RESPONSE CONTROL OF STRUCTURES BY TUNED MASS DAMPERS AND THEIR GENERALIZATIONS

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ABSTRACT
This paper presents the results of a study performed on tuned mass dampers (TMD) and their generalizations - multi-tuned mass dampers (MTMD) and active tuned mass dampers (ATMD). Correspondence between the design of a TMD for a SDOF structure and a certain mode of a MDOF structure is drawn to simplify TMD design to control a single mode of a damped multi-modal structure. An example is given to illustrate the design procedure. Investigations are made for controlling multiple structural modes using MTMD. Regarding ATMD, a recently proposed acceleration- and velocity-feedback algorithm is simplified with practical modifications.

KEYWORDS
Tuned mass dampers; Multi tuned mass dampers; Active tuned mass dampers; Acceleration feedback; Velocity feedback; Parametric study; Design.

INTRODUCTION
The concept of tuned mass dampers (TMD) dates back to year 1909, when Frahm invented a vibration control device called dynamic vibration absorber. Ormondroyd and Den Hartog (1928) showed that by introducing damping in Frahm's absorber, its performance can be significantly improved. Den Hartog derived closed form expressions for damper parameters which optimize the steady-state response of an undamped single degree of freedom (SDOF) structure-absorber system subjected to harmonic excitation acting on the structure.

TUNED MASS DAMPERS
A schematic representation of a damped vibration absorber or so-called tuned mass damper (TMD) is shown in Fig. 1. The equations of motion of this SDOF structure-TMD mechanism are given as:

\[ M \ddot{X}(t) + KX(t) - [c(\dot{X}(t) - \ddot{X}(t)) + k(x(t) - X(t))] = P(t) \]

\[ m\ddot{x}(t) + c(\dot{x}(t) - \dddot{X}(t)) + k(x(t) - X(t)) = p(t) \]

where \( \mu \): Damper mass to main mass ratio, \( \mu = \frac{m}{M} \). 

\( P(t) \): Force acting on main mass. For base excitation with acceleration \( \dddot{x}_g(t) \), \( P(t) = -M\dddot{x}_g(t) \).  

\( p(t) \): Force acting on damper mass. It is given as:

\[ p(t) = \begin{cases} 
\frac{m}{M} P(t) & \text{for base (earthquake-type) excitation} \\
0 & \text{for main mass (wind-type) excitation}
\end{cases} \]

To facilitate further discussion, the following additional notations are introduced:
ω: Frequency of a harmonic excitation.
Ω: Natural frequency of main mass, \( \Omega = \sqrt{\frac{k}{M}} \).
\( \omega_a \): Natural frequency of damper mass, \( \omega_a = \sqrt{\frac{k}{m}} \).
g1: Ratio of excitation frequency to main mass natural frequency, \( g_1 = \frac{\omega}{\Omega} \). For MDOF structures, \( g_1 = \frac{\omega}{\Omega_1} \), where \( \Omega_1 \) is the first modal frequency of the structure.
f: Frequency ratio, \( f = \frac{\omega}{\Omega} \).
\( \zeta_d \): Damping ratio of TMD.
\( \zeta \): Damping ratio of main mass.

![Absorber Mass](image)

**Fig. 1. Damped Vibration Absorber Suggested by Den Hartog (1928)**

For undamped main structure, optimum values of TMD parameters are given as:

<table>
<thead>
<tr>
<th>TMD parameter</th>
<th>Harmonic main mass excitation</th>
<th>Harmonic base excitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{opt} )</td>
<td>( \frac{1}{1+\mu} )</td>
<td>( \frac{1}{1+\mu} )</td>
</tr>
<tr>
<td>( \zeta_{d,opt} )</td>
<td>( \sqrt{\frac{8(1+\mu)}{1+\mu}} )</td>
<td>( \sqrt{\frac{8(1+\mu)}{1+\mu}} )</td>
</tr>
</tbody>
</table>

**TMD Design for Damped Structures**

In the presence of damping in the main mass, the equation of motion of the main mass, Eq. (1) is modified by adding the term \( C \ddot{x}(t) \) to the left-hand side. No closed-form expressions can be derived for the optimum damper parameters, however, they may be obtained by numerical trials. In a numerical trial approach, several combinations of damper parameters \( \zeta_d \) and \( f \) are investigated in a systematic manner until the combination which minimizes the higher response peak is reached. This approach was used by Randall et al. (1981) to develop design graphs for obtaining optimum damper parameters for damped SDOF structures under main mass excitation. Warburton and Ayorinde (1980) tabulated numerically searched optimum values of absorber parameters for certain values of structure parameters. Rana (1995) developed tables of numerically searched optimum damper parameters covering a practical range of structural parameters for harmonic main mass and base excitations. For a general \( N \) DOF damped structure-TMD system with a TMD placed at the \( j \)th floor of the structure, the equations of motion, for a typical \( r \)th floor mass, can be written as:

\[
M_r \dddot{x}_r(t) + C_r \dddot{x}_r(t) + K_r x_r(t) - \delta_{rj} [c(\dot{x}(t) - \dot{x}_r(t)) + k(x(t) - x_r(t))] = P_r(t)
\]

where
\[
\delta_{rj} = \begin{cases} 
0 & r \neq j \\
1 & r = j 
\end{cases}
\]

and for the damper mass:

\[
m \dddot{x}(t) + c(\dot{x}(t) - \dot{x}_r(t)) + k(x(t) - x_r(t)) = p(t)
\]

where
\[
p(t) = \begin{cases} 
\frac{m}{M_j} P_j(t) & \text{for base excitation} \\
0 & \text{for main structure excitation}
\end{cases}
\]

If TMD is to be designed to control \( i \)th structural mode with modal properties \( M_l \), \( K_l \) and \( C_l \), the design problem is essentially similar as that of designing a TMD to control a SDOF structure. For the \( i \)th structural mode-TMD system, equations of motion can be written as:

\[
M_l \dddot{y}_i(t) + C_l \dddot{y}_i(t) + K_l y_i(t) - \phi_i [c(\dot{x}(t) - \dot{X}_i(t)) + k(x(t) - X_i(t)))] = P_i(t)
\]
\[ m\ddot{z}(t) + c(\dot{z}(t) - \dot{X}_j(t)) + k(z(t) - X_j(t)) = \Delta p(t) \]  

(6)

where \( \Delta \) is zero for main mass excitation and one for base excitation and

\[ P_i(t) = \phi_i^T \mathbf{P}(t), \quad \mathbf{P}(t) \text{ being the load vector acting on structure.} \]

\[ y(t) \quad \text{Generalized displacement of } i\text{th mode.} \]

Comparing Eqs. (5) and (6) to equations of motion for SDOF structure-TMD system, Eqs. (1) and (2), one can observe that these two pairs of equations differ on two accounts, namely, the presence of the term \( \phi_{ij} \) in Eq. (5) and the presence of term \( X_j(t) \) in stead of \( y(t) \) in both Eqs. (5) and (6). However, if the structure's \( i \)th mode shape vector is normalized with respect to its \( j \)th element which corresponds to the TMD location \( (j \)th floor), \( \phi_{ij} \) becomes unity and \( X_j(t) = \phi_{ij} y(t) = y(t) \), and Eqs. (5) and (6) reduce to the same form as Eqs. (1) and (2). Thus if \( \phi_{ij} \) is unity, expressions for calculating steady-state \( i \)th modal response and damper response in a MDOF structure-TMD system will be exactly the same as those for main mass and damper mass responses respectively in a SDOF structure-TMD system. In which case, design aids (e.g., Randall et al., 1981; Warburton and Ayorinde, 1980; Rana, 1995), developed for designing a TMD for a SDOF structure can be directly used to design a TMD for a certain structural mode of a MDOF structure. Design procedure using this approach is illustrated by designing a TMD to control the first mode of a flexible and lightly damped 3 DOF structure with properties given in Table 1:

<table>
<thead>
<tr>
<th>Table 1. Properties of the 3 DOF Structure Considered</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M(Kg) =</strong></td>
</tr>
</tbody>
</table>
| 10 0 0 | \[
\begin{bmatrix}
1000 & -1000 & 0 \\
-1000 & 2000 & -1000 \\
0 & -1000 & 2000 \\
\end{bmatrix}
\]
| 0 10 0 |
| 0 0 10 |
| **C(N-s/m) =** | **\Phi =** |
| \[
\begin{bmatrix}
1.519 & 0.272 & 0.097 \\
0.272 & 1.343 & 0.176 \\
0.097 & 0.176 & 1.246 \\
\end{bmatrix}
\]
| 0.737 & -0.591 & -0.328 |
| 0.328 & 0.737 & -0.591 |
| **\Omega_1 = 0.708306 Hz, \Omega_2 = 1.98463 Hz, \Omega_3 = 2.86787 Hz** |
| **\zeta_1 = 2.0\%, \zeta_2 = 0.5\%, \zeta_3 = 0.3\%** |

Observing the structural modal matrix \( \Phi \) in Table 1, it can be said that in the first mode, the top floor will undergo largest steady-state deflection under a harmonic excitation. Therefore the TMD should be placed at the top floor for best control of the first mode. Since the TMD will be placed at the top floor, mode-shape vector \( \Phi_1 \) should be normalized with respect to its first element to calculate the structure's first modal mass. Therefore, the normalized \( \Phi_1 \) is given as:

\[ \Phi_{1n} = [1.000 \quad 0.892 \quad 0.445]^T \]

The first-mode modal mass is given as: \( M_1 = \Phi_{1n}^T M \Phi_{1n} = 18.41 \text{ Kg} \). If the damper mass is taken to be 2\% of the entire building mass, then \( m = 0.6 \text{ Kg} \). Therefore, the damper mass to the structure's first-mode modal mass ratio is: \( \mu_1 = \frac{0.6}{18.41} = 0.03259 \). The first-mode modal damping ratio is known to be \( \zeta_1 = 2\% \). Using these known values of \( \mu_1 \) and \( \zeta_1 \), the optimum damper parameters, \( f_{opt} \) and \( \zeta_{dopt} \), can be found from a numerical search. One obtains

\[ f_{opt} = 0.952, \quad \zeta_{dopt} = 0.11. \]

This completes the design of the TMD tuned to the first mode of a 3 DOF structure.

**MULTI-TUNED MASS DAMPERS**

A single TMD can only control the mode for which it is designed. For controlling an additional mode a separate TMD can be used. Therefore, the concept of multi-tuned mass dampers (MTMD), i.e., having a separate TMD for every structural mode appears to be worth investigating. Much of the research in
the area of multi-tuned mass dampers (e.g., Xu and Igusa, 1992; Yamaguchi and Harponchait, 1993; Kareem and Kline, 1995) has been done with the aim of controlling a single mode only. In this paper, analyses are carried out with the purpose of controlling multiple modes. The design method illustrated earlier is used to tune each damper.

Assuming that 2% of the building mass is the total available mass for all the dampers, an appropriate mass distribution among the various TMDs must first be determined. A response analysis of the structure with harmonic base excitation of frequencies varying over a range which covers all three natural frequencies was done. Obtained peak response ratios are, in first mode - 50.0 (at the top floor), in second mode - 7.5 (at the first floor) and in third mode - 2.5 (at the middle floor). These response values indicate the relative importance of various modes in determining the overall structural response and TMD masses are distributed in the ratio of 50.0 : 7.5 : 2.5 for first-, second- and third-mode TMD, respectively.

**Harmonic Analysis of Modal Interaction**

When a TMD is installed in the structure to control a particular mode, properties of the finally obtained system differ from those of the original structure. Now, if an additional TMD tuned to another mode is also to be installed, it may not perform as expected because of this effective change in structural parameters. Also, the addition of a TMD may affect the performance of TMD(s) already present. This problem of modal interaction is discussed with the help of a harmonic base excitation analysis. The parameters of the various TMDs used are given in Table 2. Top-floor responses of the structure are shown in Fig. 2 and the following observations are taken:

- To effectively control any particular mode, a separate TMD, specifically tuned to that mode, must be provided.
- The structural response of the first controlled mode is marginally increased due to the presence of TMDs tuned to other modes while the structural response of second and third controlled mode is marginally reduced due to the presence of TMDs tuned to other modes.

![Fig. 2. Top Floor Response Showing Effect of TMD(s) on Various Modes](image)

**Time-History Analysis**

It was observed in the harmonic analysis that, the presence of higher-mode TMDs causes some deterioration in the first-mode response. At the same time, to control higher modes, a separate TMD must be provided. Therefore, a MTMD is designed and investigated with the help of time-history analyses under El Centro and Mexico earthquakes using one, two and three TMDs, respectively. Results of these analyses are summarized in Table 2.

It is clear that the designed MTMD does not result in appreciable response reduction in addition to what is already possible by a first-mode TMD. Based on the above harmonic and time-history analyses, it can be concluded that, effect of controlling the higher modes is getting nullified by a marginal increase in the first mode response.
### Table 2. 3 DOF Structure-MTMD System Time-History Analyses Results

<table>
<thead>
<tr>
<th>TMD(s) description</th>
<th>Floor no.</th>
<th>Peak(cm)</th>
<th>RMS(cm)</th>
<th>Peak(cm)</th>
<th>RMS(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuned to (location) Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st mode: m=0.5 Kg (3rd floor)</td>
<td>3rd</td>
<td>13.29</td>
<td>4.58</td>
<td>-22.33</td>
<td>5.60</td>
</tr>
<tr>
<td>2nd mode: k=9.11 N/m (3rd floor)</td>
<td>2nd</td>
<td>9.87</td>
<td>3.61</td>
<td>-17.76</td>
<td>4.52</td>
</tr>
<tr>
<td>3rd mode: c=0.43 N-s/m (TMD-1)</td>
<td>1st</td>
<td>5.26</td>
<td>2.12</td>
<td>-10.20</td>
<td>2.59</td>
</tr>
<tr>
<td>1st mode: as above (3rd floor)</td>
<td>3rd</td>
<td>13.62</td>
<td>4.57</td>
<td>-22.22</td>
<td>5.62</td>
</tr>
<tr>
<td>2nd mode: m=0.075 Kg (1st floor)</td>
<td>2nd</td>
<td>9.66</td>
<td>3.61</td>
<td>-17.89</td>
<td>4.53</td>
</tr>
<tr>
<td>3rd mode: k=11.53 N/m (TMD-1)</td>
<td>1st</td>
<td>5.14</td>
<td>2.06</td>
<td>-10.20</td>
<td>2.59</td>
</tr>
<tr>
<td>1st mode: as above (2nd floor)</td>
<td>3rd</td>
<td>13.63</td>
<td>3.61</td>
<td>-17.91</td>
<td>4.54</td>
</tr>
<tr>
<td>2nd mode: as above (1st floor)</td>
<td>2nd</td>
<td>9.69</td>
<td>3.61</td>
<td>-17.91</td>
<td>4.54</td>
</tr>
<tr>
<td>3rd mode: m=0.025 Kg (TMD-2)</td>
<td>1st</td>
<td>5.15</td>
<td>2.07</td>
<td>-10.21</td>
<td>2.60</td>
</tr>
<tr>
<td>1st mode: as above (2nd floor)</td>
<td>3rd</td>
<td>13.63</td>
<td>3.61</td>
<td>-17.91</td>
<td>4.54</td>
</tr>
<tr>
<td>2nd mode: as above (1st floor)</td>
<td>2nd</td>
<td>9.69</td>
<td>3.61</td>
<td>-17.91</td>
<td>4.54</td>
</tr>
<tr>
<td>3rd mode: k=8.09 N/m (TMD-3)</td>
<td>1st</td>
<td>11.41</td>
<td>4.28</td>
<td>-19.23</td>
<td>4.83</td>
</tr>
<tr>
<td>1st mode: as above (2nd floor)</td>
<td>3rd</td>
<td>15.65</td>
<td>6.45</td>
<td>-23.47</td>
<td>5.66</td>
</tr>
<tr>
<td>2nd mode: as above (1st floor)</td>
<td>2nd</td>
<td>11.02</td>
<td>5.13</td>
<td>-19.07</td>
<td>4.61</td>
</tr>
<tr>
<td>3rd mode: c=0.02 N-s/m</td>
<td>1st</td>
<td>6.18</td>
<td>2.94</td>
<td>-10.92</td>
<td>2.64</td>
</tr>
</tbody>
</table>

### ACTIVE TUNED MASS DAMPERS

The concept of active TMD or ATMD, has been an area of interest to researchers for some time (Morison and Karnopp, 1973; Chang and Soong, 1980). An ATMD offers benefits over a TMD like controlling multiple structural modes and reduced damper mass and stroke requirement. At present there exist several examples of real-life implementations of active mass dampers (Soong et al., 1994). Considering the difficulties involved in the accurate measurement of floor displacements due to the lack of absolute reference during a seismic event, algorithms have recently been proposed (e.g., Dyke et al., 1993; Nishimura et al., 1992a, b) which require only acceleration or acceleration and velocity measurements but not displacement measurements. The algorithm proposed by Nishimura et al. (1992a, b) essentially utilizes the same methodology of optimization, as originally proposed by Den Hartog (1956). This algorithm is further studied in this paper. The equations of motion of a SDOF structure-active mass damper system can be written as follows:

\[ m\ddot{x} + k(x - X) + c(\dot{x} - \dot{X}) = \Delta \mu P(t) + u(t) \]  
\[ M\ddot{X} + KX - k(x - X) + C\dot{X} - c(\dot{x} - \dot{X}) = P(t) - u(t) \]

where \( u(t) \) is the control force. Assuming an undamped SDOF structure, Nishimura et al. (1992a, b) proposed the following expression for control force calculation:

\[ u(t) = -MG_a\dot{X} - c_{opt}G_v(\dot{x} - \dot{X}) \]

Where \( G_a \) and \( G_v \) are acceleration- and velocity-feedback gains respectively. \( G_a \) should always be less than unity since \( G_a \geq 1.0 \) results in a complete annihilation of inertial resistance causing instability. In the presence of inherent damping in the main structure, the optimum damper parameters can be found by a numerical search. However for an undamped SDOF structure, closed-form expressions for optimal parameters have been derived:

<table>
<thead>
<tr>
<th>Harmonic main mass excitation</th>
<th>Harmonic base excitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{opt} )</td>
<td>( \frac{1}{1+\mu}\sqrt{1 - G_a} )</td>
</tr>
<tr>
<td>( \zeta_{opt} )</td>
<td>( \approx \sqrt{\frac{3(\mu + G_a)}{2(1 + \mu)}} )</td>
</tr>
</tbody>
</table>

\( \approx \sqrt{\frac{3(\mu + G_a)}{2(1 + \mu)}} \sqrt{\frac{2}{2 - \mu}} \)
Following above discussion, one can obtain optimum damper parameters when the damper is activated using acceleration- and velocity-feedback control force. However, it is preferable to activate the TMD only selectively while using it in passive mode for most of the times. Therefore, there is a need to study the performance of an active damper whose parameters (frequency ratio $f$ and damping ratio $\zeta_d$) are maintained at their optimum values of the passive mode. In this practical situation, $G_a$ and $G_v$ are the parameters which should be optimized.

**Harmonic Analysis**

Figure 3 is obtained by performing steady-state harmonic base excitation analyses on a SDOF structure-active damper system. This figure shows the plots of the peak response ratio values for different $G_a$-$G_v$ combinations with changing structural parameters. It is seen that at higher $\mu$ and/or $\zeta$, the benefit of activating the TMD becomes less significant. From this figure, it is also clear that:

- Increasing $G_a$ and accordingly adjusting $G_v$ results in significant reduction of peak response ratios. For parameters $\mu = 0.02$ and $\zeta = 0.02$, for example, the peak response ratio reduces from 7.6 (passive TMD) to 2.4 (at $G_a = 0.8$ and $G_v = 4.0$). The effect of increasing $G_a$ diminishes at higher $G_a$ values.

- For each $G_a$ value, there exists an optimum $G_v$, denoted here as $G_{\text{opt}}$. $G_{\text{opt}}$ increases with increasing $G_a$. Adjusting $G_v$ to its optimum value is important in response reduction.

Performing a similar harmonic analysis on the 3 DOF structure considered, Fig. 4 is obtained. This figure indicates that a significant response reduction is possible in all three modes. Although the value of $G_{\text{opt}}$ is different for different modes, the reduction in the response of higher structural modes by using a $G_v$ which is optimum for the first mode would still be significant.

**Time-History Analysis**

Figures 5 and 6 summarize the results of the time-history analyses performed on the 3 DOF structure under El Centro and Mexico earthquakes respectively. Under El Centro earthquake, significant additional reduction is observed. Under Mexico earthquake, the structure derives a relatively smaller benefit.
from active control. Regarding the selection of suitable values of $G_a$ and $G_v$, it is noted from above analyses that increasing $G_a$ in general results in a further response reduction. However, this effect is smaller at higher $G_a$ values. For the structure analyzed here, a $G_a$ of 0.6 seems to be reasonably good. Also, it is to be remembered that a $G_a \geq 1.0$ will result in an unstable system. The value of $G_{v_{opt}}$ depends on the value of $G_a$. Although $G_{v_{opt}}$, as observed in the time-history results, usually differs from harmonic excitation analysis results, providing a $G_{v_{opt}}$ as suggested by harmonic excitation analysis usually proves to be a good, if not the best, choice.

![Graphs showing response and actuator force for various $G_a$-$G_v$ combinations.](image)

Fig. 5. Top Floor Response and Actuator Force for Various $G_a$-$G_v$ Combinations: El Centro

![Graphs showing response and actuator force for various $G_a$-$G_v$ combinations.](image)

Fig. 6. Top Floor Response and Actuator Force for Various $G_a$-$G_v$ Combinations: Mexico

**Harmonic Detuning Analysis**

To determine the additional benefit that can possibly be obtained by adjusting the damper parameters to their optimum active mode values rather than keeping them fixed at their optimum passive mode values, detuning analysis using steady-state harmonic base excitation analysis is done on a SDOF structure-active damper system. The peak response ratio values so obtained are plotted in Fig. 7. In this analysis, optimum $G_v$ which is obtained by harmonic analysis (Fig. 3) has been used.

In addition to noting the significant additional response reduction offered by ATMD, it is observed that the benefit of adjusting parameter $f$ is not significant. Since damper parameter $\zeta_d$ is already adjusted to its active state optimum value by using the appropriate $G_v$, a change in $\zeta_d$ causes performance deterioration. From Fig. 7, it can be concluded that it is permissible to let the damper parameters remain fixed at their optimum values of passive state even when the damper is switched to the active mode. This is significant in simplifying the implementation of such a control system without compromising control efficiency.
CONCLUSIONS

A study performed on TMD and their generalizations is presented. A simplified procedure for TMD design is illustrated. Control of multiple structural modes using a MTMD is not found to be effective due to modal contamination in the 3 DOF structure considered. A recently proposed active TMD algorithm is simplified and it is shown to be effective in response control. Selection of appropriate values of feedback gains is also explained.

REFERENCES


