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TWO DIMENSIONAL ANALYTICAL MODEL OF REINFORCED CONCRETE SHEAR WALLS

J. I. MILEV

Department of Reinforced Concrete Structures, Faculty of Civil Engineering,
University of Architecture, Civil Engineering and Geodesy,
1 Christo Smirnenski blvd., Sofia 1421, Bulgaria

ABSTRACT

Two-dimensional analytical macro model of reinforced concrete (RC) shear walls capable to describe their inelastic response is introduced. Monotonic and reversed cyclic types of loading are considered. The standard one story wall member is modeled as a group of a 2-D nonlinear plane element and two outside truss elements. Some numerical studies were carried out to check the reliability and the effectiveness of the wall macro model. The experimental data of T-shaped shear walls tested at Yokohama National University is analyzed. The comparison between numerical and experimental results shows the ability of the proposed macro element to predict, with reasonable accuracy, the response of structural walls for monotonic and reversed cyclic loading. The proposed model is simple enough to be efficiently incorporated in a practical inelastic response analysis of RC frame-wall structures.

KEYWORDS

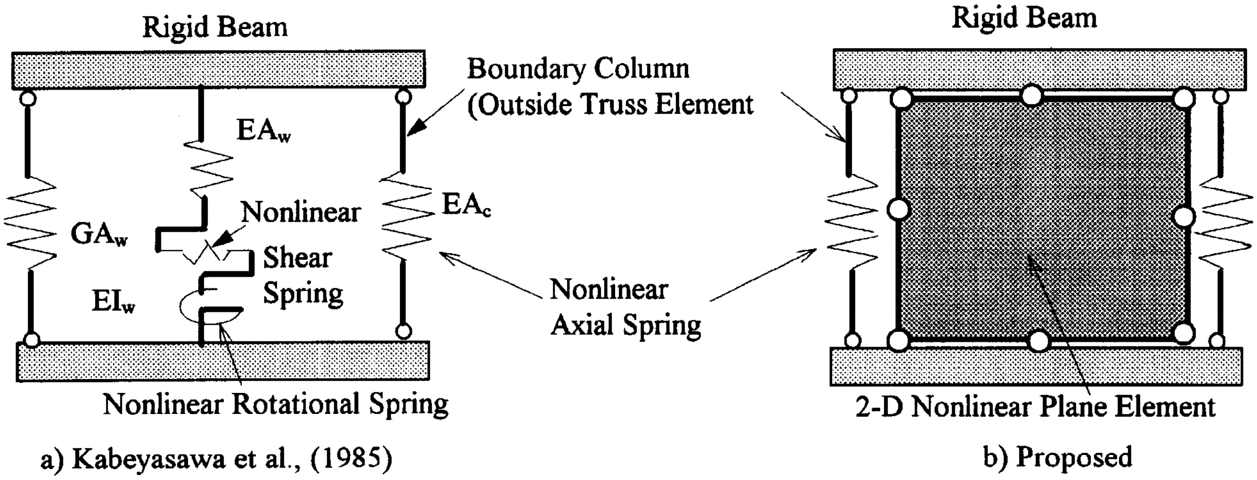
Analytical model; frame-wall structures; hybrid structures; inelastic response; macro model; monotonic; reversed cyclic; shear wall.

INTRODUCTION

The ability of a structural system to behave stable in inelastic range is taken into account in Eurocode 8 by the possibility to analyze it for the forces smaller than those induced by linear elastic response. In order to avoid the need of inelastic response analysis for design purposes, the reliable force reduction (behavior) factors have to be employed. The structural behavior factor (**q**-factor) is the ratio of the values of the elastic acceleration spectrum to those of inelastic design spectrum. This factor reflects the energy dissipation capacity of the structure through ductile behavior. Some basic values of the behavior factor are given (Eurocode 8, 1994) for the frame-wall structures. However it is necessary to correct them for many practicable design cases. Moreover, there is some lack of provisions for so called hybrid structures. Steel frames are connected through rigid floor diaphragms to wall or core elements in such building construction. There are two ways to estimate the behavior factor: experimental assessment and analytical methods. The experimental assessment is limited for small range of structures by the practical considerations. Therefore the analytical methods have to be used to obtain the reliable prediction of the inelastic seismic response of reinforced concrete (RC) frame-wall and hybrid structures. However this requires analytical models that are capable of reproducing the nonlinear behavior of each structural component with reasonable accuracy and simple enough to allow economical numerical solution for the case of multistory structures.

THE WALL MACRO ELEMENT

The direct application of a microscopic finite element modeling of shear walls for the nonlinear analysis of complete multistory RC frame-wall structures is practically impossible. This is the reason for proposal of various macro models for predicting the inelastic response of RC structural walls. The three-vertical-line-element-model (TVLEM - Fig. 1^a) was pointed in some studies (Vulcano and Bertero, 1987) as the most suitable for the prediction of the overall inelastic behavior of RC structural walls for monotonic and reversed cyclic loading. This element was formulated (Kabeyasawa *et al.*, 1985) to describe a standard one story wall member. It consisted of infinitely rigid beams at top and bottom floor levels, two outside truss elements that represented the axial stiffness of boundary columns and central one dimensional element that was a combination of vertical, horizontal and rotational springs. In some earlier studies (Vulcano and Bertero, 1987; Colloti, 1993) features and limitations of this model have been discussed and some extensions of it have been proposed.



a) Kabeyasawa et al., (1985)

b) Proposed

Fig. 1. Wall Member Model

The main modification of TVLEM in this work is that the central element (vertical, horizontal and rotational spring) is substituted by one two-dimensional nonlinear plane element (Fig. 1^b). Moreover the envelope curve of the proposed (Kabeyasawa *et al.*, 1985) Axial-Stiffness Hysteresis Model, describing the axial force-deformation relationship of the outside truss elements is modified according to recommendation of CEB-FIP Model Code 1990 - Final Draft, 1993.

Central 2-D Nonlinear Plane Element

The material constitutive model is based on macroscopic smeared crack approach. Because of this powerful higher order elements could give adequate results with greater efficiency. Quadrilateral 'serendipity' elements (Zienkiewicz and Taylor, 1991) with three- to nine-nodded isoparametric shape functions are used (Fig. 1^b). The 3*3 Gauss integration rule is applied.

Constitutive Model for Concrete. Some constitutive models have been proposed (Vecchio and Collins, 1986; Hsu, 1993; etc.) for predicting the response of RC membrane element containing an orthogonal grid of reinforcement parallel to the edges in the case of monotonic loading. However only a few complete studies have been reported (Stevens *et al.*, 1988; Okamura and Maekawa, 1990) in the case of reversed cyclic loading. The constitutive model used in this study is an adopted version of modified compression field theory (Vecchio and Collins, 1986) with the modification proposed (Stevens *et al.*, 1988) for the case of reversed cyclic loading. However the constitutive equations are extended by the shear transfer model. Additionally the rotating smeared crack approach and the perfect bond between concrete and uniformly distributed reinforcing bars is assumed.

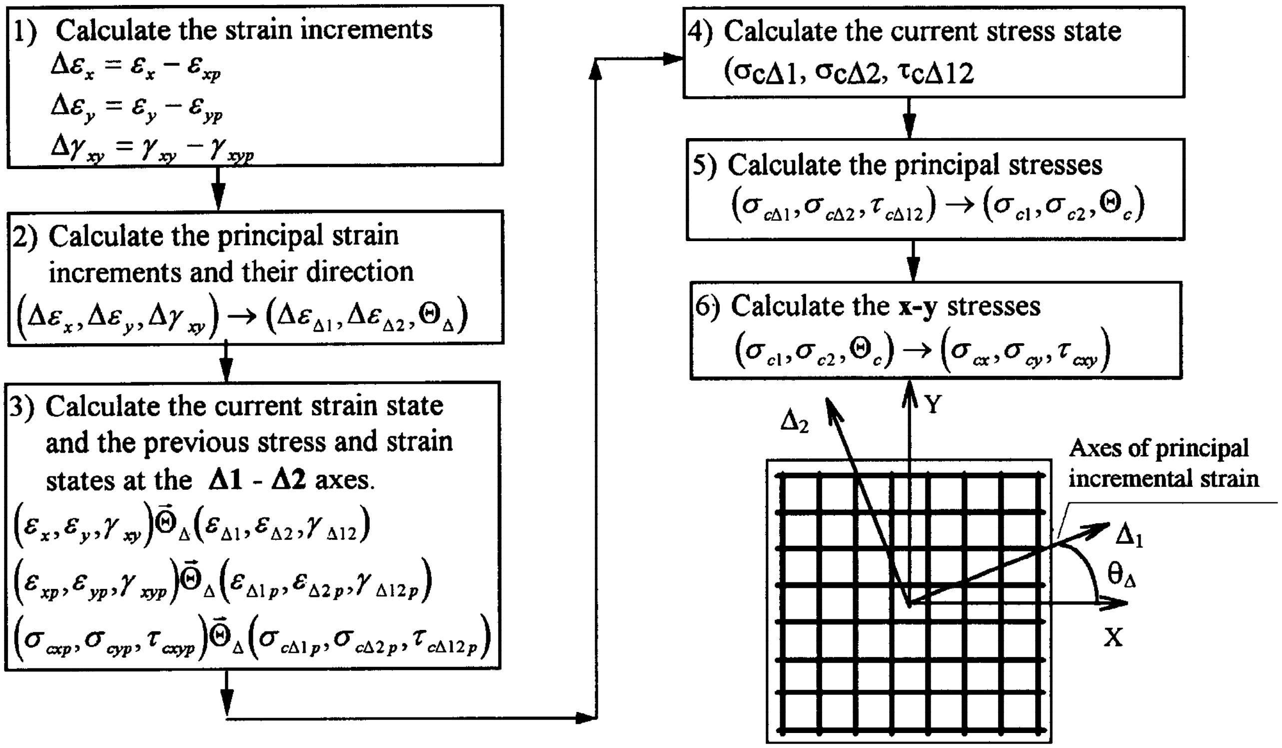


Fig. 2. The Procedure for Obtaining the Concrete Stress State for a Given Strain State

The procedure for solution of the problem, to find the concrete stress state $(\sigma_{cx}, \sigma_{cy}, \tau_{cxy})$ for a given strain state $(\varepsilon_x, \varepsilon_y, \gamma_{xy})$ as a function of the stress and strains at the end of previous load stage $(\sigma_{cxp}, \sigma_{cyp}, \tau_{cxyp})$, $(\varepsilon_{xp}, \varepsilon_{yp}, \gamma_{xyp})$ and strain history, is shown on Fig. 3. In order to make step 4 of the procedure (Fig. 3) an appropriate concrete stress-strain relationship that considers the strain history and the total strain state is adopted (Stevens, 1987). However the shear transfer across the cracks in the basic model (Stevens, 1987) is omitted and substituted by adjusting the principal concrete stresses in the permissible region. In this study the concrete stress-strain relationship is extended by adopted shear transfer model (Okamura and Maekawa, 1990) in the case of reversed cyclic loading (Fig. 4).

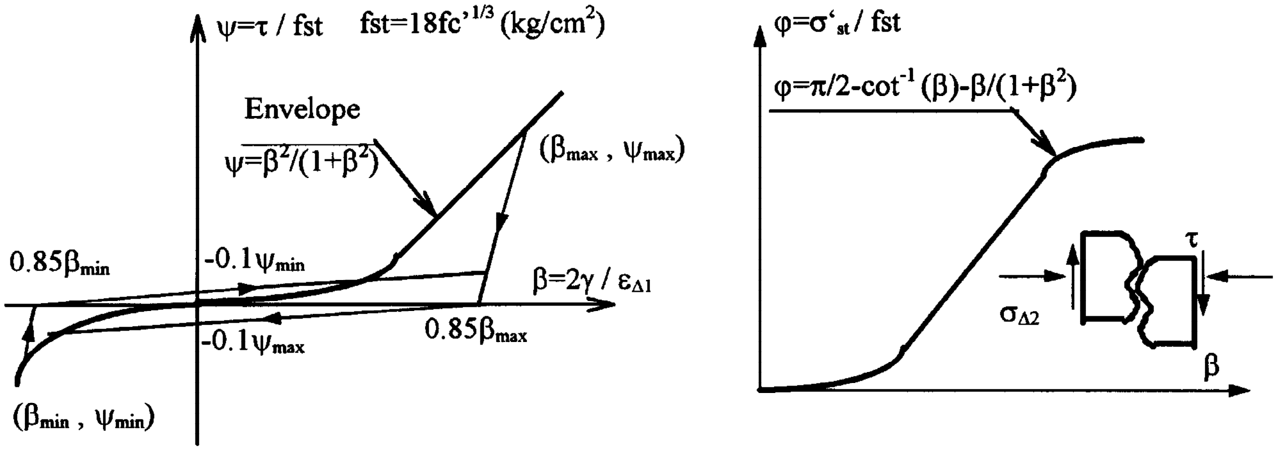
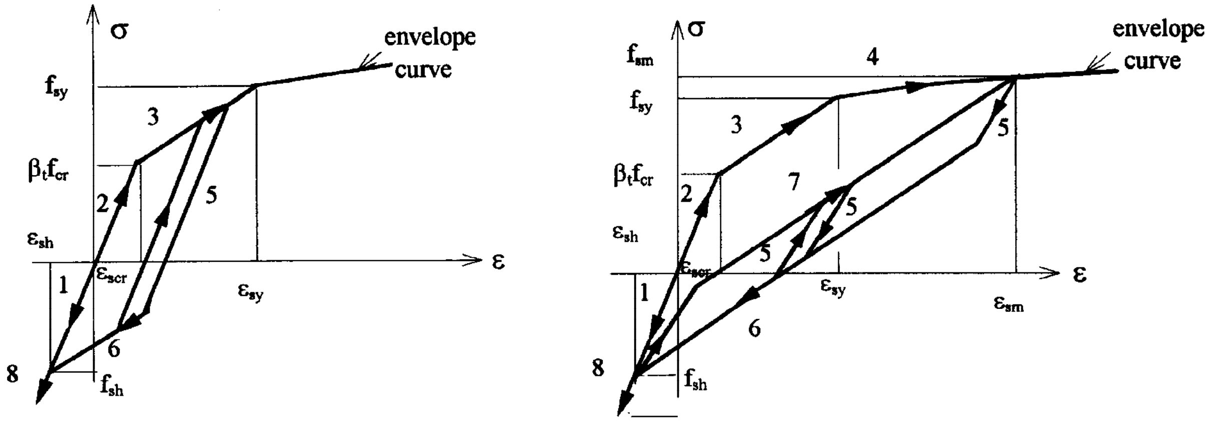


Fig. 3. Shear Transfer Model (Okamura and Maekawa, 1990)

Constitutive Models for Steel. An average stress versus average strain relationship for reinforcing steel embedded in concrete is adopted (Stevens, 1987). This model gives the steel stiffness as a function of the stress level, and stress history. The function cannot be directly integrated to give $\sigma_s(\varepsilon_s)$. Therefore, to get the stress at a particular strain an iteration procedure is adopted.

Outside Truss Element

According to some recommendations (CEB-FIP Model Code 1990 - Final Draft, 1993) the stiffening effect may be taken into account by a modified stress-strain relation of the embedded reinforcement. Following these recommendations the envelope curve of the Axial Stiffness Hysteresis Model (Kabeyasawa et al., 1985) is modified (Fig. 4) in order to obtain better response. The hysteresis rules are given in the Table 1.



a)before yielding

b)after yielding

Fig. 4. Modified Axial Stiffness Hysteresis Model

Table 1. Axial Stiffness Hysteresis Model (modified) Rules

Rule No	Description	Stiffness K_i
1	Initial compression	$K_1 = A_1 E_c$
2	Tension before cracking.	$K_2 = A_1 E_c$
3	Tension cracking	$K_3 = \dfrac{f_{sy} - \beta_t f_{cr}}{\varepsilon_{sy} - \beta_t \Delta\varepsilon_{sr} - \dfrac{\beta_t f_{cr}}{E_c A_1}} A_s$
4	Tension yielding	$K_4 = \dfrac{E_s}{0.2 + 0.8 \dfrac{f_{cr}}{f_{sy}}} A_s$
5	Unloading in tension	$K_5 = K_1$ if $\varepsilon_{sm} \le \varepsilon_{sy} - \beta_t \Delta\varepsilon_{sr}$ $K_5 = K_1 \dfrac{(f_{sm} - f_{sh})(\varepsilon_{sy} - \beta_t \Delta\varepsilon_{sr} - \varepsilon_{sh})}{(f_{sy} - f_{sh})(\varepsilon_{sm} - \varepsilon_{sh})}$ if $\varepsilon_{sm} > \varepsilon_{sy} - \beta_t \Delta\varepsilon_{sr}$
6 & 7	Reloading	$K_6, K_7 = \dfrac{K_3}{K_1} K_4$
8	Loading in compression	$K_8 = K_1$

Notation in the Table 1:

A_c , A_s - the area of the concrete section and of the tensile reinforcement;
E_c, E_s - the Young's modules of the concrete and of the steel;
f_{sy}, ε_{sy} - yielding of the tensile reinforcement;
f_{sm}, ε_{sm} - tensile peak point;
f_{cr} - tensile cracking of the reinforcement;
$f_{sh} = -f_{sy}$, $\varepsilon_{sh} = f_{sh} / K_1$ - point H, at which compression stiffness regained;

$\beta_t = 0.4$ for short term loading;
$\beta_t = 0.25$ for long-term or repeated loading (pure tension);
$A_1 = A_c + A_s E_s / A_c E_c$ - transformed section (state I);
$\Delta\varepsilon_{cr} = f_{cr} A_1 / E_s A_s - f_{cr} / E_c$

Once the stiffness K_i is found the normal force in the outside truss element may be obtained as follows:

$$N = K_i \varepsilon_{sm} \tag{1}$$

SOLUTION OF NONLINEAR EQUATION

The computer program RCFEM for IBM PC is coded on the basis of software PCFEAP (Zienkiewicz and Taylor, 1991). The load is applied to a structure as a series of load increments. The modified Newton-Raphson method combined with the 'line search' algorithm (Zienkiewicz and Taylor, 1991) for acceleration of the convergence is used for each load increment. For the case of cyclic loading it is virtually impossible to formulate the exact tangent matrix explicitly. Therefore the concrete material stiffness matrix is evaluated numerically by applying appropriate increments to εx, εy, γxy and calculating corresponding increments in the stresses σcx, σcy, τcxy. The concrete material stiffness matrix in the principal directions is non-symmetric (Stevens, 1987) for the 2-D Nonlinear Plane Element. Usage of the modified compression field theory extended by the shear transfer model is the reason for this. Therefore it was decided to implement non-symmetric solver based on the frontal approach.

The concrete material stiffness matrix in global coordinates is determined by standard rotational transformation. The total tangent stiffness material matrix is simply the sum of the concrete stiffness matrix and the stiffness contributions from the reinforcing bars. For bars in the X and Y directions:

$$[D] = \begin{bmatrix} D_{xx} + \rho_{sx} E_{sx} & D_{xy} & D_{x\gamma} \\ D_{yx} & D_{yy} + \rho_{sy} E_{sy} & D_{y\gamma} \\ D_{vx} & D_{vy} & D_{v\gamma} \end{bmatrix} \tag{2}$$

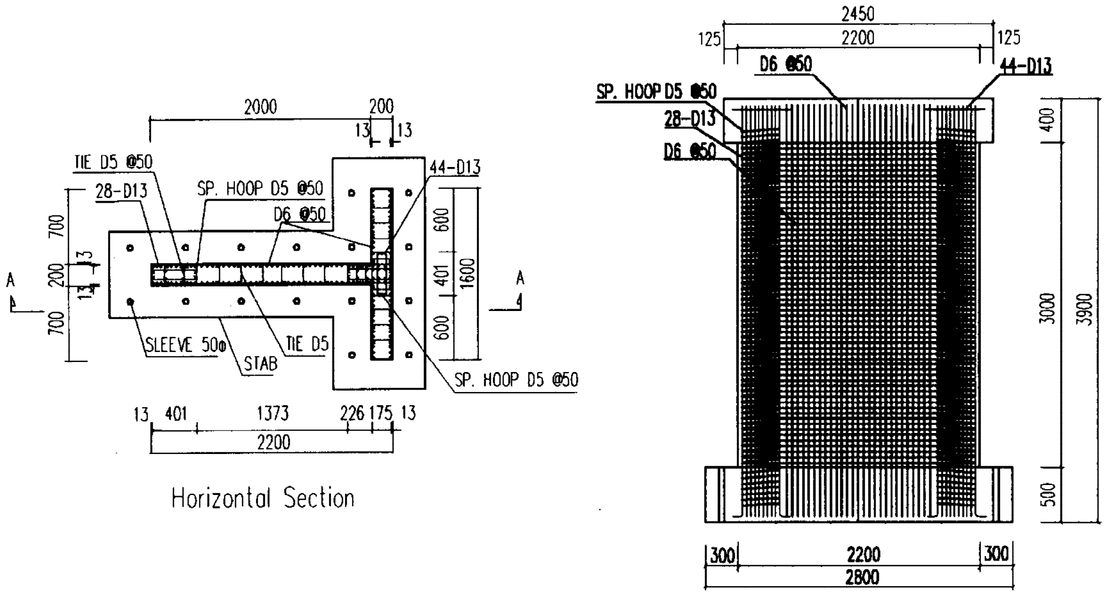
NUMERICAL STUDIES

Description and Modeling of the Test Structure

A 1/3 - scale test specimen, previously tested at the structural laboratory of Yokohama National University, October through November 1994 by Dr. T. Kabeyasawa, is analyzed (Kabeyasawa et al., 1995). The tests were conducted as a part of US-Japan cooperative research program on hybrid structural systems. The specimens represented a coupled core wall in 12-story prototype hybrid structural system with non seismic steel frames. Detailed wall cross sections and loading setup of the specimen are shown on the Fig. 5 and Fig. 6^a. The specimen HW1 is analyzed by usage of proposed macroscopic model. The analysis is carried out for the cases of monotonic and reversed cyclic loading. The web of the test specimen is modeled by idealizing as one macro element (Fig. 6^b). The flange is modeled as elastic 2D element with reduced stiffness.

Comparison of Experimental and Analytical Curves.

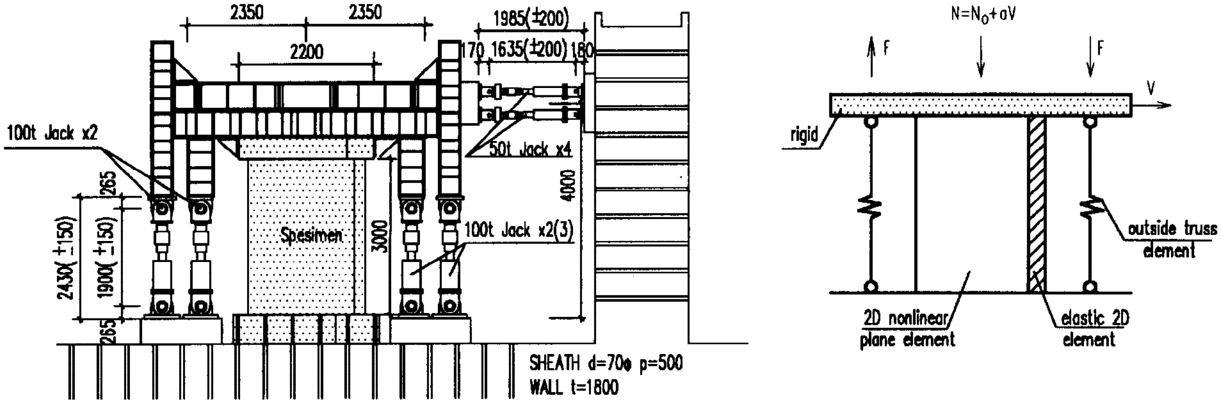
Experimental and computed responses are shown in Fig. 7 and 8. The correlation of experimental and analytical curves is apparently good for plots of base shear **V** versus top displacement **δ.** However much more comparison of experimental and analytical results is necessary for both cases of monotonic and cycling loading in order to improve the reliability of the proposed macro model. This will be a topic of further studies.



Horizontal Section

A Direction

Fig.5. Reinforcement Details of T-Shaped Wall Specimen HW1



a) Loadig Sistem

b) Analytical Model

Fig.6. Lodig Setup

Table 2. Material Properties

Concrete			
Age	Elastic Stiffness [MPa]	Compressive Strength [MPa]	Strain at Compressive Strength [MPa]
99	27 200	35.5	0.0020
Steel			
Nominal Diameter [mm]	Yield Strength [MPa]	Tensile Strength [Mpa]	Usage
D6	402	548	Panel, Wall
D13	362	538	Main Bar
D5	1375	1446	Spiral, Tie

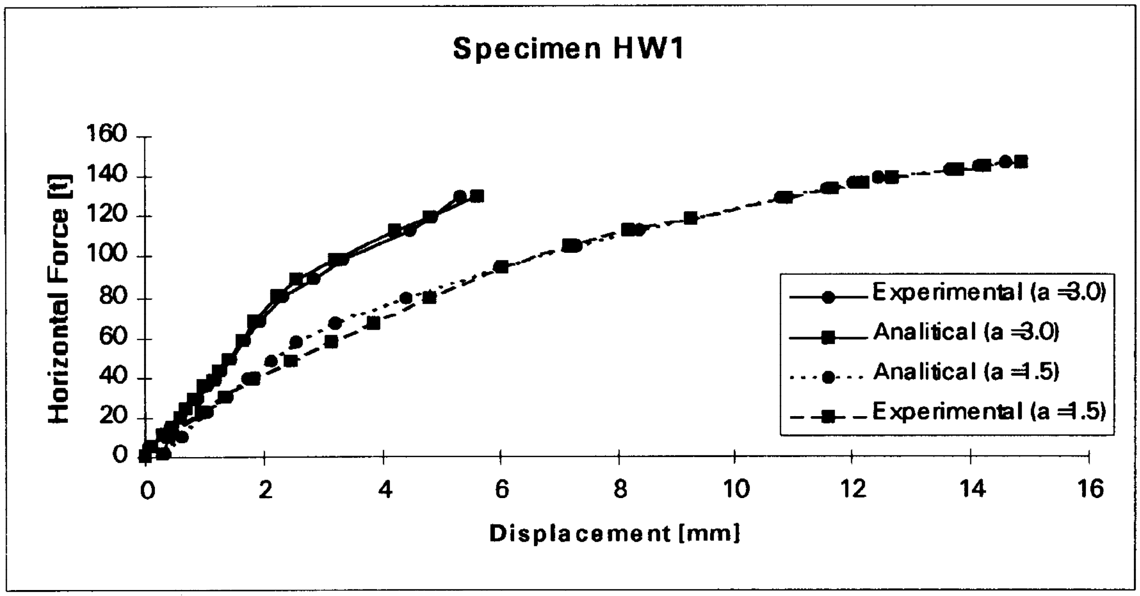
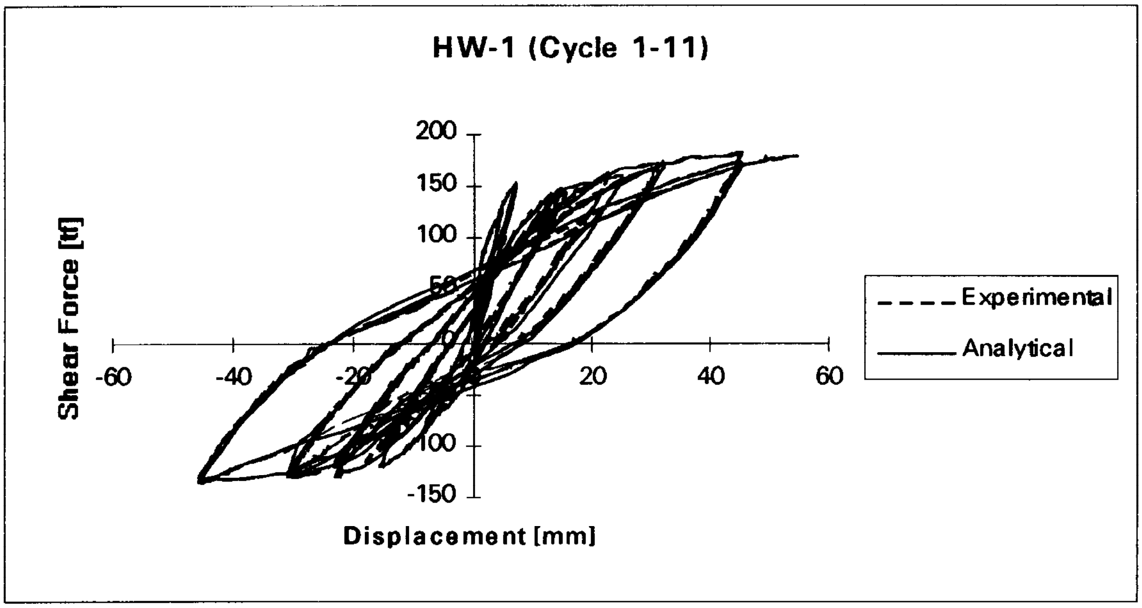
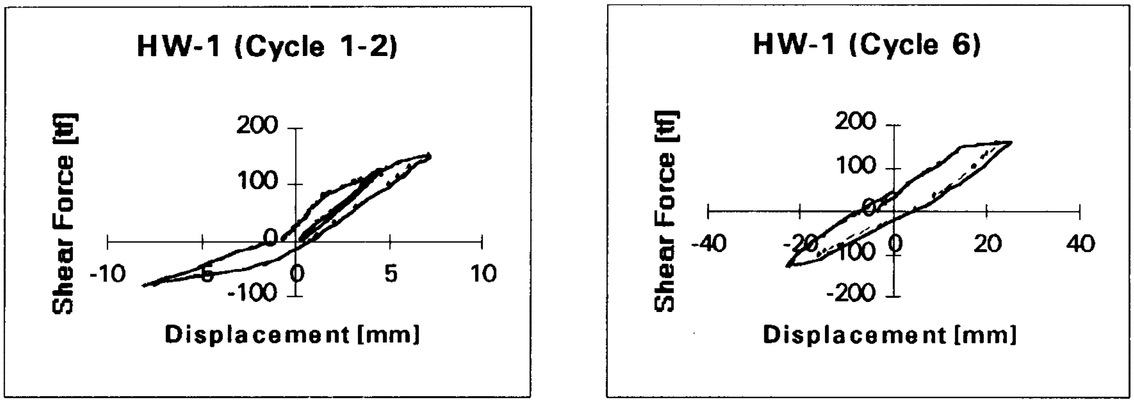


Fig. 7. Fig. 8 Base Shear Versus Net Top Displacement (Monotonic Loading)



a) Cycle 1-11



b) Cycle 1-2

c) Cycle 6

Fig. 8 Base Shear Versus Net Top Displacement (Cyclic Loading)

CONCLUSIONS

1)The proposed macro element is relatively simple and, therefore, it can be efficiently incorporated in the software for inelastic analysis of multistory reinforced concrete frame-wall systems (e.g. Kabeyasawa, 1983).

2)Under monotonic loading the proposed model provides very well simulation of the behavior of test specimens.

3)However some problems are expected under reversed cyclic loading for describing the response of the specimens for high shear stresses. The possible solution of the problem is to use elements with cubic displacement distribution and to improve the numerical solution technique.

4)Other problem for analyzing the test specimen for cyclic loading is modeling of the flange. Elastic 2-D element with reduced stiffness seems to be good approximation for monotonic loading but for cyclic loading more refined model is necessary. However it requires usage of out-of-plane loaded 2-D nonlinear elements (plate elements) which could be a topic of other study.

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