11 WCEE

Paper No. 240. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

TRANSIENT RESPONSES OF CIRCULAR ARCHES FOR BASE EXCITED MOTIONS

C.S. HUANG, T.J. TENG, C.H. YEH, and C.H. CHO

National Center for Research on Earthquake Engineering,
National Taiwan University, Taipei, Taiwan

ABSTRACT

This paper studies the in-plane stochastic transient responses of circular arches subjected to multiple horizontal and vertical excitations. The multiple support motion in the horizontal or vertical direction can be fully correlated, correlated with phase shift only, or fully uncorrelated. The solution for the transient responses of arches are analytically formulated in the Laplace domain. Then, the inverse of the Laplace transform is accurately obtained by means of a numerical technique. The variations of the mean maximum responses of the tangential displacement, radial displacement, moment, shear force, and axial force along arches with fixed-fixed, fixed-hinged, and hinged-hinged ends are given.

KEYWORDS

Arch, transient response, multiple base excitation, Laplace transform

INTRODUCTION

Curved structural elements may be found frequently in practical situations such as arch bridges, roof structures and aerospace structures. Because of their importance, a vast literature has been published on the dynamic behavior of planar curved structural elements. Much of this literature has been summarized in review articles (Markus and Nanasi, 1981; Laura and Maurizi, 1987; Chidamparam and Leissa, 1993). In particular, the recent article by Chidalparam and Leissa (1993) lists over 400 references on this subject.

Of this vast literature, much less research has focused on situations involving base excitation, which is particularly of interest in earthquake engineering. Mau and Williams (1988) utilized a Green's function approach to analyze the response of circular arches subjected to uniform and non-uniform base excitations in the frequency domain. In this approach, a so-called pseudo-static solution has to be found, which can be troublesome. Hao (1993, 1994) studied the effects of spatially correlated multiple earthquake ground excitation on the stochastic responses of incompressible circular arches by using a modal superposition technique with a quasi-static solution found explicitly. In order to completely eliminate the difficulty of finding the quasi-static solution, Yeh *et al.* (1995) applied Betti's law in the process of using the modal superposition method to transform the equations of motion into ordinary differential equations. However, this solution does not exactly satisfy the motions at the boundaries, so that accurate solutions for the axial force, moment, and shear force are very difficult to obtain. Recently, Huang *et al.* (1995) proposed an accurate solution for the in-plane transient response of a circular arch by applying the Laplace transform to the time variable. Then, an analytical solution was formulated in the s-domain. Finally, an efficient and accurate numerical technique was applied to obtain the inverse of the Laplace transform. Huang's approach gives highly accurate results not only for displacement components, but also for stress resultants, which are functions of higher derivatives of the displacement. Furthermore, it is not necessary to find the vibration modes or an auxiliary solution (such as a quasi-static solution), which are required in most of the approaches given in previous technical publications, considering the case of base excitations.

The main purpose of this study was to comprehensively study the effects of multiple base excitations with phase shifts only on the transient responses of circular arches having various opening angles and with various types of end conditions. This goal was achieved by using Huang's approach (Huang *et al.*, 1995). The earthquake accelerograms were simulated by multiplying banded white noises by a time enveloping function that represents general characteristics of earthquakes occurred in the south part of Taiwan. The time history of displacement was transformed to the Laplace domain as the input in Huang's solution. After accomplishing the Laplace inverses, displacements and stress resultants at various positions of an arch subjected to multiple support motion were obtained. The mean maximum quantities using twenty events were obtained to investigate the variations of responses due to various excitations.

GOVERNING EQUATIONS AND SOLUTION

Shown in Fig. 1 is a uniform circular arch of thickness h with radius R of the centroidal axis. The angular coordinate of a point on the centroidal axis is represented by θ, as measured from the right support. The tangential and radial displacements are denoted by v and w, respectively. The sign conventions for the positive displacements, moment (*M*), shear force (*Q*), and axial force (*N*) are also given in Fig. 1. A set of governing equations for a circular arch subjected to base excitations only are (cf. Veletsos *et al.* , 1972)

$$\frac{\partial^2 v}{\partial\theta^2}-\frac{\partial w}{\partial\theta}=\frac{R^2}{EA}(\rho A\frac{\partial^2 v}{\partial t^2}); \qquad \frac{\partial v}{\partial\theta}-(1+\gamma)w-2\gamma\frac{\partial^2 w}{\partial\theta^2}-\gamma\frac{\partial^4 w}{\partial\theta^4}=\frac{R^2}{EA}(\rho A\frac{\partial^2 w}{\partial t^2}), \tag{1}$$

where E is the Young's modulus, ρ is the mass per unit volume, A is the area of the cross section, and $\gamma = h^2/12R^2$.

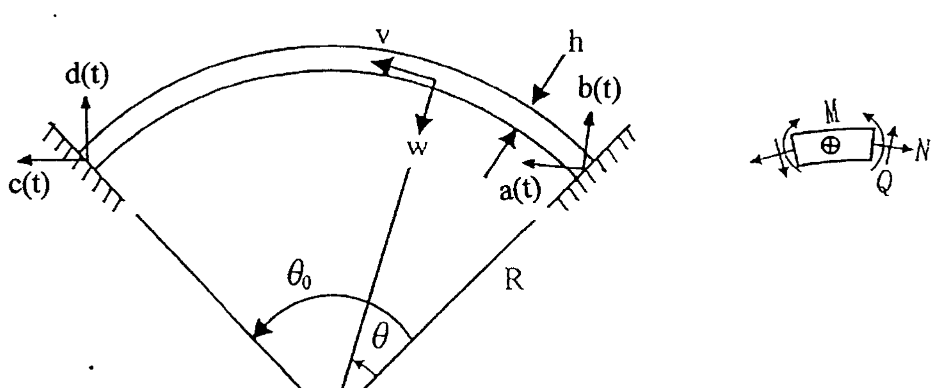


Fig. 1. Defining sketch for a circular arch

The governing equations (1) are derived from Flugge's equations for cylindrical thin shells by eliminating the longitudinal variable of the shell. The effects of rotary inertia and shear deformation are neglected. The moment (*M*), shear force (*Q*), and axial force (*N*) are expressed in terms of displacements and are nondimensionlized as follows:

$$M^*=\frac{R^2}{EIh}M=-\frac{1}{h}(w+\frac{\partial^2 w}{\partial\theta^2}); \quad Q^*=\frac{R^3}{EIh}Q=-\frac{1}{h}(\frac{\partial w}{\partial\theta}+\frac{\partial^3 w}{\partial\theta^3}); \quad N^*=\frac{R}{EAh}N=\frac{1}{h}(\frac{\partial v}{\partial\theta}-w-\gamma(w+\frac{\partial^2 w}{\partial\theta^2})), \tag{2}$$

where I is the moment of inertia of the cross section.

Six boundary conditions are needed to obtain a complete solution for eqs.(1). For the case of base excitations, four among the six boundary conditions are specified as follows:

$$w(0,t)=w_0(t),\ \ v(0,t)=v_0(t),\ \ w(\theta_0,t)=w_1(t),\ \ v(\theta_0,t)=v_1(t)\,. \tag{3}$$

The other two boundary conditions will be

$$\frac{\partial w}{R\partial\theta}-\frac{v}{R}=0 \ \text{ at } \theta=0 \text{ and } \theta=\theta_0 \text{ for a fixed - fixed arch;} \tag{4a}$$

$$\frac{\partial w}{R\partial\theta}-\frac{v}{R}=0 \text{ at } \theta=0 \text{ and } M=0 \text{ at } \theta=\theta_0 \text{ for a fixed - hinged arch;} \tag{4b}$$

$$M=0 \text{ at } \theta=0 \text{ and } \theta=\theta_0 \text{ for a hinged - hinged arch.} \tag{4c}$$

To obtain an accurate solution for eqs.(1) subjected to the boundary conditions given in eqs.(3) and (4), performing a Laplace transformation on eqs. (1) and assuming zero initial conditions yields a set of ordinary differential equations with the spatial variable, θ, in the s-domain. Then, the solution of these differential equations satisfying the boundary conditions, which are also transformed to the s-domain, can be formulated analytically without any difficulty. Finally, the numerical technique proposed by Durbin (1974) is applied to obtain, accurately and efficiently, the inverse of the Laplace transform. The details of the solution procedure have been given in the paper by Huang *et al.* (1995).

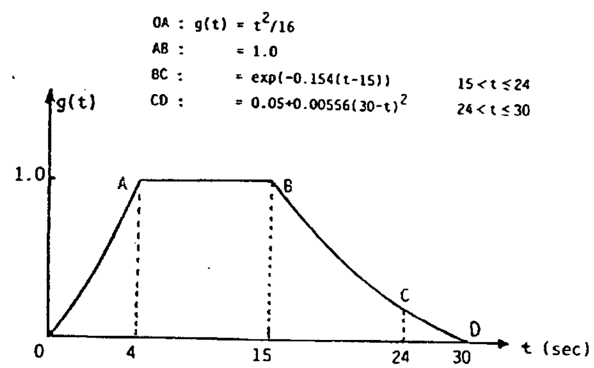
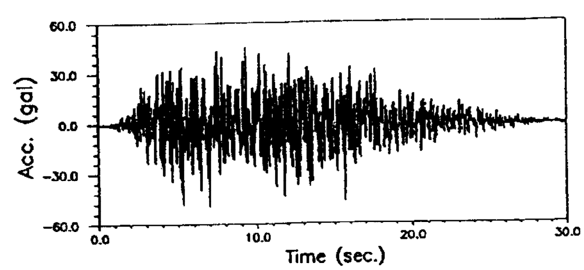
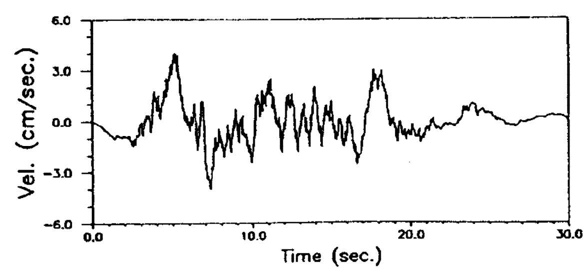


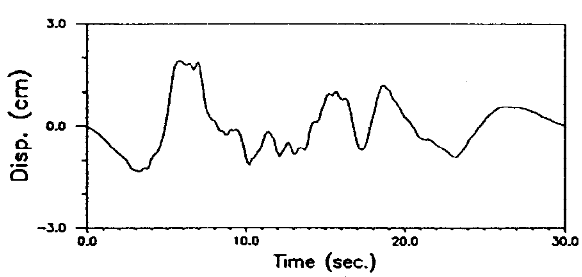
Fig. 2. Time enveloping function
(from Loh. et al., 1993)



(a)



(b)



(c)

Fig. 3. A simulated earthquake record

BASE EXCITATION INPUT

In this work, simulated earthquake records were used as the base excitation input. The simulation of earthquake accelerograms was accomplished by generating white noise with a power spectrum equal to 1 and 1/4 in the horizontal and vertical directions, respectively. The accelerograms were multiplied by a time enveloping function, shown in Fig. 2, that expresses the general characteristics of earthquakes in the southern part of Taiwan (Loh *et al.*, 1993). Then, the accelerograms were filtered so that their components with frequencies higher than 25 Hz and lower than 0.1 Hz were almost eliminated. A typical acceleration record after this step is shown in Fig. 3a. Finally, the velocity and displacement records could be obtained as shown in Figs. (3b) and (3c) by integrating the filtered acceleration data with base-line correction (Cho, 1996).

Twenty earthquake records in the horizontal and vertical directions, respectively, were generated for the studies shown in the following section. These records were statistically independent. It should be pointed out that the horizontal and vertical displacements had to be decomposed into tangential and normal displacements to match the prescribed boundary conditions given in eqs.(3) and (4a). Then, these records had to be transformed to the s-domain through a discrete Laplace transform scheme (Beskos and Narayanan, 1983) to fit the solution procedure described in the previous section.

RESULTS AND DISCUSSION

The material property and the geometry of the arches under consideration were $\sqrt{E/\rho}$ = 5000 m/sec, R=100 m, and h=1 m. Two opening angles were considered, namely, $\theta_0 = 60°$ and $\theta_0 = 90°$. Three types of arches with different end conditions were considered, namely, fixed-fixed, fixed-hinged, and hinged-hinged arches. The first three natural frequencies of the arches under consideration are listed in Table 1.

Table 1. The first-three natural frequencies, f (Hz), for the arches under consideration

Mode	Fixed-Fixed		Fixed-Hinged		Hinged-Hinged	
	$\theta_0 = 90°$	$\theta_0 = 60°$	$\theta_0 = 90°$	$\theta_0 = 60°$	$\theta_0 = 90°$	$\theta_0 = 60°$
1	0.51972	1.2344	0.41049	0.98627	0.31618	0.77243
2	0.99174	2.2611	0.86488	1.9859	0.74392	1.7194
3	1.7969	4.1192	1.5995	3.6677	1.4166	3.2522

To investigate the effects of the various ground motions on the arch responses, four base excitation input cases were considered. Uniform input means that a(t)=c(t) and b(t)=d(t) (see Fig. 1). Multiple inputs with phase shift only are defined by delayed time, τ, so that c(t-τ)=a(t) and d(t-τ)=b(t). Two different values for τ, namely, τ=0.2 seconds and 0.4 seconds, were considered. Uncorrelated input means that a(t) and b(t) are uncorrelated with c(t) and d(t), respectively. It should be pointed out that in all cases, the horizontal displacement records, i.e. a(t) and c(t), were not correlated with the vertical displacements records, i.e. b(t) and d(t).

Considering horizontal base excitation inputs, Figs. 4 to 6 show the mean maximum responses at various positions of arches with fixed-fixed, fixed-hinged, and hinged-hinged ends, respectively. The mean values were computed from twenty events. Except for the tangential displacement, the other quantities are significantly underestimated for uniform input cases. It should be mentioned that a uniform input excites only the anti-symmetric modes for fixed-fixed and hinged-hinged arches so that the normal displacement, moment, and axial force at mid span are zero. It is interesting to observe that the responses of the moment, shear force, and axial force calculated for uncorrelated input and for phase shift input with τ=0.2 sec are generally larger than those calculated for phase shift input with τ=0.4 sec. These differences seem to

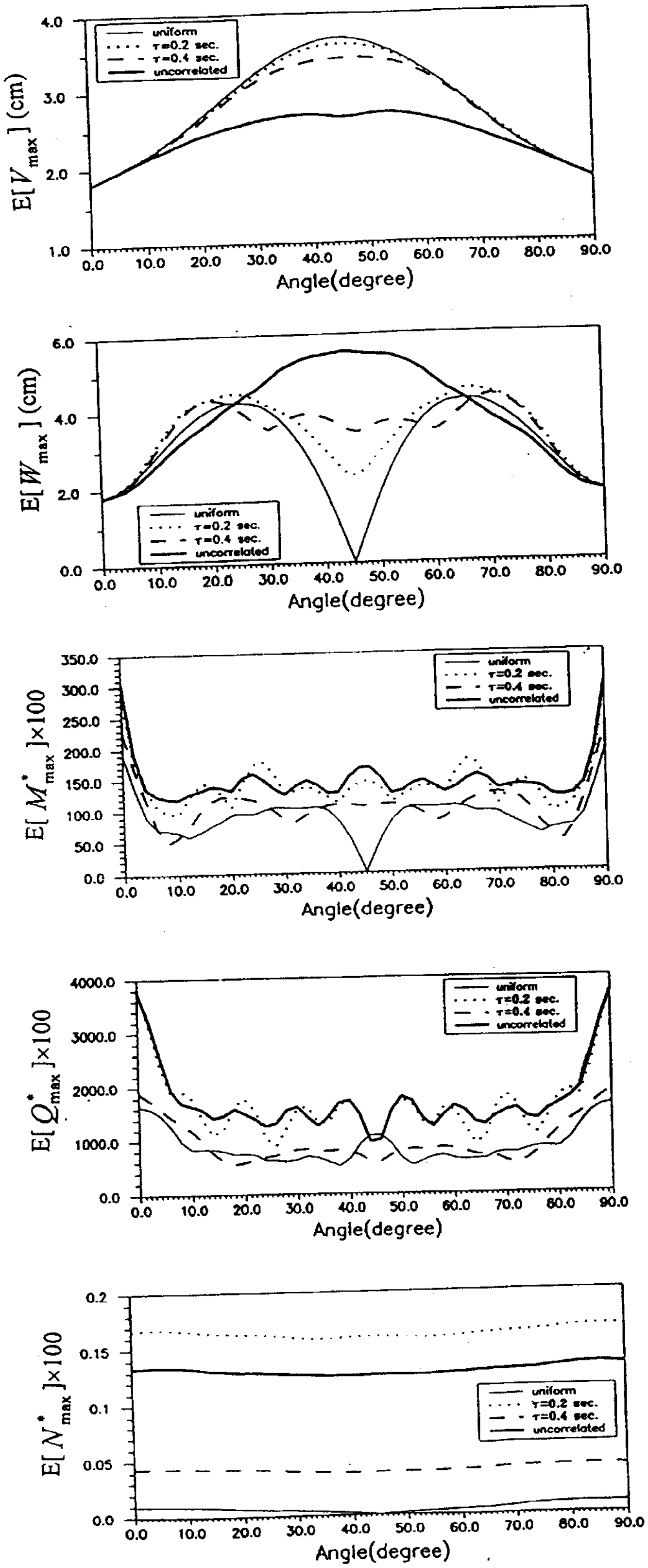


Fig. 4. Mean maximum responses for a fixed-fixed arch with $\theta_0 = 90^o$ subjected to horizontal base excitation

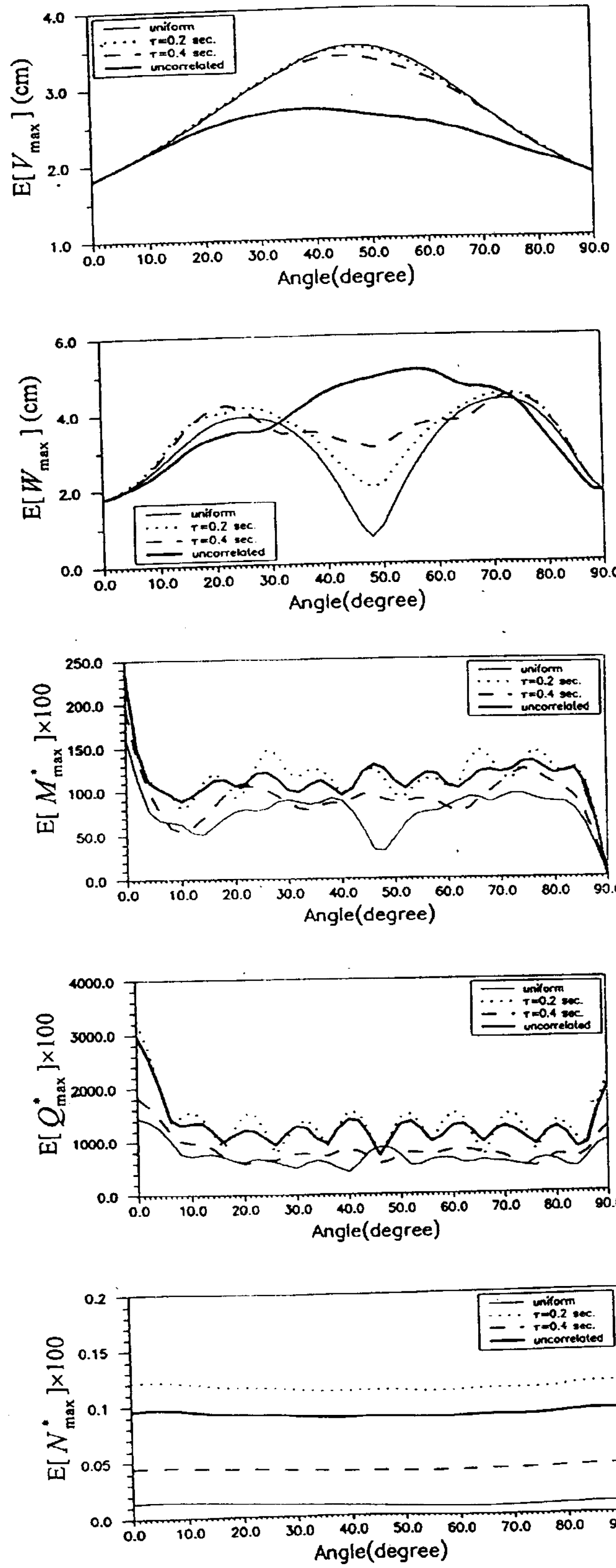


Fig. 5. Mean maximum responses for a fixed-hinged arch with $\theta_0 = 90^o$ subjected to horizontal base excitation

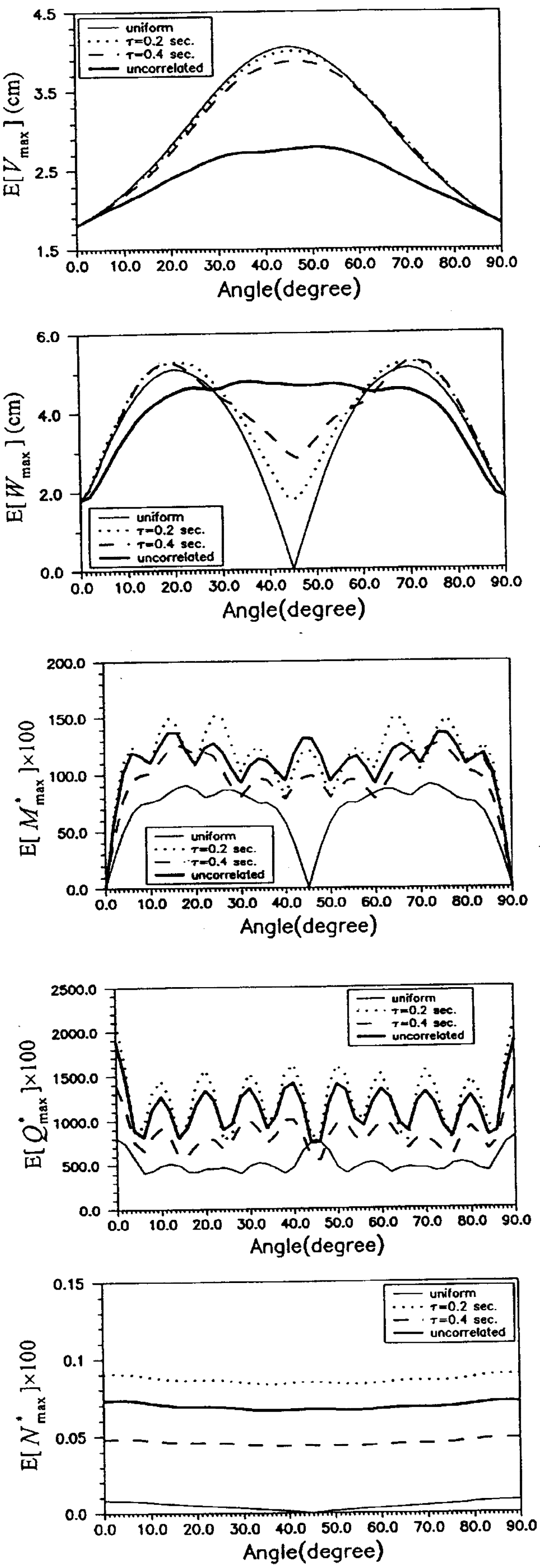


Fig. 6. Mean maximum responses for a hinged-hinged arch with $\theta_0 = 90^o$ subjected to horizontal base excitation

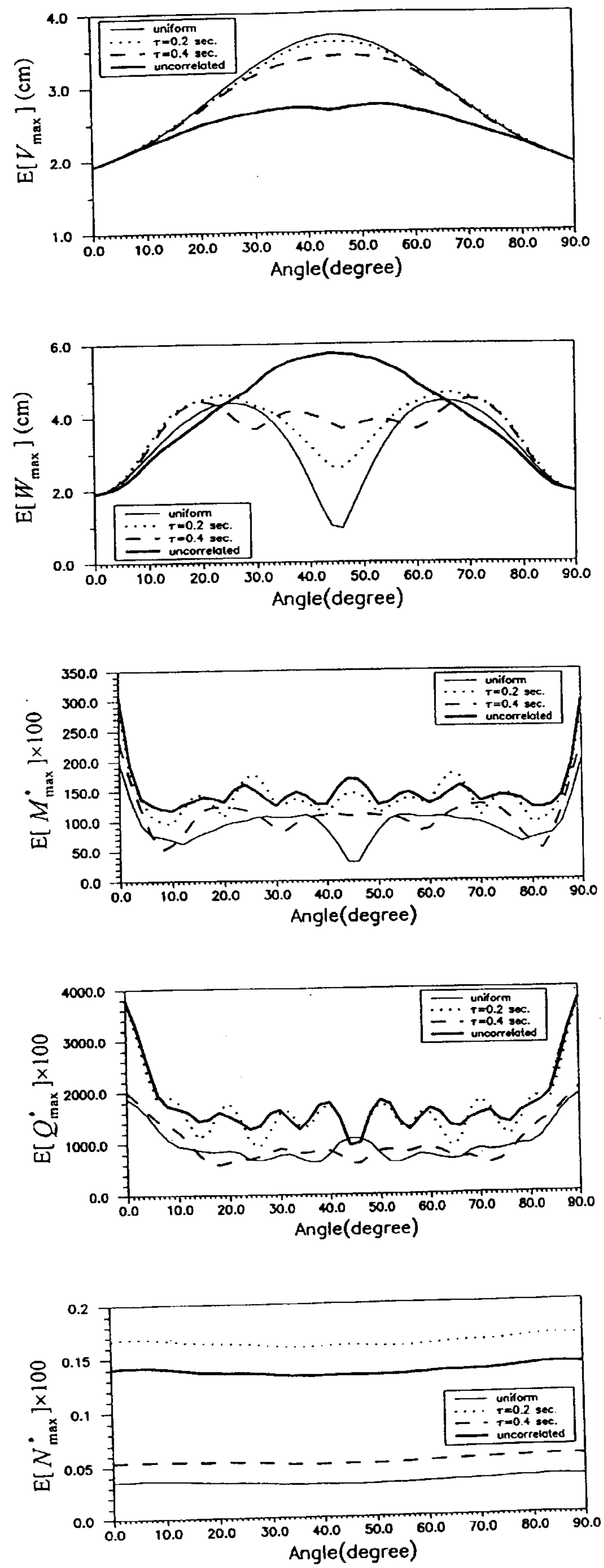
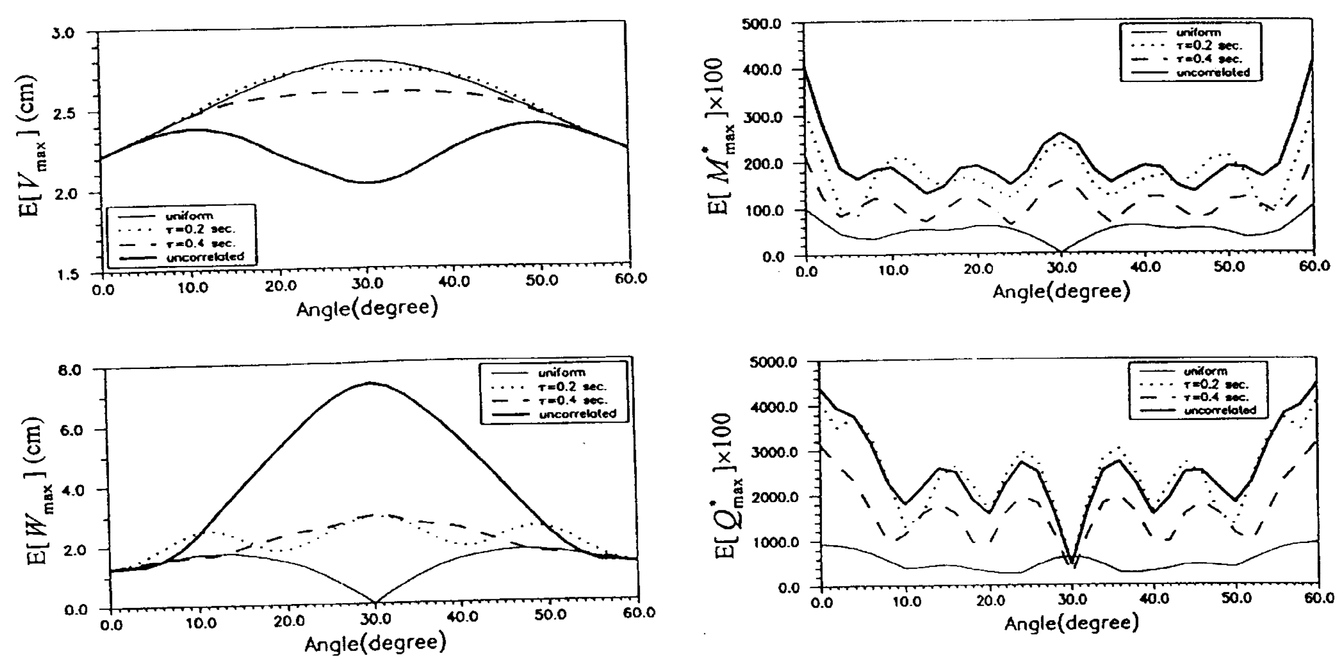


Fig. 7. Mean maximum responses for a fixed-fixed arch with $\theta_0 = 90^o$ subjected to vertical and horizontal base excitation

become smaller for more flexible structures like a hinged-hinged arch. For the same base excitation inputs, it is reasonable to find that a stiffer structure like a fixed-fixed arch has larger responses of the moment, shear force, and axial force.

Figure 7 shows the mean maximum responses at various positions of a fixed-fixed arch with $\theta_0 = 90°$ subjected to horizontal and vertical base excitation simultaneously. Comparison with Fig. 4 reveals that there are no significant differences except for the results for the uniform input case. This phenomenon is also observed in the results for fixed-hinged and hinged-hinged arches and also for arches with $\theta_0 = 60°$, which results are not shown here. The differences in the results for uniform input cases in fixed-fixed and hinged-hinged arches exist mainly because symmetric as well as antisymmetric modes are excited for horizontal and vertical base excitation inputs while only antisymmetric modes are excited for horizontal base excitation.

Figure 8 shows the mean maximum responses at various positions of a fixed-fixed arch with $\theta_0 = 60°$ subjected to horizontal base excitation only. Although the responses shown in Figs. 4 and 8 resulted from the same horizontal displacement records, they had different tangential and normal component inputs due to the different opening angles of the arch. Fig. 8 has larger tangential displacement input but less normal displacement input compared to Fig. 4. However, the responses of tangential and normal displacements in the region near the mid span in Fig. 8 are smaller than those in Fig. 4 except for the uncorrelated input case. The same phenomenon is also observed in fixed-hinged and hinged-hinged arches, which results are not shown here. Comparison of the responses of the moment, shear force, and axial force in Fig. 8 with those in Fig. 4 reveals that the results for the uniform input case in Fig. 8 are smaller than those shown in Fig. 4, while the results for other input cases are generally greater than those shown in Fig. 4. This trend is also observed in the results for a fixed-hinged arch.



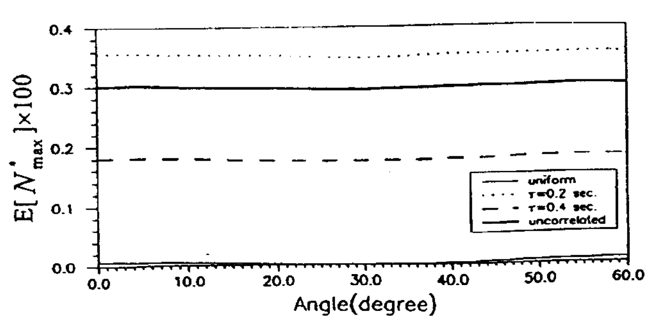


Fig. 8. Mean maximum responses for a fixed-fixed arch with $\theta_0 = 60°$ subjected to horizontal base excitation

CONCLUSIONS

This paper has applied a systematic and accurate procedure incorporating the Laplace transform to calculate the transient response of arches subjected to base-excitation inputs. The main advantages of this approach over the other existing analytical methods are that no quasi-static is needed and that not only displacement components, but also stress resultants can be accurately calculated. In addition, unlike using the Fourier transform, there is no need to take care of poles when obtaining the Laplace inverse.

This study has also demonstrated the significant effects of multiple base excitation inputs with phase shift on the mean maximum responses of arches. Uniform support motion generally results in significant underestimation of the responses for normal displacement, moment, shear force, and axial force but not for tangential displacement. Except for uniform input cases, the mean maximum responses of various types of arches subjected to independent vertical and horizontal base excitation simultaneously are slightly different from those obtained for horizontal base excitation only. In all cases considered here, there is little variation of the mean maximum amplitudes of the axial force along the arches.

ACKNOWLEDGE

This research was supported by the National Science Council, R.O.C. (NSC 85-2711-3-319-001). Much appreciation is extended to colleagues in the National Center for Research on Earthquake Engineering for their valuable discussion

REFERENCES

Beskos, D. E. and G. N. Narayanan (1983). Dynamic response of frameworks by numerical Laplace transform. Computer Methods in Applied Mechanics and Engineering 37, 289-307.

Chidamparam, P. and A. W. Leissa (1993). Vibrations of planar curved beams, rings, and arches. Applied Mechanics Reviews 46(9), 467-483.

Cho, C. H. (1996). In-plane transient response of circular arches subjected to base excitation. M.S. thesis, National Chung-Hsiung University, Taiwan.

Durbin, F. (1974). Numerical inversion of Laplace transforms: an efficient improvement to Dubner and Abate's method. Computer Journal 17, 371-376

Hao, H.(1993). Arch responses to correlated multiple excitations. Earthquake Engineering and Structural Dynamics 22, 389-404.

Hao, H. (1994). Ground-motion spatial variation effects on circular arch responses. Journal of the Engineering Mechanics Division,ASCE, 120 , 2326-2341.

Huang, C. S., T. J. Teng, and A. W. Leissa (1995). An accurate solution for the in-plane transient response of a circular arch. Journal of Sound and Vibration (accepted).

Laura, P. A. A. and M. J. Maurizi (1987). Recent research on vibrations of arch-type structures.The Shock and Vibration Digest 19 (1), 6-9.

Loh, C. H., I. C. Tsai, R. Y. Tan, C. S. Yeh, and Y. T. Yeh (1993). Report of design study for Kaohsiung metropolitan area mass rapid transient system. Research report No. CEER-R82-02, Earthquake Engineering Center in National Taiwan University.

Markus, S. and T. Nanasi (1981). Vibration of curved beams. The Shock and Vibration Digest 13 (4), 3-14.

Mau, S. T. and A. N. Williams (1988). Frequency response functions of circular arches for support motions. Engineering Structures 10, 265-271.

Veletsos, A. S., W.J. Austin, C.A.L. Pereira and Shyr-Jen Wung (1972). Free in-plane vibration of circular arches. Journal of the Engineering Mechanics Division,ASCE, 98 (EM2), 311-329.

Yeh, C. S., T. J. Teng, and J. S. Wang (1995). Transient response of a circular arch subjected to multiple support motions. Proceedings of the Fifth East Asia-Pacific Conference on Structural Engineering and Construction, Gold Coast, Queenland, Australia.