier damage to the secondary elements.
- At input accelerations of $a_{\text{max}} = 0.60$ g (corresponding to a return period of 1000 years), the original model will experience failure, whereas the strengthened model will suffer heavy damage to the top structure and the walls but will still be far below the ultimate bearing capacity.
It can be concluded that such an analytical modeling of the dynamic behaviour of the original and the strengthened model corresponds thoroughly to what was observed during the experimental tests.

Table 2 the summarizes the results of the nonlinear dynamic analysis of the original and the strengthened church model in the form of input parameters that define the envelope points of the Q-δ diagram for the hysteretic IZIIS model, depending on the initial stiffness (K), storey height (H), and the ultimate storey shear force (Q). In this way, knowing the values of K and Q, it is possible to analyze the dynamic response of real structures belonging to this or similar types.

<table>
<thead>
<tr>
<th></th>
<th>original model</th>
<th>strengthened model</th>
</tr>
</thead>
<tbody>
<tr>
<td>point C</td>
<td>Kc=K</td>
<td>Qc=25-30% Q</td>
</tr>
<tr>
<td>point U</td>
<td>Du=1.5-2‰ H</td>
<td>Qu= Q</td>
</tr>
<tr>
<td>point M</td>
<td>Dm=2.5-3.5‰ H</td>
<td>Qm=20-30% Q</td>
</tr>
<tr>
<td>point F</td>
<td>Qf=10-15‰ Q</td>
<td></td>
</tr>
<tr>
<td>point S</td>
<td>Ds=0.1‰ H</td>
<td>Qs=5-10‰ Q</td>
</tr>
</tbody>
</table>

APPLICATION OF GAINED KNOWLEDGE IN PRACTICAL ANALYSIS

The bearing capacity in the form of ultimate shear force, Q, and initial storey stiffness, K, could be defined from the storey Q-δ diagram obtained by summing up of the Qi-δi diagrams for all the individual walls, Qi representing the referent bearing capacity of the horizontal section of each wall and δi representing the corresponding displacement in respect to the geometrical stiffness of the wall. When compared to the equivalent seismic force, this ultimate shear force yields the safety factor against failure. The need for possible strengthening of the structure is based on the value of this safety factor.

Using the results of the analysis of the bearing capacity and the relationships given in Table 2, the existing and the strengthened structures of the prototype church of St. Nikita were analyzed. Figure 7 shows the input parameters for the nonlinear dynamic analysis of the existing and the strengthened state of the church structure using the IZIIS hysteretic model. The output results are presented in the form of required ductility obtained for certain types of earthquakes and intensities of 0.12g, 0.20 g, 0.34g and 0.39g (Table 3). These intensities correspond to the maximal expected accelerations with return periods of 100, 200, 1000 and 10000 years, which are the seismic parameters for the terrain of the church of St. Nikita.

![Existing Structure](image1)

![Strengthened Structure](image2)

**Fig. 7.** Input parameters for the analysis of St. Nikita church (existing and strengthened state)
Table 3. Required ductility for different input accelerations

<table>
<thead>
<tr>
<th>earthquake</th>
<th>original structure</th>
<th>strengthened structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petrovac</td>
<td>N-S</td>
<td>E-W</td>
</tr>
<tr>
<td>0.80</td>
<td>1.90</td>
<td>4.20</td>
</tr>
<tr>
<td>0.60</td>
<td>1.14</td>
<td>1.40</td>
</tr>
<tr>
<td>1.00</td>
<td>1.31</td>
<td>2.59</td>
</tr>
<tr>
<td>El Centro</td>
<td>N-S</td>
<td>E-W</td>
</tr>
<tr>
<td>0.52</td>
<td>1.00</td>
<td>3.00</td>
</tr>
<tr>
<td>0.54</td>
<td>1.11</td>
<td>1.40</td>
</tr>
<tr>
<td>0.98</td>
<td>1.30</td>
<td>1.50</td>
</tr>
</tbody>
</table>

The results obtained lead to the conclusion that, unlike the existing state, the strengthened structure of St. Nikita church has a sufficient bearing capacity and deformability and its behaviour complies with the safety criteria defined for certain earthquake intensities. The analysis performed for the maximal expected earthquake with a return period of 10000 years points to the applicability of the IZIIS model for states of high non-linearity as well. The diagram obtained for this input acceleration (0.39g) shows some stiffness and strength degradation, but dissipation of hysteretic energy is still possible. This describes the state immediately prior to failure of the structure, whereas the residual strength is due to friction.

CONCLUSIONS

On the basis of the results obtained during the analytical study, it may be concluded that use of the trilinear hysteretic model was successful for modeling the response of this historic monument structure in all the phases of its behaviour under seismic effects: elastic range (no cracks), nonlinear range (occurrence of cracks), sliding range and range of heavy damages resulting in failure.

The study enabled gaining of knowledge on the bearing capacity and deformability of the church model structure for both its existing state and the state of being strengthened in the originally proposed way. These results could be applied to the prototype St. Nikita church, as well as to similar types of historic monuments.

The analytical studies performed in this project resulted in a simplified procedure for the practical analysis of the seismic stability of this type of Byzantine church structure, as well as a seismic retrofitting method for strengthening during the process of conservation and restoration.

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