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KINEMATIC HARDENING MODEL BASED ON GENERAL PLASTICITY FOR RC STRUCTURES

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ABSTRACT

A plasticity-based global approach is proposed to simulate the nonlinear behavior of RC frames subjected to earthquakes. The yielding surface in generalized variables is defined by an interaction diagram (*M,N*) at cross section level. The kinematic rule describes the plastic strain rate beyond concrete cracking in the hardening and softening regime. The direction of plastic flow is given by a new rule which takes into account the elastic trial stress and the updated kinematic center of the yield surface. Hardening and damage parameters are identified by an equivalent cyclic law which has been calibrated by a uniaxial approach.

KEYWORDS

Reinforced concrete; kinematic rule; non-associated plasticity; hysteretic behavior; hardening; softening.

INTRODUCTION

Most of the constitutive relations for RC structures have been derived from experimental results from certain class of loading paths. Furthermore, the complexity of the overall involved phenomena has been eliminated by simplified models assuming a decoupled effect of normal force, shear and flexion. However, for more general load histories this assumption is not valid and leads to inaccurate and unreliable results. The principal objection of uniaxial orthotropic models is that, in general, they do not satisfy the form-invariance condition for initially isotropic materials as reinforced concrete. Within the framework of the theory of plasticity the dependence of the stresses on the history of the material and their interaction, hardening and degradation can be described by means of internal variables. Since the internal variables describe irreversible material behavior, their evolution is governed by rate equations characterized by a yield surface.

Multi yield surfaces, non-associated flow and softening characterize the integration algorithms for elastoplastic constitutive relations for concrete and geomaterials. A major attention should be paid to loading/unloading increments when multiple yield conditions occur at a stage of the solution process (corners problem). The correct identification of active surfaces leads to a general solution for simple or multiple surface models. If several surfaces are activated, the ambiguity of plastic flow is removed by considering the contribution of each individual loading surface separately. Furthermore, potentially involved surfaces should be activated one by one, instead of being deactivated, in order to have a valid solution for hardening and softening regimes (Pramono and Willam 1989).

Due to nonlinear behavior of materials, it is accepted the existence of a limit point of load history which divides the structural behavior in elastic and plastic regime. During the elastic regime all unloading rends the structure to a pre-existing state in the constitutive law. In contrast, beyond this limit named yield point, residual strains appear at unloading and the previous state of the material is irrecoverable.
The theory of plasticity defines "when" and "how" the plastic strain appears. In the first case, the existence of an elastic limit function $f(\Gamma,V)$ is accepted. This function is generally represented in a stress space where internal variables are included. The stress Γ defines the occurrence of elastic or plastic behavior, whereas the internal variables V characterize the plastic flow of material where $f(\Gamma,V)>0$ is unacceptable. Such a function defines for all the points one of the following results:
a) $f(\Gamma,V) < 0$ which implies elastic behavior, consequently the elastic hypothesis is correct, and strains and stresses are updated.
b) $f(\Gamma,V) > 0$ which implies plastic behavior, consequently the elastic trial stress should be returned to the yield surface to verify $f(\Gamma,V)=0$, and plastic behavior arises.

Once the plastic behavior is detected, the 'incremental plastic consistency' and the 'plastic strain velocity' define "how" the plastic flow appears. The first condition is expressed by eqns (1) and (2) below:

$$f(\Gamma,V)=0 \tag{1}$$

$$\frac{\partial f}{\partial \Gamma}:d\Gamma+\frac{\partial f}{\partial V}:dV=0 \tag{2}$$

where (1) defines the existence of the plastic strain velocity and (2) returns the elastic predictor to the yield surface (consistency condition). The second condition named 'normality rule' is expressed by:

$$\dot{\varepsilon}^{p}=\dot{\lambda}_{p}\frac{\partial f}{\partial \Gamma} \tag{3}$$

where $\dot{\varepsilon}^{p}$= increment of plastic strain, $\dot{\lambda}_{p}$= plastic multiplier which defines the magnitude of plastic strain and $\partial f/\partial \Gamma$ is the gradient of the yield surface which defines the direction of flow. In addition to previous conditions, the commonly assumed additive decomposition of strains into elastic and plastic components (Drucker-Prager's law) completes the set of evolution equations.

KINEMATIC HARDENING RULE

The classical theory of plasticity involves a unique yield surface which moves, expands and distorts due to the plastic flow. In order to simplify the plastic evolution, two major rules are used:
a) expansion and distortion are replaced by a homogeneous deformity of the yield surface characterizing the isotropic hardening. The plastic flow is given by a scalar variable R which defines the increase of the surface size (Prandtl-Reuss's rule):

$$f(\Gamma,R)=F(\Gamma)-\Gamma(R) \tag{4}$$

b) motion is characterized by a translation of rigid body as a kinematic hardening. The plastic flow is given by a tensor variable X which defines the current position of the yield surface (Prager's rule) and K represents a constant yield stress:

$$f(\Gamma,X)=F(\Gamma-X)-K \tag{5}$$

In order to take into account the nonlinear evolution of plastic flow in a simple way, a set of nested loading surfaces surrounding the yield surface was proposed (Mroz, 1969). In this multiple surface model the nonlinear plastic flow is described by a set of constant hardening modulus corresponding to each engaged surface. Two-surface models (Dafalias and Popov 1975) considered as a simplified Mroz model have been widely used. In both families, the condition of same tangent at the contact point of involved surfaces leads to a strong limitation in the description of plastic flow due to independence of loading path and flow direction.

The isotropic hardening seen as a cyclic hardening effect can be introduced into the kinematic variable considering the cumulative plastic strain through the evolution of the kinematic center (Lemaitre and Chaboche 1984). In this way, and keeping in mind the interest of a single surface model, different kinematic rules are reviewed:

- In the kinematic rule proposed by Prager (1955), the yield surface follows the outside normal direction at contact stress Γ_C. The kinematic hardening is given by the normality rule with a hardening variable α, identical to the current plastic strain (linear kinematic hardening). This means that the incremental evolution of the center yield surface is linear dependent on the plastic strain increment, and is given by:

$$\dot{\alpha} = -\dot{\lambda}\frac{\partial f}{\partial X} = \dot{\lambda}\frac{\partial f}{\partial \Gamma} = \dot{\varepsilon}^p \tag{6}$$

$$\dot{X} = C_0 \dot{\varepsilon}^p \tag{7}$$

with $\dot{\varepsilon}^p$ given by equation (3) and the plastic multiplier λ is given by the consistency condition, where C_0 is the hardening modulus and $H(f)$ is the Heaviside function:

$$\dot{\lambda} = \frac{H(f)}{C_0} \frac{\frac{\partial f}{\partial \Gamma}:\dot{\Gamma}}{\frac{\partial f}{\partial \Gamma}:\frac{\partial f}{\partial \Gamma}} \tag{8}$$

- The model proposed by Ziegler (1959) leaves the normality rule as a direction parameter. In this case the direction of flow is given by the current kinematic center X and the contact stress Γ_C.

- A different kinematic rule with a new direction of plastic flow has been proposed by Mroz (1969). The translation of the yield surface lies on the line connecting two contact stress points which outside normal has the same direction. The first point Γ_C corresponds to the reached surface, whereas the second one Γ_{l+1} corresponds to the next load surface to be engaged.

In the preceding models the direction of flow is independent of loading path. As it was noted by Jiang (1994) the independence of the direction of plastic flow leads to a poor estimation of cyclic behavior which is more accentuated when non-proportional loadings are involved.

In order to account for the applied stress in the direction of plastic flow (Jiang 1994) and based on the preceding kinematic laws, a new translation rule is proposed. In this case, the final position of the kinematic center lies on the line connecting the elastic trial stress Γ^t and the current kinematic center X_c. The consistency condition applied to this new variation law of $\dot{X}$ leads to the corresponding expression of the kinematic multiplier $\dot{\mu}$. The updated position of the kinematic center for subsequent plastic loading characterize the proposed kinematic evolution.

Table 1 shows the expressions for the translation of the kinematic center and the kinematic multiplier of these three rules.

Table 1. Evolution law and kinematic multiplier for three different kinematic rules

$$\dot{X} = \left(\Gamma_g - X\right)\dot{\mu} \qquad \text{with} \qquad \dot{\mu} = H(f)\frac{\dfrac{\partial f}{\partial \Gamma_g}\dot{\Gamma}_g}{\left(\Gamma_g - X\right):\dfrac{\partial f}{\partial \Gamma_g}}$$

Model	Γ_g	X
Mroz	$\Gamma_{\ell+1}$	Γ_c
Ziegler	Γ	X_c
proposed	Γ^t	X_c

Similarly to existing models, the plastic multiplier is given by equation 8. Figure 1 shows the direction of plastic flow for these kinematic rules. The proposed direction describes a plastic strain growth rate that monotonically decreases and finally approaches a constant value for non-symmetrical loading and zero for symmetric loading when the steady state is reached. That means that non symmetrical loading causes unlimited plastic strain accumulation, while symmetrical loading only causes bounded plastic strain growth. This phenomenon is illustrated in Fig. 2.

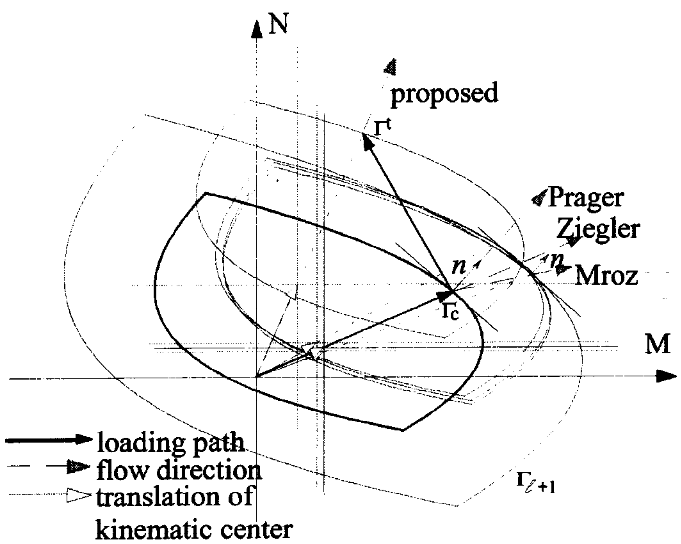


Fig. 1. Direction of plastic flow

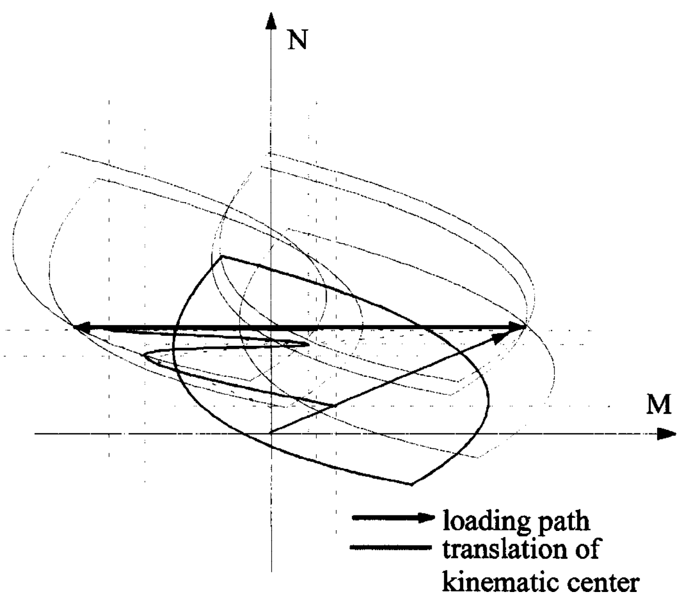
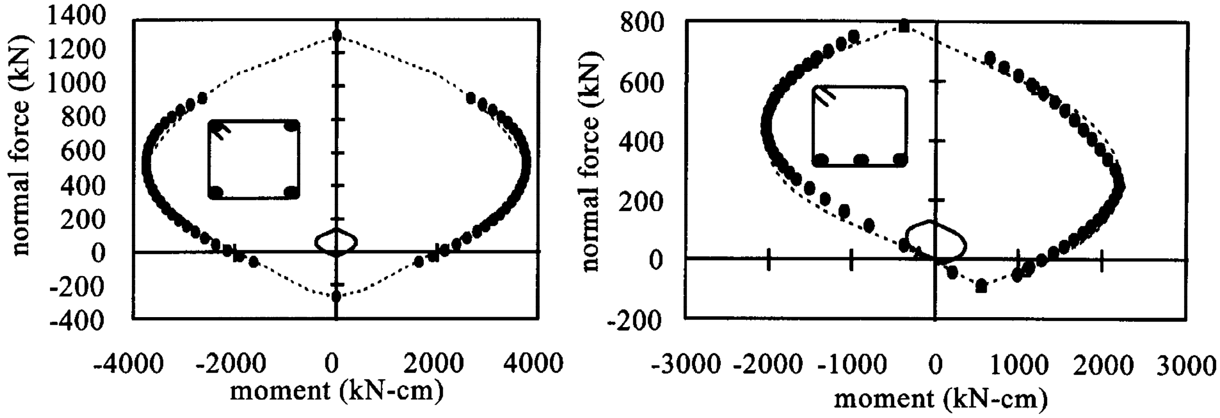


Fig. 2. Cyclic translation of yield surface for non-symmetrical loading.

Yield Surface

The yield function $f(\Gamma, V)$, in terms of generalized variables (N,V,M) inserted to essential postulates of plasticity (normality rule and consistency condition) leads to similar elasto-plastic constitutive equations to those obtained for yield functions formulated in a local stress field (σ_1 , σ_2 , σ_3). In this work, the yield surface is defined by an interaction diagram of a RC section. A limit analysis for five strain conditions at section level in two directions of flexion are used to build a set of three polynomial functions which describe completely the interaction diagram. Each direction is composed of two straight lines and a polynomial of fifth degree. The final size of the yield surface is evaluated in such a way that maximum strength in tension coincides with cracking strain of concrete. Figures 3a and 3b show two yield surfaces for symmetrical and non symmetrical RC sections.



a) Symmetrical RC section

b) Asymmetrical RC section

Fig. 3. Interaction diagram: ● polynomial - - - limit analysis ___ yield function.

RETURN MAPPING STRATEGY

The central issue in computational plasticity is the determination of the plastic multiplier, in order to return the trial stress state to the final yield condition when plastic loading takes place. The plastic multiplier is evaluated with the aid of the consistency condition which assures that the hardening/softening law, as well as the yield condition are both satisfied by the final state of stress and plastic deformation. Two general groups of return mapping algorithms were identified (Ortiz and Popov 1985); *the generalized trapezoidal rule* and the *generalized midpoint rule*, in which the linearized consistency condition requires a sequence of linear steps for advancing the elastic process from stage t_n to stage t_{n+1} with the new state variables Γ_{n+1}, V_{n+1} (*cutting plane algorithms*). A more direct solution is given by the non-linearized consistency condition in the state t_{n+1} (Simo and Taylor 1985). In the same category of non-linearized consistency condition, a new single step return method named *generalized midstep rule* has been proposed (Pramono and Willam 1989). This method was introduced for plasticity models which exhibit hardening/softening within the framework of non associated flow.

In this work, with regard to the proposed kinematic rule, the kinematic center is updated accordingly to the elastic trial stress. Furthermore, taking into account the main idea of the single step return method (Pramono and Willam 1989) which infers that the direction of plastic strain remains constant during the entire return step, the expressions of the plastic multiplier formulated in a generalized stress space $\Gamma = (M, N)$, are defined as follows: in a general form, the yield function is given by the moment M and a polynomial of degree n in N as normal force, where corrected trial stress is expressed as a function of plastic multiplier by:

$$\Gamma = \Gamma_0 + \dot{\Gamma} \tag{9}$$

where the first term in the right hand side is the initial stress state at the current incremental step and the second is the corrected trial stress defined by:

$$\dot{\Gamma} = \dot{\Gamma}^t - \dot{\lambda}\frac{\partial f}{\partial \Gamma}C(\Gamma) \tag{10}$$

where $C(\Gamma)$ denotes the kinematic hardening modulus. The evolution of this parameter is guided by an equivalent stress-strain relationship $(\overline{\Gamma}, \overline{\varepsilon})$. This behavioral law is based on a moment-curvature uniaxial cyclic law (Miramontes et al 1995) which is expressed here in terms of equivalent magnitudes. If plastic flow occurs in an intersection of two yield functions, in a general solution for hardening/softening behavior (Pramono and Willam 1989), the projection of the trial stress onto each yield function is carried out one by one. If the projected stress for a first function violates a second one, the plastic multiplier is evaluated for this second yield function. The overall plastic strain rate can be decomposed in:

$$\dot{\varepsilon}^p = \dot{\lambda}_1 \frac{\partial f_1}{\partial \Gamma} + \dot{\lambda}_2 \frac{\partial f_2}{\partial \Gamma} \tag{11}$$

CYCLIC BEHAVIOR LAW

The principal phenomena described by the hysteretic rules are: the strength reduction and stiffness degradation due to cyclic loading, the pinching of hysteretic loops due to shear strains and the softening behavior at failure. In order to account for strength deterioration due to cyclic loading, in both pre-peak strain range and post-peak strain regime, a new parameter of cyclic evolution (β) has been proposed. This parameter is used to define a focal point associated with the maximal strain level $\beta\,\overline{\varepsilon}_{max}$ and depends on a damage index D_{max}, on an accommodating factor A_{max} and on the number of cycles n experienced up to the maximum strain level (figure 4):

$$\beta = 1 + \sum_i^n \left[D_{max} A_{max}\right]^i = 1 + \sum_i^n \left[\frac{E_{max}}{E_u} \frac{M_{max}\overline{\varepsilon}_{max}}{M_u \overline{\varepsilon}_u}\right]^i \tag{12}$$

where E_{max} and E_u are the total absorbed energy at the $\overline{\varepsilon}_{max}$ and $\overline{\varepsilon}_y$ strains respectively. Every time the maximum strain ever experienced $\overline{\varepsilon}_{max}$ is updated, i and β are initialized to zero and one respectively, while D_{max} and A_{max} remain constant throughout the cyclic loading.

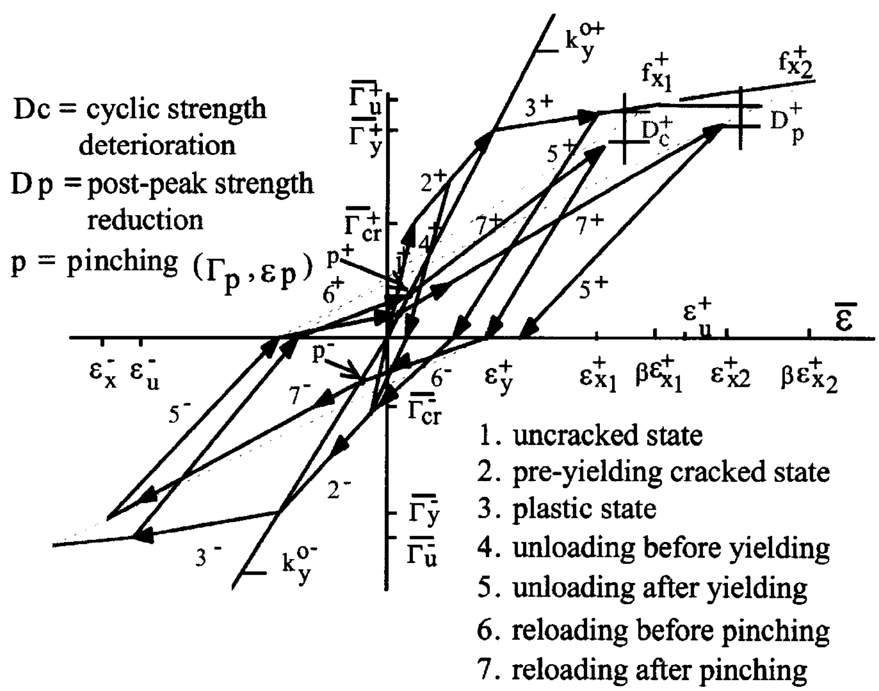


Fig. 4. Equivalent hysteretic law

VALIDATION OF THE UNIAXIAL LAW

Beam-Column joint subassembly

The numerical investigation presented here concerns a beam-column subassembly subjected to reverse cyclic loading. The specimen considered is an interior joint of a frame with 4 m bays and 2 m storey heights. The geometric characteristics and the reinforced layout are described by Miramontes *et al* (1995). The vertical displacements of the beam ends are prevented and an alternating horizontal displacement is imposed on the base of the column simulating the seismic action. The global response of the subassembly concerns the history of the horizontal displacement *d* at the base of the column, versus the horizontal load *H*. This example shows the complete description of the nonlinear behavior during the cyclic loading with good agreement with the experimental results in terms of the degradation process during the pre-peak and post-peak regime (table 2).

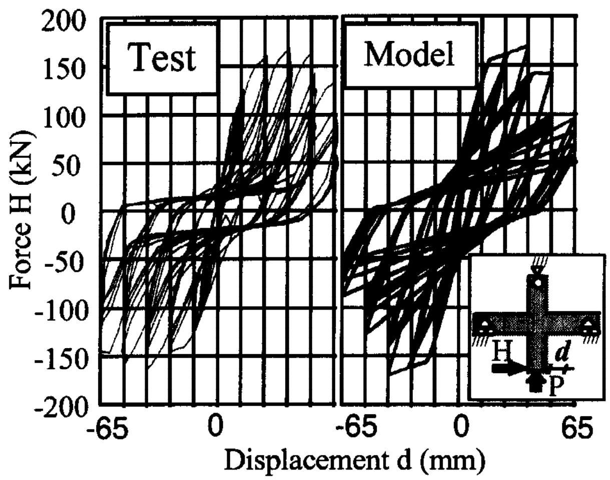


Fig. 5. Beam-column cyclic test

Table 2. Residual strains and strength deterio ration at different cycle levels

cyclic displacement (mm)	residual strain (mm) test	residual strain (mm) calc	strength reduction % test	strength reduction % calc
26	10.21	7.69	12.5	7.4
39	22.29	17.71	27.0	23.7
52	33.43	29.70	37.6	42.9
65	46.42	42.71	43.0	44.4

Four storey RC full scale frame

An experimental test on a four storey full-scale RC frame was carried out in the European Laboratory for Structural Assessment at Ispra Joint Research Center in Italy. Details of the structure, reinforcement layout and experimental results are reported in Negro *et al* (1994). The time history of storey displacements are compared in figure 7. The global response of a predictive analysis (non cracked sections) agrees well with experimental results (table 3). In general, between 0 and 4s (Fig. 6) the dispersion of the predicted response decreases as the intensity of the seismic excitation increases. That means that the more the nonlinear response becomes relevant the less important are the assumptions made in what concerns the initial stiffness and the behavior should be controlled by a correct nonlinear description of the members.

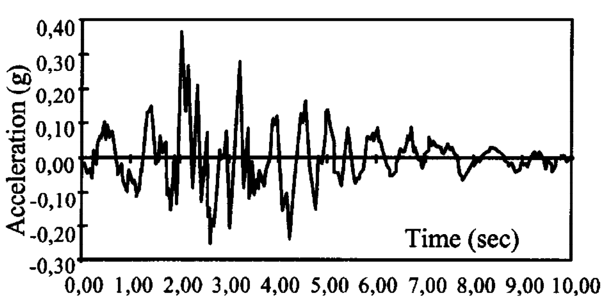


Fig. 6. Friuli-like generated accelerogram S7

Table 3. Global response comparison

measurement	experiment	proposed model
d_{max}/time step	21.0 / 3.02	17.45 / 3.0
averaged frequency	0.820	0.905
base shear (kN)	1440	1490

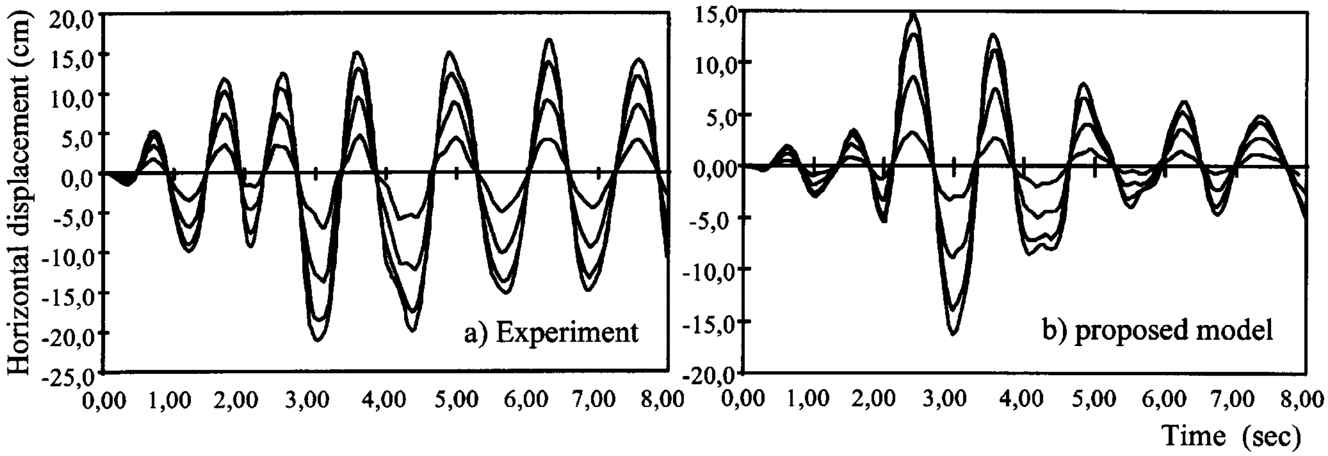


Fig. 7. History of experimental and analytical story displacements

CONCLUSIONS

In order to take advantage of efficient uniaxial models and considering the interaction of different effects such as normal force and flexion, a plasticity approach formulated in generalized variables has been proposed. A plasticity-based formulation for reinforced concrete involves multiple yield functions, degradation process and softening. The major aspects included in the present formulation are the load dependence into the definition of plastic flow direction and its evolution for cyclic loading by a new kinematic rule. The efficiency of the proposed model is assured by using a unique yield surface defined by an interaction diagram. Strength deterioration and stiffness degradation due to cyclic loading, pinching of hysteretic loops due to shear strain, hardening and softening are included into the evolution of internal variables with the aid of a calibrated uniaxial relationship which global response agrees quite well with the experimental results.

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