



## SIMPLIFIED EFFECTIVE STRESS PROCEDURE FOR EVALUATING SEISMIC RESPONSE OF EARTH DAMS IN 3-D

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### ABSTRACT

Earth dams in narrow canyons are complicated from many aspects, such as 3-D effect, non-linear material properties, and seismic pore water pressure generation. Rigorous analysis of their seismic response is a formidable task. In order to give preliminary estimation and parameter studies of different design schemes, this paper presents a simplified effective stress procedure for evaluating seismic response of earth dams in 3-D. The procedure is very simple and enables to determinate the first natural frequency and crest acceleration of dam in 3-D due to a specified earthquake loading by means of hand calculation even without computer.

### KEYWORDS

Effective stress procedure; seismic response; earth dam; 3-D; crest acceleration.

### INTRODUCTION

In recent two decades the analytical procedure of dynamic response for earth dam have been developed (Seed *et al.*, 1975, Mejia *et al.*, 1982). Particularly, the application of FEM in dynamic analysis of soils offers a useful means for resolving many complicated engineering problems in connection with soil liquefaction. But the dynamic analysis of earth dam with conventional methods is a troublesome expensive job. Thus, in 1979, a simplified procedure for evaluating embankment response is advanced (Makdisi *et al.*, 1979). Using only hand calculation, the procedure can offer an evaluation of max. crest acceleration induced at crest of dam by the earthquake and the natural period of vibration of earth dam in 2-D with sufficient accuracy for practical purpose. The method also allows, through iteration,

the use of strain dependent material properties. However, this method does not take into account the generation of seismic pore water pressure under earthquake and the 3-D boundary conditions of earth dam in triangular canyons. In the following the procedure will be extended to include the effects of the seismic pore water pressure and 3-D conditions of earth dam in triangular canyons.

### BASIC THEORIES AND FORMATIONS

#### Calculation of Max. Crest Acceleration and Natural Period of Dam

Figure 1 shows the max. longitudinal section and max. transversal section of dam. It is assumed that these sections are symmetrical triangular. Other assumptions inherented in the shear wedge analysis of

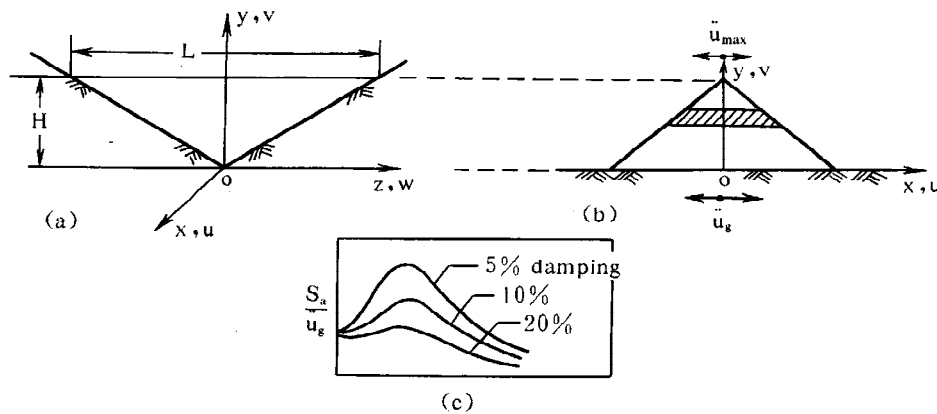


Fig. 1. Earth dam in triangular canyon  
 (a) longitudinal section; (b) transversal section;  
 (c) earthquake acceleration response spectra

earth dam are as follows: (1) the canyon walls are perfectly rigid; (2) the direction of ground motion is horizontal and parallel to the canyon walls and there are no displacements in other direction; (3) the dam is homogeneous and the dam materials are linearly elastic; (4) interaction between water in the reservoir and the dam is negligible; (5) Only shear deformation is taken into account. After shear wedge analysis, the acceleration  $\ddot{u}(y,z,t)$  in any point of dam can be approximately given by (Xu, 1993)

$$\ddot{u}(y,z,t) \approx \ddot{u}_1(y,z,t) = \eta_1 \Phi_1(y,z) \omega_1 V_1(t) \tag{1}$$

in which (effects of higher modes on response have been neglected)  $\Phi_1$  is first mode function

$$\Phi_1 = \frac{1}{H^4} \left( y + \frac{2Hz}{L} \right) \left( y - \frac{2Hz}{L} \right) \left( y - 2H + \frac{2Hz}{L} \right) \left( y - 2H - \frac{2Hz}{L} \right) \tag{2}$$

$\omega_1$  is first natural frequency

$$\omega_1 = \frac{v_s}{H} \sqrt{\frac{15}{4} + 20 \frac{H^2}{L^2}} \tag{3}$$

where L is length of dam crest, H height of dam,  $v_s = \sqrt{G/\rho}$  is shear wave velocity of the material, G shear modulus and  $\rho$  density of the material. The value  $V_1(t)$  known as Duhammel's intergral is given

by the expression

$$V_1(t) = \int_0^t \ddot{u}_g e^{-\lambda_1 \omega_1 (t-\tau)} \sin \omega_1' (t - \tau) d\tau \quad (4)$$

in which  $\omega_1' = \omega_1 \sqrt{1 - \lambda_1^2} \approx \omega_1$  for small value of  $\lambda_1$ ,  $\lambda_1$  is critical damping ratio of first mode and  $\ddot{u}_g$  is acceleration of rigid canyon in x-direction,  $t$  the time,  $\eta_1$  is first mode participation coefficient

$$\eta_1 = \frac{\int_0^H \int_0^{Ly/2H} \Phi_1(H - y) dz dy}{\int_0^H \int_0^{Ly/2H} \Phi_1^2(H - y) dz dy} = 1.839$$

The subscripts '1' of symbols  $\omega_1$ ,  $\omega_1'$ ,  $\lambda_1$ ,  $\Phi_1$ ,  $u_1$ ,  $V_1$ ,  $\dots$ , in the following will be omitted for simplification.

At the crest of central cross section of dam,  $y=H$ ,  $z=0$ ,  $\Phi_1(H, 0) = 1$ . Therefore, the value of acceleration at the crest of this section can be obtained by the expression

$$\ddot{u} \approx \eta \Phi(H, 0) \omega V(t)$$

and the max. value of crest acceleration of central cross section is as follows

$$\ddot{u}_{max} \approx \eta \Phi(H, 0) \omega S_v \approx \eta S_a \quad (5)$$

in which  $S_v$ , known as the spectral velocity, is the max. value of  $V(t)$  and is a function of  $\omega, \lambda$  and the characteristics of the ground motion  $\ddot{u}_g(t)$ . For small values of  $\lambda$  the spectral acceleration  $S_a$  is approximately equal to  $\omega S_v$ .

At the crest of  $\pm L/4$  section of dam,  $y=H$ ,  $z = \pm L/4$ ,  $\Phi(H, \pm L/4) = 0.563$ , and the max. value of crest acceleration is

$$\ddot{u}_{max} \approx \eta \Phi(H, \pm \frac{L}{4}) \omega S_v \approx 0.563 \eta S_a \quad (6)$$

### Determination of Average Shear Strain and Average Shear Stress

To estimate the strain compatible material properties, an expression for the average shear strain over the entire dam body should be determined. From the shear slice theory, the expression for shear strain at any point in the dam as a function of time is given by:

$$\gamma_{yx}(y, z, t) = \eta \frac{H^2}{v_s^2} \frac{4L^2}{45L^2 + 80H^2} \Phi_y'(y, z) \omega V(t) \quad (7)$$

in which  $\Phi_y' = \frac{1}{H^4} (4y^3 - 12Hy^2 + 8H^2y - 16\frac{H^2}{L^2}yz^2 + 16\frac{H^3}{L^2}z^2)$  is the first derivative of function  $\Phi$  for  $y$ .

Thus from eq. (7) the max. shear strain at any point may be written as

$$\gamma_{yx,max} = \eta \frac{H^2}{v_s^2} \frac{4L^2}{45L^2 + 80H^2} \Phi_y'(y, z) S_a \quad (8)$$

The average max. shear strain for the entire dam body may be determined by an average value  $(\Phi_y')_{ave}$  of the  $\Phi_y'(y, z)$  in Fig. 2, from which the average value  $(\Phi_y')_{ave} \approx 1.02/H$ . The average max. shear strain in dam body is given as

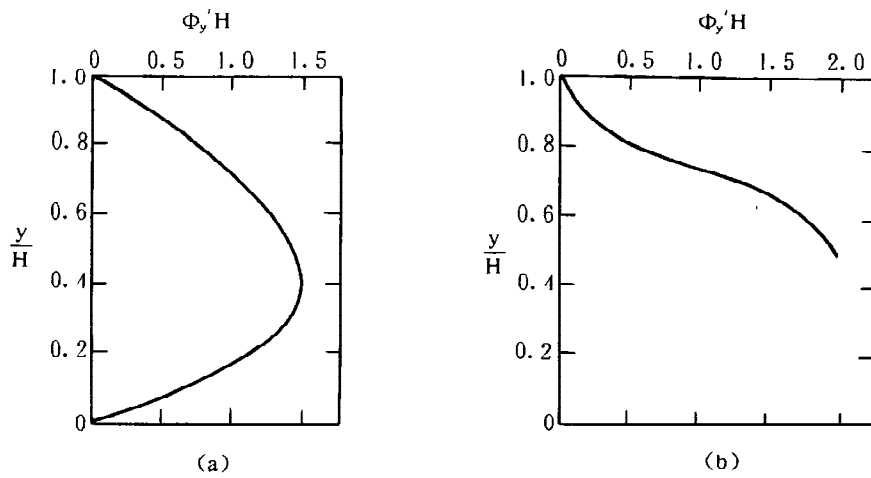


Fig. 2. Vibration of shear strain participation factor (first mode) in dam  
(a) central section; (b) L/4 section

$$(\gamma_{yx,ave})_{max} = \eta \frac{H^2}{v_s^2} \frac{4L^2}{45L^2 + 80H^2} (\Phi_y')_{ave} S_a \quad (9)$$

Assuming the equivalent cyclic shear strain is approx 65% of max. average shear  $(\gamma_{yx,ave})_{max}$ , then

$$(\gamma_{yx,ave})_{eq} = 0.65 \times 1.02 \eta \frac{H^2}{v_s^2} \frac{4L^2}{45L^2 + 80H^2} S_a \quad (10)$$

and the equivalent cyclic shear stress can be expressed as

$$(\tau_{yx,ave})_{eq} = G (\gamma_{yx,ave})_{eq} \quad (11)$$

### Evaluation of Basic Dynamic Properties

Assuming soils as equivalent visco-elastic solid, the following empirical relationships for shear modulus and damping ratio (Hardin et al., 1972) can be used

$$G = \frac{G_{max}}{1 + \gamma_b} \quad (12)$$

$$\lambda = \lambda_{max} \frac{\gamma_b}{1 + \gamma_b} \quad (13)$$

where

$$G_{max} = 220 k_{z,max} (\sigma_m')^{1/2} \quad (\text{in kPa}) \quad (14)$$

$$\gamma_b = \frac{\gamma}{\gamma_r} \left[ 1 + a \exp(-b \frac{\gamma}{\gamma_r}) \right] \quad (15)$$

in which  $\gamma$  is amplitude of dynamic shear strain,  $\gamma_r = \tau_{max}/G_{max}$ ,  $\tau_{max}$  is shear strength,  $k_{z,max}$  soil constant,  $\lambda_{max}$ ,  $a$  and  $b$  are parameters related with soil type and loading frequency.

### Estimation of Vibration Pore Water Pressure

The following Seed's empirical formula for vibration pore water pressure will be used in this paper.

$$p_g = \frac{2\sigma_v'}{\pi} = \arcsin\left(\frac{N}{N_L}\right)^{1/2\theta} \quad (16)$$

where  $p_g$  is generated pore water pressure,  $\sigma_v'$  is initial effective stress,  $N$  is number of loading cycles,  $N_L$  is number of cycles to cause liquefaction,  $\theta$  is constant.

## PROCEDURE OF COMPUTATION

Following steps are included in the procedure of computation :

Step 1 Determination of static stresses in dam. Static stresses may be calculated by FEM or by simplified method used in engineering design. Then the initial vertical effective stresses, horizontal effective stresses and mean effective stresses are obtained.

Step 2 According to average vertical effective stress in dam and the curve of shear stress ratio versus cycles to liquefaction, and assuming an average shear stress  $\tau_{yx,ave}$ , the number of loading cycles to liquefaction  $N_L$  can be obtained. For different earthquake magnitude the equivalent number of cycles  $N_{eq}$  can be taken from Table 1.

Table 1. Guideline for determining equivalent number of cycles

earthquake magnitude	5.5—6	6.5	7	7.5	8
equivalent number of cycles $N_{eq}$	5	8	12	20	30

Step 3 After  $N_L$  and  $N_{eq}$  are obtained, the seismic pore water pressure  $p_g$  may be determined by eq. (16).

Step 4 When  $p_g$  and initial mean effective stress  $\sigma_{m0}'$  in dam are known, the mean effective stress  $\sigma_m'$  can be calculated as  $\sigma_m' = \sigma_{m0}' - p_g$  and then the max. shear modulus  $G_{max}$  is got from eq. (14).

Step 5 By assuming average shear strain  $\gamma_{yx,ave}$  and basing on the formulas of  $\tau_{max}$  and  $G_{max}$ , the  $\gamma_r$ ,  $\gamma_h$  etc. can be calculated and then  $G$ ,  $\lambda$  can be determined. For sandy soils,  $\tau_{max} = \sigma_m' \sin \varphi$ , where  $\varphi$  is the angle of internal friction of soils.

Step 6 Calculation of shear wave velocity  $v_s = \sqrt{G/\rho}$ .

Step 7 Calculation of first natural frequency  $\omega$  and natural period  $T = 2\pi/\omega$ .

Step 8 On the basis of obtained  $\lambda$  and  $T$ ,  $S_a$  can be determined from earthquake response spectrum and max. crest acceleration  $\ddot{u}_{max}$  at crest of central cross section and at crest of  $L/4$  cross section of dam

can be computed by eq. (5) and by eq. (6) respectively.

Step 9 Computation the value of average shear strain  $(\gamma_{yx,ave})_{eq}$  by eq. (10) and average shear stress  $(\tau_{yx,ave})_{eq}$  by eq. (11).

Step 10 If the obtained  $(\gamma_{yx,ave})_{eq}$  and  $(\tau_{yx,ave})_{eq}$  are indential with the above assumed  $\gamma_{yx,ave}$  and  $\tau_{yx,ave}$ , the iterative computation is finished. If otherwise the obtained new values of  $(\gamma_{yx,ave})_{eq}$  and  $(\tau_{yx,ave})_{eq}$  are used for next iteration, the steps 2—9 will be repeated until the condition of convergency is satisfied.

### COMPUTED EXAMPLE

Suppose that symmetrical earth dam in trianguar canyon as shown in Fig. 1 is subjected to a transversal earthquake (Taft record in 1952, magnitude 7.7, adjusted to have a max. acceleration of 0.2g, its normalized acceleration response spectra is shown in Fig. 3). The max. height of dam  $H=46m$ , crest length  $L=184m$ . The properties of dam material are; cohesion  $c=0$ , angle of internal friction  $\varphi=30^\circ$ , saturated unit weight  $\gamma=2.1 \times 10^4 N/m^3$  (effective unit weight  $\gamma'=1.1 \times 10^4 N/m^3$ , density  $\rho=0.21 \times 10^4 kg/m^4 \cdot s^2$ ), Poission' s ratio  $\mu=0.3$ . The dynamic properties of dam material are;  $k_{z,max}=44$ ,  $\lambda_{max}=0.25$ ,  $a=0$ ,  $b=1$ ,  $\theta=0.7$ . The curve of shear stress ratio versus cycles to liquefaction is shown in Fig. 4. Determination of the natural period and max. crest accelerations in central and  $L/4$  sections of dam is demanded.

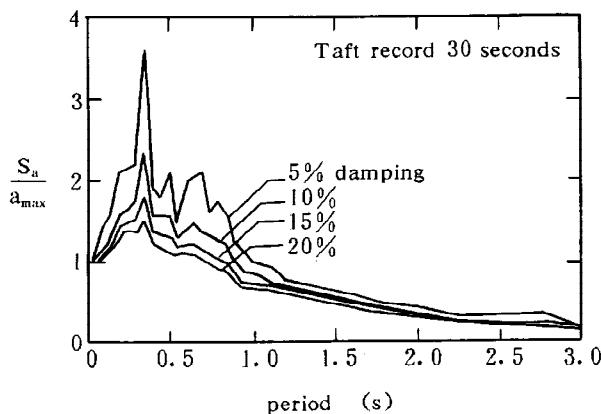


Fig. 3. Normalized acceleration response spectra- Taft record (N—S component)

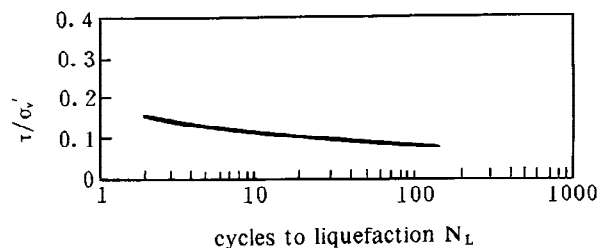


Fig. 4. Curve of shear stress ratio versus cycles to liquefaction

Solution; In order to approximately compute initial static stress, we take the average value of stresses at half depth on central section and  $\pm L/4$  section as an average stress in dam. Average vertical effective stress  $(\sigma'_y)_{ave}$ , average horizontal effective stresses  $(\sigma'_x)_{ave}$ ,  $(\sigma'_z)_{ave}$  and average mean effective stress  $(\sigma'_{mo})_{ave}$  are computed respectively as follows:

$$(\sigma_y')_{ave} = \frac{1.1 \times 10^4 \times 23 + 2 \times 1.1 \times 10^4 \times 11.5}{3} = 169 \text{ kPa}$$

$$(\sigma_x')_{ave} = (\sigma_z')_{ave} = \frac{\mu}{1-\mu} (\sigma_y')_{ave} = \frac{0.3}{1-0.3} \times 169 = 72.4 \text{ kPa}$$

$$(\sigma_{mo}')_{ave} = \frac{72.4 + 169 + 72.4}{3} = 105 \text{ kPa}$$

Iteration No. 1: Assume  $\tau_{yx,ave} = 15 \text{ kPa}$ . Therefore  $\tau_{yx,ave}/(\sigma_y')_{ave} = 15/169 = 0.089$ . From Fig. 4, cycles to liquefaction  $N_L = 65$ . On the basis of  $M = 7.7$ , from Table 1,  $N_{eq} = 25$ . Then the seismic pore water pressure  $p_g$  is calculated as

$$p_g = \frac{2}{\pi} \arcsin\left(\frac{25}{65}\right)^{\frac{1}{2 \times 0.7}} \times 169 = 56.8 \text{ kPa}$$

$$\sigma_m' = 105 - 56.8 = 48.2 \text{ kPa}$$

$$\tau_{max} = 48.2 \times \sin 30^\circ = 24.1 \text{ kPa}$$

$$G_{max} = 220 \times 44 \times \sqrt{48.2} = 0.67 \times 10^5 \text{ kPa}$$

Assuming average shear strain  $\gamma_{yx,ave} = 0.1\%$ , shear modulus  $G$  and damping ratio  $\lambda$  are computed respectively by eq. (12) and eq. (13) as follows

$$G = \frac{0.67 \times 10^5}{1 + \frac{0.1\% \times 0.67 \times 10^5}{24.1}} = 0.18 \times 10^5 \text{ kPa}$$

$$\lambda = 0.25 \times \frac{\frac{0.1\% \times 0.67 \times 10^5}{24.1}}{1 + \frac{0.1\% \times 0.67 \times 10^5}{24.1}} = 18.3\%$$

$$v_s = \sqrt{\frac{0.18 \times 10^5 \times 10^3}{0.21 \times 10^4}} = 92.5 \text{ m/s}$$

Thus the natural frequency and natural period are

$$\omega = \frac{92.5}{46} \times \sqrt{\frac{45}{4} + 20 \times \frac{46^2}{184^2}} = 7.1 \text{ rad/s} \quad T = \frac{2\pi}{7.1} = 0.88 \text{ s}$$

The value of spectral acceleration for  $T$  are obtained from Fig. 3 for  $\lambda = 18.3\%$ ,  $S_a/a_{max} = 0.75$ , thus  $S_a = 0.2 \times 0.75 = 0.15g$ . From eq. (5) and eq. (6),

$$\ddot{u}_{max} \approx 1.839 \times 0.15g = 0.275g \quad (\text{central section})$$

$$\ddot{u}_{max} \approx 1.839 \times 0.563 \times 0.15g = 0.155g \quad (\pm L/4 \text{ section})$$

The average equivalent shear strain  $(\gamma_{yx,ave})_{eq}$  and the average equivalent shear stress  $(\tau_{yx,ave})_{eq}$  are obtained as follows

$$(\gamma_{yx,ave})_{eq} = 0.65 \times 1.839 \times 1.02 \times \frac{46}{92.5^2} \frac{4 \times 184^2}{45 \times 184^2 + 80 \times 46^2} = 0.077\%$$

$$(\tau_{yx,ave})_{eq} = 0.18 \times 10^5 \times 0.077\% = 13.9 \text{ kPa}$$

Repeating the same procedure for Iteration No. 2, No. 3 and No. 4, we get the results as shown in Table 2.

Table 2 Results of iterative computation

Iteration No.	max. crest acceleration in central section (g)	max. crest acceleration in L/4 section (g)	first natural period T (s)	$(\gamma_{yx,ave})_{eq}$ (%)	$(\tau_{yx,ave})_{eq}$ (kPa)
1	0.275	0.155	0.88	0.077	13.9
2	0.294	0.165	0.82	0.066	14.5
3	0.285	0.160	0.77	0.069	14.3
4	0.300	0.169	0.75	0.068	14.2

### CONCLUSION

Based upon the principle of effective stress, a simplified procedure for evaluating seismic response of earth dam in 3-D is given. By using this procedure not only the non-linear behavior of dynamic shear modulus and damping ratio, both of which depend upon the dynamic shear strain, are taken into account, but also the influence of seismic pore water pressure generation on the shear modulus and 3-D boundary condition of dam are considered. The procedure enables the approximate determination of max. crest acceleration and natural period of earth dam in 3-D due to a specified earthquake loading to be made by hand calculation even without computer.

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