SIMPLIFIED RESPONSE ANALYSIS OF EARTH DAMS TO SPATIALLY VARYING EARTHQUAKE GROUND MOTION

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ABSTRACT

Recent research (Harichandran and Vanmarcke, 1986; and many others) has shown that significant variation of earthquake ground motion exists over the base dimensions of large structures. The response analysis of an earth dam subjected to spatially varying earthquake ground motion (SVEGM) generally requires a 3-D finite element model in order to accurately represent the spatially varying excitation at the base and sides of the dam. Since SVEGM is usually characterized using probabilistic models, the seismic analysis of the dam is best performed using random vibration analysis, as long as linear response is assumed. However, a 3-D finite element-based random vibration analysis can be extremely time-consuming. This paper presents an approximate method for obtaining “equivalent” excitations in reduced space from the general excitation along the dam/rock interface, such that the response of the dam may be computed reasonably accurately at locations away from the base using a 2-D shear beam model. The maximum shear stress at the base of the dam, which can be large due to the ground strain caused by SVEGM and the high stiffness of the stream-bed, is largely due to the so-called pseudo-static response of the dam. Reasonably accurate estimates of the shear stresses at the base requires the pseudo-static responses to be computed through a 3-D finite element model, which is nevertheless much quicker than performing a dynamic analysis.

KEYWORDS

earth dam, spatial variation, random vibration, three-dimensional, finite element, simplified excitation

INTRODUCTION

Random vibration analysis or deterministic dynamic analysis of 3-D finite element earth dam models is costly and time-consuming. In current practice engineers prefer to use simpler 1-D or 2-D shear beam models, or 2-D plane strain models (Gazetas, 1987). While deterministic dynamic analysis is possible, for SVEGM a random vibration analysis is preferable if linear behavior is assumed. The authors have conducted a detailed study of the responses of the Santa Felicia earth dam located in California to SVEGM (Chen, 1995; Chen and Harichandran, 1995). Soil-structure interaction effects, which are not significant for a dam situated on bedrock, were neglected. The seismic responses of the Santa Felicia dam have been studied extensively using more conventional ground motion models (Abdel-Ghaffar and Scott, 1979; Prevost et al., 1985). The dam is made of a cen-
tral impervious core and pervious shell upstream and downstream resting on a stiff layer of gravel and sand down to bedrock. In this study, a SVEGM model which accounts for both incoherence and wave passage effects is used to specify the base motions in the upstream/downstream direction (Harichandran and Vanmarcke, 1986). The cross section of the dam and the form of the cross spectral density function (SDF) between the ground accelerations at two locations $A$ and $B$ are given in the companion paper by Harichandran and Chen (1996).

It is not possible to represent SVEGM in a reasonable way for a 1-D shear beam model of the dam. For a 2-D plane strain model, SVEGM can be represented only in the width direction but not in the length direction. For upstream-downstream excitation and response, the effect of SVEGM in the length direction is much more significant than that in the width direction. Hence a 2-D plane strain model is unsuitable for studying the effect of SVEGM on earth dams. For a 2-D shear beam model, SVEGM can be included in the length direction but not in the width direction, making the use of such a model feasible. This paper presents a method of obtaining an equivalent reduced excitation model for use with a 2-D shear beam model of the dam, and examines the accuracy of using the reduced excitation in place of the general SVEGM. It should be noted that a 2-D shear beam model was not actually used; the reduced excitation and the general SVEGM were applied to a 3-D finite element model and the responses are compared. For the reduced excitation, the base of the dam was divided into strips to simulate the division in a 2-D shear beam model, and all base nodes of the 3-D model located in a given strip were assumed to have identical excitations. The intent of the analysis was to assess the effect of simplifying the excitation model. If the simplified excitation model is acceptable, then it could be used with a 2-D shear beam model of the dam.

**REDUCED EXCITATION FOR 2-D SHEAR BEAM MODEL**

For a 2-D shear beam model of an earth dam, the base consists of a number of truncated wedges, as shown in Fig. 1. The base of each wedge is a strip along the upstream to downstream width of the dam. The strips each have a single degree of freedom along which ground motion excitation is applied. As a result, all points within a strip have identical excitations. However, the ground motion at two points on different strips need not be identical and may be partially incoherent. One method of obtaining an equivalent excitation for each strip is to assume that it is the average of the correlated excitations at each point within the strip (i.e., to assume that each strip is rigid). The coherency between the equivalent excitations at different strips would then be the coherency between each average excitation. However, the assumption of a rigid strip is questionable, and the calculations required to obtain the SDFs of the average excitations and the coherency between them is tedious. In this paper, the coherency decay along the width of the dam is neglected, and the SDF of the equivalent excitation for each strip is taken to be the point SDF given by (2) in the companion paper by Harichandran and Chen (1996). Further, the coherency between the excitations at two different strips is taken to be the coherency evaluated at the
average separation between all points on one strip and all points on the other strip. For any pair of horizontal strips $i$ and $j$, as shown in Fig. 1, the average separation between a point on strip $i$ and a point on strip $j$ is

$$\bar{V}_{ij} = \frac{1}{\Delta x_1 \Delta z_1 \Delta x_2 \Delta z_2} \int_{x_1=0}^{x_1=\Delta x_1} \int_{z_1=0}^{z_1=\Delta z_1} \int_{x_2=0}^{x_2=\Delta x_2} \int_{z_2=0}^{z_2=\Delta z_2} \sqrt{(x_2-x_1)^2 + (z_2-z_1)^2} \; dx_1 \; dz_1 \; dx_2 \; dz_2$$

(1)

where $\Delta x_1$ and $\Delta z_1$ are the width and length of strip $i$, $\Delta x_2$ and $\Delta z_2$ are the width and length of strip $j$, and $L_1$ and $L_2$ are separations between the lower left corners of these two strips in the $x$ and $z$ directions, respectively. The analytical expression in (1) cannot be evaluated in closed-form and becomes more complicated when inclined strips along the sides of the valley are involved. However, $\bar{V}_{ij}$ may be estimated by taking the average separation between a sufficient number of points simulated on strip $i$ and on strip $j$. Using $n$ randomly simulated points on strip $i$ and $m$ points on strip $j$, the average separation between strips $i$ and $j$ may be estimated by

$$\bar{V}_{ij} = \frac{1}{nm} \sum_{k=1}^{n} \sum_{l=1}^{m} \sqrt{(x_{ki}-x_{lj})^2 + (y_{ki}-y_{lj})^2 + (z_{ki}-z_{lj})^2}$$

(2)

The coherency between the equivalent excitations on strips $i$ and $j$ were obtained by using $\bar{V}_{ij}$ in place of $v$ in (3) of the companion paper by Harichandran and Chen (1996). For the general excitation model, incoherence in the base excitation in both length and width directions was considered for all pair of nodes.

For most of the analyses, the base of the dam was divided into 15 strips for the model with reduced excitation, as shown in Fig. 2. The $i$th strip is denoted by $S_i$. Inclined boundary regions were approximated by rectangular strips as shown in Fig. 2. In addition, sensitivity of the earth dam responses to the number of strips at the base was investigated by considering 9 and 21 strips in addition to the 15-strip case.

RESPONSES

The method of computing the dam responses to correlated random base excitation is described elsewhere (Chen, 1995; Chen and Harichandran, 1995), and only the results are reported herein. Three types of excitations were considered: (a) general excitation given by (1)–(4) and Table 1 of the companion paper (Harichandran and Chen, 1996); (b) reduced excitation along strips as described above; and (c) identical excitation at all nodes.

The three standard deviation ($3\sigma$) x-displacement responses for the reduced and general excitation models are shown in Fig. 3(a) and (b), respectively (the mean x-displacement is zero). The responses are shown as contours on the vertical XY cutting plane at the mid-length of the dam. It is apparent that the $3\sigma$ x-displacement inside the dam as well as on the top surface due to the reduced excitation agree well with those due to the general excitation. Using the reduced excitation, the maximum $3\sigma$ x-displacement is only 1.3% higher than that using the general excitation. For identical excitation, the maximum $3\sigma$ x-displacement is 8.1% larger than that due to general excitation.

The mean plus three standard deviation ($\mu+3\sigma$) maximum shear strain ($\gamma_{max}$) responses using reduced and general excitations are shown in Figs. 4(a) and (b). The $\gamma_{max}$ contours are displayed on the vertical XY cutting plane at the mid-length of the dam and are quite similar to each other. The maximum $\mu+3\sigma$ $\gamma_{max}$ response is about 1.9% larger for the reduced excitation compared to that for the general excitation. For identical excitation, the maximum $\mu+3\sigma$ $\gamma_{max}$ response is 10.3% higher than that for the general excitation.

Figures 5(a) and (b) show the $\mu+3\sigma$ maximum shear stress ($\tau_{max}$) response contours for the reduced and general excitation models, respectively, on the vertical XY cutting plane at the mid-length of the dam. The $\tau_{max}$ response near the base due to the reduced excitation is considerably larger than that due to general excitation.
The $\mu + 3\sigma \tau_{\text{max}}$ responses at the base are shown in Figs. 6(a) and (b) for the reduced and general excitations, respectively, and these are quite dissimilar. The maximum $\tau_{\text{max}}$ response is 50% larger for the reduced excitation compared to that due to general excitation. Although the reduced excitation model does not predict $\tau_{\text{max}}$ well near the base, the predictions at locations away from the boundary are reasonable. Figures 7(a) and (b) display the $\tau_{\text{max}}$ contours on the horizontal cutting plane at the mid-height of the dam, and show that the two contour patterns match each other very well. The $\tau_{\text{max}}$ responses in the core clay region are shown in Figs. 8(a)

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**Fig. 2.** Illustration of 15 strip division

**Fig. 3.** Elevation view of the $3\sigma$ displacements (mm) using the (a) reduced and (b) general excitations
Fig. 4. Elevation view of the $\mu+3\sigma \gamma_{\text{max}}$ responses ($\times 10^{-3}$) using the (a) reduced and (b) general excitations.

Fig. 5. Elevation view of the $\mu+3\sigma \tau_{\text{max}}$ responses (MPa) using the (a) reduced and (b) general excitations.

Fig. 6. $\mu+3\sigma \tau_{\text{max}}$ responses (MPa) at the base using the (a) reduced and (b) general excitations.

and (b) which display vertical YZ cutting planes at mid-width. The $\tau_{\text{max}}$ response due to the reduced excitation is very similar to that due to general excitation over the entire dam except near the base.
As the number of strips on the base is reduced, the accuracy of the reduced excitation would be expected to become poorer. Analyses with 9 and 21 strips on the base were performed to assess this effect. The horizontal and two inclined portions of the base were divided into 3 strips each for the 9-strip case, and 7 strips each for the 21-strip case, respectively. In general, the displacements do not vary significantly between the 9-, 15- and 21-strip cases. The values of the critical $x$-displacement response for all excitation cases used in this study are given in Table 1, and indicate that slightly better results are obtained when more strips are used. However, the use of 9 strips, which appears minimal, seems adequate, and the improvement with 15 and 21 strips is marginal. In the limit, as fewer and fewer strips are used on the base, the reduced excitation model approaches the identical excitation model.

The maximum $\mu+3\sigma \gamma_{max}$ responses for the cases of identical, reduced and general excitations are listed in Table 2 and indicate that the reduced excitation yields better $\gamma_{max}$ values than identical excitation. As with the

<table>
<thead>
<tr>
<th>Identical excitation</th>
<th>Reduced excitation</th>
<th>General excitation</th>
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<tbody>
<tr>
<td></td>
<td>9 strips</td>
<td>15 strips</td>
</tr>
<tr>
<td>207.4</td>
<td>194.93</td>
<td>194.26</td>
</tr>
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</table>
Table 2. Critical $\mu+3\sigma \tau_{\text{max}}$ responses ($\times 10^{-3}$)

<table>
<thead>
<tr>
<th>Identical excitation</th>
<th>Reduced excitation</th>
<th>General excitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 strips</td>
<td>15 strips</td>
<td>21 strips</td>
</tr>
<tr>
<td>2.133</td>
<td>1.979</td>
<td>1.972</td>
</tr>
</tbody>
</table>

Table 3. Critical $\mu+3\sigma \tau_{\text{max}}$ responses (kPa) using simplified and general excitation models

<table>
<thead>
<tr>
<th>Location</th>
<th>Identical excitation model</th>
<th>Reduced excitation</th>
<th>General excitation</th>
</tr>
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<tbody>
<tr>
<td>At the base</td>
<td>498</td>
<td>2,607</td>
<td>2,749</td>
</tr>
<tr>
<td>Horizontal cutting plane at mid-height (also vertical cutting plane at mid-width)</td>
<td>410.57</td>
<td>379.1</td>
<td>378.2</td>
</tr>
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displacement response, the accuracy of the reduced excitation model becomes better as the number of strips is increased, but the improvement beyond 9 strips is marginal.

The maximum $\tau_{\text{max}}$ responses are given in Table 3 for each excitation model. Reduced excitation yields stress responses that are only 1.5% to 2% larger than those due to general excitation at the mid-height of the dam, with marginal improvement as the number of strips is increased from 9 to 21. However, stresses predicted at the base using the reduced excitation is poor (over-conservative) and becomes worse as more strips are used. For the $\tau_{\text{max}}$ response in the base gravel region, the pseudo-static response component usually dominates the total response (Chen and Harichandran, 1995). Figures 9(a) and (b) show the closeness between the total response and the pseudo-static component of $\tau_{\text{max}}$ at the base. Thus, for design use, the pseudo-static component of $\tau_{\text{max}}$ should suffice at the base.

It should be noted, however, that the stress response in the stiff gravel material at the base is sensitive to the coherency model used. The use of several coherency models was investigated (Chen, 1995), and for most models the pseudo-static component of $\tau_{\text{max}}$ dominates the response. However, for the coherency model proposed

Fig. 9. $\mu+3\sigma \tau_{\text{max}}$ (MPa) contours at the base for (a) total response and (b) static response component.
by Abrahamson (1993), the contribution of the pseudo-static component of $\tau_{\text{max}}$ at the base is small and consequently the effect of SVEGM on the dam response is much less significant. Pseudo-static stresses near the base occur due to incoherence in the ground displacements. The ground displacement is dominated by motions with frequencies less than about 0.5 Hz. Abrahamson’s model is highly coherent at this very low frequency range while most other models display some incoherence; hence the discrepancy.

CONCLUSIONS

For the dynamic analysis of earth dams, the 2-D shear beam model usually results in slightly higher natural frequencies and responses than a 3-D model. Use of the reduced ground motion model developed in this paper gives rise to slightly larger displacement, stress and strain responses than use of the general excitation model. As a consequence, the use of the 2-D shear beam model in conjunction with the proposed reduced excitation model would be expected to yield reasonable and slightly conservative results at locations away from the base.

For the prediction of stresses in the lower part of the dam, especially in the stiff gravel, the reduced excitation significantly over-estimates the response and is unacceptable when ground displacements are not highly coherent. On the other hand, the use of identical excitation significantly under-estimates the stress response. Hence, a 3-D model is required in order to reasonably predict stresses generated by SVEGM at the base of the dam. However, the computation of the pseudo-static response component, which is quick, is sufficient for the stress response in the stiff gravel region, and time-consuming 3-D dynamic analysis can be avoided.

ACKNOWLEDGEMENT

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REFERENCES


