DAMPING DETERMINATION OF SANDS UNDER DIFFERENT LOADINGS

X.J. ZHANG1 and M.S. AGGOUR2

1Watcher-Sayre, Inc., 905 Southlake Boulevard, Richmond, VA 23236, USA
2Department of Civil Engineering, University of Maryland, College Park, MD 20742, USA

ABSTRACT

Experiments were conducted on Ottawa 20-30 air dry sand specimens in a resonant column device using three types of loading: sinusoidal, random and impulse, to study the effect of loading type on the damping of sands. The specimens were excited torsionally with one of the three types of loading at different strain levels from low to high. Each specimen was tested at three confining pressures of 5, 10 and 40 psi. From the measured resonant frequencies and responses, the damping ratios of the sand specimens under different types of loading at different shear strain levels were obtained. The damping ratios from different loading tests were compared and analyzed. It was found that for each type of loading, the damping ratio under different confining pressures could be unified using a normalized shear strain with the reference shear strain. Loading type had a significant effect on the damping ratio. At high shear strain levels the sand specimens had the highest damping under impulse loading and lowest damping under sinusoidal loading. At low strain levels, damping was the same for all types of loading. Equations for the determination of the damping ratio of sand for sinusoidal, random and impulse loading were developed.

KEYWORDS

Damping; sand; resonant column; different loading: sinusoidal, random; impulse; shear strain.

INTRODUCTION

Damping ratio is considered to be one of the primary dynamic properties of soils. In general it is evaluated from laboratory testing. The damping ratio of soils under sinusoidal loading has been studied extensively, however, research on the damping ratio of soils under other types of loading are very limited. Earthquakes, wind, ocean waves, and certain man-made forces do not provide a sinusoidal pattern of excitation. In order to establish meaningful results that represent field conditions, nonperiodic loading should be used in laboratory testing. The objective of this research was the determination of the damping ratio of sand under various loading conditions using the resonant column technique.

Experiments were conducted on Ottawa 20-30 sand using three types of loading: sinusoidal, random and impulse. Sand specimens were excited torsionally with one of the three types of loading at different strain levels from low to high. Each specimen was tested at three different confining pressures. The damping ratio of the sand specimens under different types of loading at different shear strain levels were determined from
the measured resonant frequencies and responses. Equations for the determination of the damping ratio of sand were developed for each type of loading.

TEST APPARATUS AND TEST PROCEDURES

The resonant column device used in this research was the Drnevich "fixed-free" type with solid cylindrical specimens. The specimens were fixed at the base with excitation forces applied to the top. The dimensions of the solid cylindrical specimens were 7.5 cm in length and 3.6 cm in diameter. Air-dry Ottawa 20-30 sand was used in the specimen preparation, and all specimens were prepared in four layers to a relative density of approximately 78% using a dry tapping method. A vacuum pressure of 15 psi was used for the assembly of the top platen, LVDT and pressure chamber. After the assembly, a confining pressure of 5 psi was applied and the vacuum pressure was disconnected.

Each test sequence was composed of several test stages from low strain level to high strain level. At each test stage, a soil specimen was excited torsionally for one minute, and the resonant frequency and response of the soil-mass system were measured. Approximately one minute of break time was maintained between each two adjacent stages. The test sequence for the confining pressure of 5 psi was finished when the test stage at the predetermined highest excitation level had been completed. After about five minutes the confining pressure was increased to 10 psi and the same test sequence was repeated, and then the confining pressure was increased to 40 psi.

The sinusoidal excitation signals were generated by a sine-wave oscillator with variable frequency and amplified by a power amplifier. Then the amplified sinusoidal signals were sent to the torsional coil for the soil excitation. The acceleration responses of the soil-mass system in the torsional direction were picked up by a transducer mounted in the top platen sitting on top of the soil column. The response signals were then amplified by a charge amplifier. The amplitudes of the excitation and response at resonance were read on a voltmeter in root-mean-square (rms) values, and the resonant frequency was read from a digital frequency meter.

A white-noise generator was used in the tests with random excitations. The signals from the white-noise generator were first filtered through a two-channel variable cut-off frequency filter, and then amplified and sent to the torsional exciting coil. The signals of random excitation and response were recorded on a tape recorder. After the tests were finished, the signals were analyzed by replaying the recorded signals to a digital analyzer (FFT analyzer). As random signals are nondeterministic, a large amount of data was necessary to establish the statistical characteristics of the random signals. Therefore, it was necessary to perform many averages on the FFT analyzer to eliminate the noise in the random signals and get a smooth response curve. Then, the values of the power spectrum density (PSD) functions of the excitation and response, and magnitude of the transfer function (MTF) were read at the resonant peak.

The impulse signals were generated by a pulse signal generator. The excitation and response signals were sent directly to the FFT analyzer. The width of a pulse signal represents the time length during which the pulse acts, and controls the characteristics of the PSD function of the pulse. The width of a pulse could be adjusted on the pulse generator. To obtain a higher strain level with impulse excitation, it was necessary to adjust the pulse width so that the peak of the PSD of a pulse was around the resonant peak.

TEST RESULTS

With the results of measurements of resonant frequency, amplitudes of excitation and response at the resonant frequency at each stage of testing, the root-mean-square (rms) of shear strain amplitude induced in the specimens and the damping ratio were determined. The rms of the sinusoidal strain amplitude was determined using the conventional method (Drnevich et al., 1978). Random vibration theory was used in
determining the rms values of shear strain amplitude under random and impulse loadings (Zhang, 1994). The Damping Calibration Factor (DCF) method, which was suggested by Hardin (1970), was used to determine the damping ratio of specimens under each of the three types of loadings because this method was found to be the most suitable and relatively insensitive to the frequency resolution of the FFT analyzer (Zhang, 1994). The damping ratio of the specimens was determined using DCF from the following equation:

$$D = \frac{1}{2} \frac{\text{DCF}}{\sqrt{H_n^2 - 2(\text{DCF})^2}}$$  \hspace{1cm} (1)

in which $H_n$ is the value of the transfer function at the resonant frequency, and DCF is the DCF of the whole system. The DCF of a resonant column system is a relative constant when the stiffness of the specimens is not drastically changed (Drnevich, 1978).

The damping ratio of sand specimens under sinusoidal loading at the three confining pressures are shown in Figure 1 with the shear strain amplitudes (rms) on a log scale. Figure 1 shows that the damping ratio of the sand specimens was low and relatively constant at a low strain range (<0.003%). With the increase of strain amplitude, the damping ratio also increased. Furthermore, the damping ratio of sand specimens was affected by the confining pressure. At any strain amplitude, the higher the confining pressure, the lower the damping ratio was.

![Graph showing damping ratio D (%) vs. shear strain γ (%) (rms) at different confining pressures from sinusoidal loading tests](image)

Fig. 1 Damping ratio D (%) vs. shear strain γ (%) (rms) at different confining pressures from sinusoidal loading tests

It was found that the test results shown in Figure 1 could be unified when a normalized scale of shear strain was used. The normalized scale of shear strain was obtained by dividing the shear strain amplitude at each stage by the corresponding reference shear strain of each test sequence, as $\gamma/\gamma_r$. The unified damping ratio of the sand specimens under sinusoidal loading is shown in Figure 2. A reference shear strain $\gamma_r$ was suggested by Hardin and Drnevich (1972) and is a parameter related to the stiffness of the specimens. For cohesionless specimens, $\gamma_r$ is determined from the equation:

$$\gamma_r = \frac{\sigma_o \sin \phi}{G_{\max}}$$  \hspace{1cm} (2)
in which \( \sigma_0 \) is the mean effective principal stress of the soil specimens, \( \phi \) is the internal friction angle of the soils and \( G_{\text{max}} \) is the shear modulus of the soil specimens at a very low shear strain amplitude \( (<10^{-4}\%) \). Therefore, the normalized shear strain, \( \gamma/\gamma_r \), which includes the shear strain applied to the soils, mean effective principal stress in the soils and the physical properties of the soils, is a main factor affecting the damping behavior of soils. It was found that this is also true for shear modulus of sands (Zhang, et al. 1995). The normalized shear modulus, \( G/G_{\text{max}} \), of sand specimens at different shear strain amplitudes and different confining pressures was unified using the normalized shear strain \( \gamma/\gamma_r \), as shown in Figure 3.

Fig. 2. Damping Ratio D (%) vs. normalized shear strain \( \gamma/\gamma_r \) from sinusoidal loading tests

Fig. 3. Normalized shear modulus \( G/G_{\text{max}} \) vs. normalized shear strain \( \gamma/\gamma_r \) from sinusoidal loading tests
The damping ratio of sand specimens under random and impulse loadings has a trend similar to that under sinusoidal loading. The damping ratio was low and relatively constant when shear strain amplitudes were low. At higher strain amplitudes, the damping ratio increased. Also, higher confining pressure induced a lower damping ratio. Further, the damping ratio obtained from the tests with random and impulse loadings could also be unified using the normalized scale of shear strain. Figure 4 shows the unified results of the damping ratio for the sinusoidal, random and impulse loading tests. Figure 4 shows that when the normalized shear strain, $\gamma/\gamma_f$, is smaller than 0.1 the damping ratio from all three types of loading are nearly the same and relatively constant. However, when $\gamma/\gamma_f$ is larger than 0.1 the damping ratio from different types of loading is significantly different. Comparing the three types of loading used in the research, at a specified $\gamma/\gamma_f$, sandy soils have the highest damping ratio under impulse loading, and the lowest damping ratio under sinusoidal loading. The damping ratio of sandy soils under random loading is between the damping ratio under impulse loading and sinusoidal loading.

![Graph showing damping ratio D (%) vs. $\gamma/\gamma_f$ from different loading tests](image)

**Fig. 4.** Damping ratio D (%) vs. $\gamma/\gamma_f$ from different loading tests

**DAMPING RATIO EQUATIONS**

From the unified damping ratio of sandy soils under each type of loading, equations of damping ratio may be developed. Damping of soils has been studied by many researchers for many years. Based on test results of damping ratio of sandy soils using sinusoidal loading, Hardin and Drnevich (1972), Tatsuoka et al. (1978) and Ishibashi (1981) proposed that the damping ratio could be expressed as a function of the normalized shear modulus, $G/G_{\text{max}}$.

Analyzing the test data of this research, it was found that the damping ratio from each type of loading could also be expressed as a function of $G/G_{\text{max}}$. A linear relationship exists between log (D) and $(G/G_{\text{max}})^b$. Hence, the following type of function was used for the damping ratio from each type of loading in this research.

$$D(\%) = A \cdot e^{-a\left(\frac{G}{G_{\text{max}}}\right)^b}$$

in which $A$, $a$ and $b$ are regression constants. Constant $A$ defines the maximum damping ratio when the ratio $(G/G_{\text{max}}) = 0$. By adjusting constant $b$, a linear relationship could be obtained between log (D) and $(G/G_{\text{max}})^b$, then constants $A$ and $a$ could be determined from the intercepts of the straight line at ratio
Fig. 5. Linear relationship between log (D) vs. \((G/G_{\text{max}})^{1.6}\) from sinusoidal loading tests

\[
D(\%) = 40 \cdot e^{-3.8(G_{\text{max}})^{1.6}}
\]  

Equations of the damping ratio of sandy soils under the three types of loading are summarized in Table 1.

<table>
<thead>
<tr>
<th>Sinusoidal</th>
<th>Random</th>
<th>Impulse</th>
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| \[
D(\%) = 40 \cdot e^{-3.8(G_{\text{max}})^{1.6}}
\] | \[
D(\%) = 40 \cdot e^{-3.8(G_{\text{max}})^{2.4}}
\] | \[
D(\%) = 40 \cdot e^{-3.8(G_{\text{max}})^{3.8}}
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It was found that a value of 40 and a value of 3.8 for constants \(A\) and \(a\), respectively, are valid for all three types of loading. Only constant \(b\) changed for different types of loading.
As discussed before, shear modulus of sandy soils under different types of loading could also be unified in $G/G_{\text{max}} \sim \gamma/\gamma_r$ space. Equations of shear modulus of sandy soils were also developed by Zhang (1994) as a function of $\gamma/\gamma_r$. By substituting the equations of $G/G_{\text{max}}$ into the equations of damping ratio listed in Table 1, the computed values of damping ratio from the equations were compared with the measured values in Figure 6. The agreement between the computed values and measured values of damping ratio from each type of loadings was very good.

**Fig. 6.** Comparison of computed values with measured values of damping ratio $D$ vs. $\gamma/\gamma_r$ under different loadings

**CONCLUSIONS**

Research on the damping ratio of sands under different types of loading, including sinusoidal, random and impulse, was conducted using a resonant column device. Sand specimens were tested at different shear strain levels at confining pressure of 5, 10 and 40 psi under each type of loading. The following conclusions were drawn.

1. The damping ratio at different confining pressures for all types of loading can be unified by using normalized shear strain, $\gamma/\gamma_r$.
2. At low strain levels ($\gamma/\gamma_r < 0.1$), the damping ratio under each type of loading is low and relatively constant. The damping ratio is not affected by the type of loading. At higher strain levels ($\gamma/\gamma_r > 0.1$), the damping ratio is affected significantly by loading type. Sands have the highest damping under impulse loading, and the lowest damping under sinusoidal loading.
3. Damping ratio can be expressed as a function of shear modulus ratio $G/G_{\text{max}}$. The exponential function of $G/G_{\text{max}}$ fits the results of damping ratio from each type of loading very well.
4. Based on this study, the damping ratio of sandy soils determined from routine sinusoidal tests should be corrected to obtain values that will be representative of actual field conditions. Such corrected values could then be used in earthquake response analysis.
REFERENCES


