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CONTROL PERFORMANCE OF AN AMD RESPONSE CONTROL SYSTEM CONSIDERING SATURATION OF THE SYSTEM DURING INTENSE SEISMIC MOTIONS

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#### **ABSTRACT**

An active mass damper is one of the systems that have been developed to ensure structural safety against the lateral force actions. In this paper, saturation of the system is examined and discussed that will be developed when a building with the system is subjected to an intense seismic motion. Major concern in this study is to utilize a response control system against strong seismic motions, where the control system will be saturated. Emphases are placed on the saturation of either control forces and actuator strokes. Through a numerical analysis, correlation of the control performance deterioration with the degree of saturation is evaluated. It is found that while the control performance deterioration due to force saturation is significantly affected by the design parameter within the control algorithms scheme, deterioration due to stroke saturation is independent of the parameter. It is concluded that the stroke saturation is not undesirable compared to the force saturation generating an excessively large reaction forces by the control system.

#### **KEYWORDS**

Response control; saturation; active mass damper; seismic response; control performance; seismic control; force saturation; stroke saturation; control efficiency; control deterioration.

### INTRODUCTION

An active mass damper (AMD) is one of the systems developed to ensure structural safety against wind and seismic actions. In this paper, saturation of the AMD system is examined and discussed that will be developed when subjected to an intense seismic motion. In the countries with high seismic activities such as Japan, it is our major concern to utilize a response control system against strong earthquake excitation. An actuator placed within the AMD system will have restrictive conditions for the control forces put from the hydraulic pressure, the stroke displacements put from the head mechanism and the stroke velocities put from the hydraulic quantities.

Within this paper, emphases are placed on the saturation of either control forces and stroke displacements of the actuator within the AMD system. Through numerical analyses, the deterioration of the control performance observed when the AMD system is revealed saturated is evaluated. Further the discussion are extended to examine the system design of an AMD control system against an intense earthquake excitation.

### ANALYTICAL MODEL AND ANALYTICAL PROCEDURE

### **Analytical Model**

For the analysis, an analytical model shown in Fig. 1 is employed. The model is composed of a seismic controlled building and a seismic controlling system. A single unit of the controlling AMD system, of which properties are identified by the specified mass m, spring k with an actuator A, is positioned at the top of the building. The controlled building is represented by a single-degree-of-freedom oscillating system identified by the specified mass M, spring K and dashpot C, which will represent a multistory building such as that designed upon the weak-beam strong-column concept having fundamental mode of which is uniquely established.

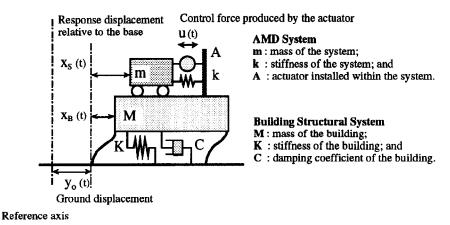


Fig. 1. Analytical model of a controlled building with a controlling AMD system.

Within this study, the mass M is specified as  $10^3$  [kg], i.e., 980 tonf in weight. The spring K and dashpot C are determined so as the fundamental period of the building and the fraction to the critical damping to equal 0.4 seconds and 0.05, respectively. The mass m and spring k of the AMD system is specified as the ratio m/M equal to 0.05 and the resultant fundamental period from m and k equal to 0.4 seconds, respectively.

# Control Theory and Algorithm

The optimal control theory is employed, in which the performance indices J linear quadratic form defined by Eq. (1) are yielded minimum (Yang, 1975, Kobori 1993):

$$J = \frac{1}{2} \int_0^{t_r} \mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) dt + \frac{1}{2} \int_0^{t_r} \mathbf{u}(t) \mathbf{R} \mathbf{u}(t) dt$$
 (1)

where  $t_r$  denotes the duration of response, matrices  $\mathbf{Q}$  and  $\mathbf{R}$  are the weighting matrices as design parameters of the control system, and  $\mathbf{x}(t)$  and  $\mathbf{u}(t)$  are the state variable vectors of the system and control force vector, respectively. Since only a single unit of AMD system is installed within the building in this study, both  $\mathbf{R}$  and  $\mathbf{u}(t)$  in Eq. (1) are given by scalar variables.

The state variable vector  $\mathbf{x}(t)$  in Eq. (1) is given by:

$$\mathbf{x}(t) = \begin{bmatrix} \dot{\mathbf{x}}_{\mathrm{B}}, \dot{\mathbf{x}}_{\mathrm{S}}, \mathbf{x}_{\mathrm{B}}, \mathbf{x}_{\mathrm{S}} \end{bmatrix}^{\mathrm{T}}$$
 (2)

in which  $x_B$  and  $x_S$  represent the response displacement relative to the base of the controlled building and the controlling AMD system as described in Fig. 1, respectively.

## Weighting Parameters in the Weighting Matrices

The elements of the weighting matrices  $\mathbf{Q}$  and  $\mathbf{R}$  are arbitrarily prescribed as the system design parameters. In this study presented herein, the matrices  $\mathbf{Q}$  and  $\mathbf{R}$  are determined to take a form described in the following.

$$\mathbf{Q} = a \times \begin{bmatrix} b \times \begin{bmatrix} 1 & 0 \\ 0 & c \end{bmatrix} & \begin{bmatrix} \mathbf{0} \end{bmatrix} \\ \begin{bmatrix} \mathbf{0} \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 0 & c \end{bmatrix} \end{bmatrix}$$
(3)

$$\mathbf{R} = [ r ] \tag{4}$$

For simplicity of analysis the weighting matrices **Q** and **R** are prescribed as diagonal matrices indicating that the coupled responses are not taken into account in evaluation of the performance index. The performance index is determined so as the sum of weighted variances of state variables and control forces to be yielded minimum. No specific considerations are paid upon the covariances obtained crosswise between any two among the state variables and control forces.

The weighting parameter a is taken unity while it has the physical dimension making the performance index a non-dimensional scalar. The parameters b and c define the relative significance of the velocity response to the displacement response and that of the responses of the controlling system to those of the controlled building, respectively. The parameter r determines the efficiency of control performance. In the following analysis, physical units are taken as meter, seconds and Newton, respectively.

Previous studies indicate the tendency that the control performance of the AMD controlling system is less significantly varied with variation of either parameters b or c when compared to that with variation of the parameter r (Kubo et al. 1994). For further simplicity, parameter b and c are set zeros yielding the performance index evaluated from both displacement responses of building and control forces produced by the AMD actuator.

Within this study, the parameter r is prescribed to be  $1.0 \times 10^{-19}$  with which the responses of acceleration of the controlled building yielded minimum. Another parameter r of  $1.0 \times 10^{-17}$  is specified with which the controlled acceleration responses are yielded greater, while the less control forces of actuator are required.

# CONTROL PERFORMANCE WITH SATURATION OF THE CONTROL SYSTEM

### Saturation of Control Force

The saturation of control force is expressed by specifying the control forces through:

$$|u(t)| \le u_{\text{max}} \tag{5}$$

in which the control force u(t) generated by the actuator shall not be greater than the capacity of the actuator  $u_{max}$  whatever the control forces are required from the control scheme.

Responses with Force Saturation The capacity of the actuator within the system  $u_{max}$  is prescribed. Figures 2 and 3 show the controlled responses for the cases with parameter r taken as  $1.0 \times 10^{-19}$  and  $1.0 \times 10^{-17}$ , respectively. The axis x indicates the intense level of excitation expressed by the peak acceleration in m/s<sup>2</sup>. The left and right columns of the axis y represent the variances of displacement responses of the building and the peak control forces produced by the actuator, respectively. With variation of the peak input acceleration, the degree of saturation can be discussed. The higher the level of input excitation is, the more significant the degree of saturation will be.

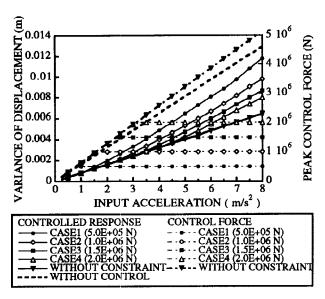


Fig. 2. Displacement responses of the building and control forces produced by the actuator obtained when subjected to the S00E component, El Centro with parameter r equal to  $1.0 \times 10^{-17}$ .

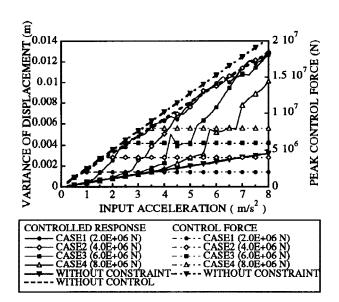


Fig. 3. Displacement responses of the building and control forces produced by the actuator obtained when subjected to the S00E component, El Centro with parameter r equal to  $1.0 \times 10^{-19}$ .

Observation on Figs. 2 and 3 leads to the evidence that the responses of the building are increased when the control force saturation is developed. It is found that the deterioration of control performance is correlated with both the degree of saturation and the parameter r determining the control efficiency of the system.

<u>Degree of Force Saturation</u> Let the force capacity ratio  $\Omega$  be defined by:

$$\Omega = \frac{u}{u_{req}} \tag{6}$$

where u and  $u_{req}$  denote the peak values of the produced control force by the actuator and the required control force determined from the control algorithms, respectively. The control force u should not be greater than the force capacity  $u_{max}$  as described in Eq. (5). The case when the force capacity ratio equals 1/2 corresponds to the condition that the installed actuator can generate control forces 1/2 times as large as required.

<u>Deterioration of Control Performance</u> Figures (a) and (b) in Figs. 4 and 5 represent the force capacity ratios associate with the variation of intensity level of ground excitation and the ratios defined by Eq. (7) representing the deterioration of response control performance associated with the control force saturation, respectively:

$$\phi = \frac{R_{\text{sat}}}{R_{\text{unsat}}} \tag{7}$$

where R<sub>sat</sub> and R<sub>unsat</sub> represent the responses obtained under the conditions with and without force saturation, respectively. The dashed line in the figures represents the ratio of response of the building without control R<sub>without</sub> compared to that with control R<sub>unsat</sub> without saturation. In Fig. 6, the correlation of the deterioration of control performance associated with the force capacity ratio is illustrated obtained in the responses subjected to the S00E component, El Centro obtained during the 1940 Imperial Valley earthquake (ELCNS), the S69E component, Taft obtained during the 1952 Kern County earthquake (TFTEW), the East-West component, Hachinohe Harbor obtained during the 1968 Tokachi-oki earthquake (HCHEW) and the North-South component, Tohoku University obtained during the 1978 Miyagi-ken-oki earthquake (THUNS). The axes x and y designate the force capacity ratio and deterioration of control performance, respectively.

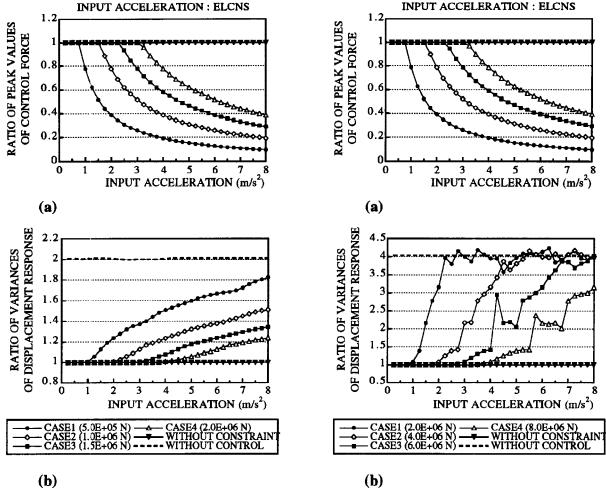


Fig. 4. Responses evaluated when the control force saturation taken into consideration obtained when subjected to the S00E component, El Centro with parameter r equal to  $1.0 \times 10^{-17}$ : (a) force capacity ratio; (b) deterioration of response control performance.

Fig. 5. Responses evaluated when the control force saturation taken into consideration obtained when subjected to the S00E component, El Centro with parameter r equal to  $1.0 \times 10^{-19}$ : (a) force capacity ratio; (b) deterioration of response control performance.

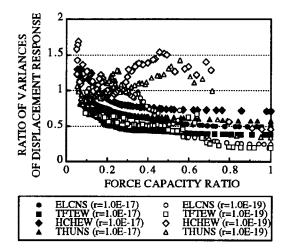


Fig. 6. Correlation of deterioration of response control performance in terms of the variance of the building displacement responses with the force capacity ratios obtained when subjected to the S00E component, El Centro, the S69E component, Taft, the East-West component, Hachinohe Harbor and the North-South component, Tohoku University with parameter r of  $1.0 \times 10^{-17}$  and  $1.0 \times 10^{-19}$ .

The saturation of actuator stroke is expressed by specifying the displacement of the AMD system through:

$$|x_S(t) - x_B(t)| \leq \delta_{sat}$$
 (8)

in which  $\delta_{sat}$  denotes the stroke of the actuator. The mathematical model of the actuator stroke saturation is represented by the load-deflection characteristics of the AMD system as described by Fig. 7, in which stiffness ratio  $\alpha$  is taken 100 in this analysis from a viewpoint of numerical stability.

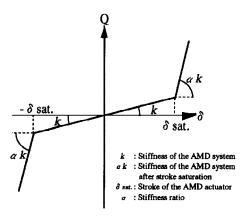


Fig. 7. Load-deflection characteristics of the AMD system when considering the stroke saturation.

Responses with Stroke Saturation Figures 8 and 9 illustrate the responses for the cases with parameter r taken as  $1.0 \times 10^{-17}$  and  $1.0 \times 10^{-19}$ , respectively. The legends of the figures are identical to those of Figs. 2 and 3, while the right column of the axis y represents the peak stroke of the AMD actuator.

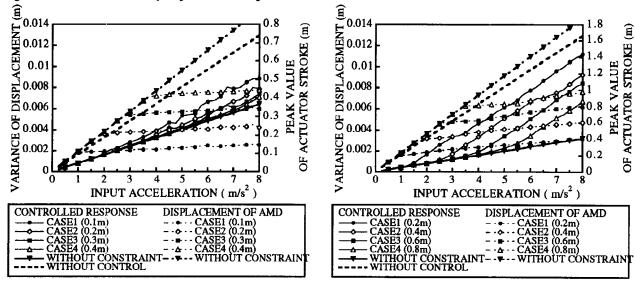


Fig. 8. Displacement responses of the building and stroke responses obtained when subjected to the S00E component, El Centro with parameter r equal to  $1.0 \times 10^{-17}$ .

Fig. 9. Displacement responses of the building and stroke responses obtained when subjected to the S00E component, El Centro with parameter r equal to  $1.0 \times 10^{19}$ .

The results obtained in the Figs. 8 and 9 reveal the fact that when the stroke is developed saturated, the control performance on the responses of the building appears deteriorated. It should be noticed that an excessively large amount of reaction forces are generated when the stroke saturation is developed.

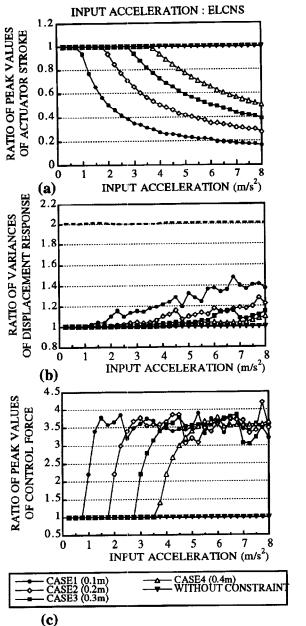
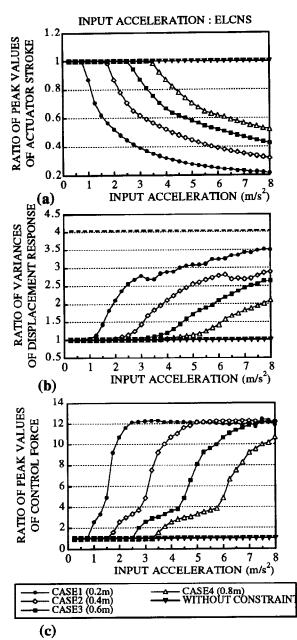


Fig. 10. Responses evaluated when the stroke saturation Fig. 11. Responses evaluated when the stroke saturation taken into consideration obtained when subjected to of response control performance; and (c) reaction forces generated by the AMD actuator.



taken into consideration obtained when subjected to the S00E component, El Centro with parameter r equal the S00E component, El Centro with parameter r equal to 1.0x10<sup>-17</sup>: (a) stroke capacity ratio; (b) deterioration to 1.0x10<sup>-19</sup>: (a) stroke capacity ratio; (b) deterioration of response control performance; and (c) reaction forces generated by the AMD actuator.

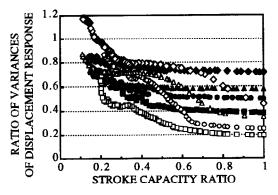


Fig. 12. Correlation of deterioration of response control performance in terms of the variance of the building displacement responses with the stroke capacity ratios.

$$\Delta = \frac{\delta}{\delta_{\text{req}}} \tag{9}$$

where  $\delta$  and  $\delta_{req}$  denote the stroke of the actuator, which should not exceed the  $\delta_{max}$  prescribed, and the peak values of the stroke determined from the control algorithms.

In Figs. 10 and 11, the results obtained when the stroke saturation of the AMD actuator taken into account are shown. Figures (a), (b) and (c) in these figures represent the results of the stroke capacity ratio associated with the variation of intensity level of ground excitation, the deterioration of response control performance associated with the stroke saturation and the peak force reaction forces generated by the AMD system to the building, respectively.

<u>Deterioration of Control Performance</u> Figure 12 summarizes the correlation of the deterioration of control performance with the stroke capacity ratio when the stroke saturation is developed. The legends of the figure are identical to those of Fig. 6 representing the correlation between the control performance deterioration and force saturation.

### CONCLUDING REMARKS

Examined and discussed the correlation of the deterioration of control performance of an AMD response control system with saturation of the system. Two types of saturation are taken into consideration, one of which is the saturation of control force, and the other the saturation of stroke of the actuator installed within the system.

The conclusions obtained within the study are summarized as follows:

- 1] When the saturation is developed, the control system reveals deterioration of control performance. Through numerical analysis, the correlation of the deterioration of control performance with saturation of control forces and that of strokes are evaluated as summarized in Figs. 6 amd 12, respectively.
- 2] The deterioration of performance associated with force saturation is significantly dependent on the parameter r determining the control efficiency of the system. When the parameter is specified so as the controlled responses yielded minimum, i.e., equal to  $1.0 \times 10^{-19}$  in the analysis, it is required to place an actuator of which capacity is large so as the resultant force capacity ratios greater than 0.7 as illustrated in Figs. 5 (b) and 6. In the case when the parameter r is taken 100 times as large as, i.e.,  $1.0 \times 10^{-17}$ , one can expect response control performance for the systems having low force capacity ratios. The stroke responses of the AMD actuator are less as well.
- 3] Stroke saturation can deteriorate control performance. The parameter r is found independent of the correlation of control performance deterioration with degree of saturation. When the stroke saturation is developed, an excessively large reaction forces will be generated from the AMD system to the building. Compared with the force saturation, the stroke saturation is concluded undesirable.
- 4] The variation of input excitation characteristics reveals less significance on the control performance deterioration caused from the saturation of the system.

### REFERENCES

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