STUDY OF THE EFFECT OF HYSTERETIC MODEL PARAMETERS ON THE NONLINEAR DYNAMIC RESPONSE OF R/C STRUCTURES EXPRESSED VIA HYSTERETIC ENERGY

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ABSTRACT

Presented within the frameworks of this paper is the effect of variation of different parameters of hysteretic models on nonlinear response of structures. The most significant analyzed parameters were: the stiffness after yielding point \( \beta \), the stiffness degradation at unloading USDP, the FYRP parameter (force yield relation parameter), the initial stiffness, the level of the first cracks as well as some specific parameters characteristic for the shear models. Investigated was the effect of the different parameters of the models on the maximal value of hysteretic energy and its index. The capacity for energy dissipation was analyzed through several real reinforced concrete structures of different structural composition and number of storeys. All the analyses were performed by application of an originally developed computer programme, INELA (INElastic Analysis).

KEYWORDS

Hysteretic energy; index of energy; hysteretic models; dissipation; stiffness degradation.

INTRODUCTION

The earthquake as a natural phenomenon is associated with an abrupt release of energy within a short time interval. Since we cannot prevent its occurrence, the only imperative in earthquake engineering is the seismic design that will provide sufficient safety and stability of structures. According to most of existing methods of seismic design, the seismic "requirements" for structures are expressed via maximal values of previously defined design parameters like the maximal displacement, the ductility and the forces in the structural elements. Here, no consideration is given to the reversible nature of the earthquake excitation which imposes the cyclic ductility, the number of transitions into the inelastic range, the energy dissipation capacity and other parameters as quantifiers of the nonlinear behaviour of structures.

An alternative of the traditional approach to seismic design is the design of seismically resistant structures based on energy concepts. The fundamentals of energy methods were given by Housner about a thirty five years ago, and were later elaborated by Uang and Bertero, Akiyama and others.

The energy approach is based on the fact that damage degree of a structure exposed to real seismic effects,
depends on the earthquake energy that is transmitted to the structure and the capacity of the structure to absorb this energy. The total input energy of the earthquake is dissipated through vibrations of the structure (kinetic energy), the mechanism of viscous damping (damping energy) and through the energy absorbed by the structure (consisting of elastic energy and hysteretic energy which is dissipated through nonlinear deformations).

The input energy is a parameter that refers to the total destructive earthquake potential, whereas the hysteretic energy is a structural parameter related to the damage degree of structures. In order that the structure behaves adequately in conditions of seismic loads, it is necessary that its ability for absorption and dissipation of energy be greater than the energy "requirements" during an earthquake.

**COMPUTER PROGRAMME INELA (INELASTIC ANALYSIS)**

The INELA computer programme has been elaborated as a result of long-term research in this field. It can be used for definition of the nonlinear response of multi-storey RC structures (modeled as "shear type" model) exposed to seismic effects, using bank of hysteretic models and different earthquake records. The created bank is composed of ten hysteretic models: six models that enable simulation of dominant bending behaviour (bilinear, Takeda, bilinear Takeda, Hisada, trilinear model with degradation, Clough model) and four models that simulate shear behaviour (bilinear-slip, Takeda-slip, peak oriented and origin-oriented). There is a possibility of different modeling per storeys of the same structure and also a possibility of extension with new hysteretic models and new earthquake records.

Using the INELA computer programme, the following information could be obtained for the analyzed structure: natural periods and mode shapes, maximal values and time histories of kinematic quantities (displacements, velocities and accelerations) for the whole time duration of the earthquake excitation, shear force - displacement relationships, maximal values and time histories of hysteretic energy and its index.

The verification of the INELA computer programme and the programme package RESIST-INELA (for evaluation of seismic resistance of RC structures), (Necevska-Cvetanova, 1991) was performed using the computer programme IDARC (Inelastic Damage Analysis of RC frame-shear wall structures), (Park et al., 1987). Comparative analyses of the results obtained by using the INELA computer programme and the results obtained by using the IDARC programme were performed for four RC structures of different structural system and number of storeys, (Petruzevska, 1995). Their response was presented via relative displacements and shear forces, in the form of time histories and maximal values, (Fig.1, Tab.1). From the performed analyses and taking into account the fact about the different modeling of the structure and the use of different hysteretic models, it may be concluded that the obtained results are satisfactory. The application of the INELA computer programme is simple and enables performance of a large number of analyses within a relatively short computer time.

![Fig.1. Time history of relative displacements and shear forces-RCS-2](image-url)
Table 1. Maximum relative displacements of analysed structures using computer programs IDARC and INELA

<table>
<thead>
<tr>
<th>Analysed structure</th>
<th>Storey</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference</td>
<td>[%]</td>
<td>[%]</td>
<td>[%]</td>
<td>[%]</td>
<td>[%]</td>
<td>[%]</td>
</tr>
<tr>
<td>SDOF</td>
<td>IDARC</td>
<td>0.149</td>
<td>0.520</td>
<td>1.230</td>
<td>2.090</td>
<td>1.970</td>
</tr>
<tr>
<td></td>
<td>INELA</td>
<td>0.153</td>
<td>2.6</td>
<td>9.6</td>
<td>8.8</td>
<td>2.020</td>
</tr>
<tr>
<td>RCS-1*</td>
<td>IDARC</td>
<td>1.21</td>
<td>1.64</td>
<td>2.39</td>
<td>2.80</td>
<td>2.40</td>
</tr>
<tr>
<td></td>
<td>INELA</td>
<td>1.16</td>
<td>4.1</td>
<td>1.48</td>
<td>12.7</td>
<td>9.7</td>
</tr>
<tr>
<td>RCS-2*</td>
<td>IDARC</td>
<td>1.45</td>
<td>1.45</td>
<td>1.36</td>
<td>1.14</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>INELA</td>
<td>1.45</td>
<td>0.0</td>
<td>1.68</td>
<td>13.7</td>
<td>1.60</td>
</tr>
</tbody>
</table>

1*-Reinforced concrete frame with beams b/d=40/60cm
2*-Reinforced concrete frame with beams b/d=36/40cm
3*-Reinforced concrete frame-wall system

PARAMETRIC ANALYSES

The obtaining of the realistic dynamic response of the structure depends on the proper selection of hysteretic models that should represent the corresponding characteristics of the analyzed elements and systems and the mode of their behaviour. Taking into account this fact, this paper deals with the effect of the different parameters of hysteretic models upon structural response expressed via hysteretic energy. Selected were six models simulating a bending behaviour (bilinear-BM, Clough model-CM, Takeda-TM, bilinear Takeda-TBM, Hisada-HM, trilinear model with degradation-TDM) and two models simulating shear behaviour (bilinear-slip-BSM and Takeda-slip-TSM).

The most important parameters by which the different hysteretic models are described and which affect the nonlinear response of the system and the shape of the hysteretic loops, i.e., the maximal dissipated energy and its index are: the stiffness after the yielding point $\beta$, the factor of stiffness degradation at unloading USDP (unloading stiffness degradation parameter), the FYRP (force yield relation parameter) parameter, the initial stiffness and the level of the first cracks in concrete (Fig. 2). The effect of some specific parameters that are characteristic only for the shear models was analyzed.

Fig. 2. Parameters of the primary curve of hysteretic models and index of energy dissipation
All the analyses were performed using the INELA computer programme. The parametric analyses refer to single-degree-of-freedom systems exposed to the Ulcinj-Albatros (Montenegro) earthquake, with maximal accelerations of 0.3g and 0.5g. The strength and strain values of the system are also selected such that a better visual presentation of the effect of the parameters of the hysteretic models upon energy characteristics is obtained.

Investigated was the effect of the variation of the considered parameters of maximal values of dissipated hysteretic energy (E_{\text{max}}) and its index (E_h). The maximal hysteretic energy was computed by summing up the surfaces enclosed by the hysteretic loops from the force-displacement relationship, during the nonlinear vibrations of the system under real seismic excitation.

The best insight into the hysteretic energy dissipated in the system during a reversible loading cycle with equal displacement amplitudes in the positive and negative direction is obtained via the surfaces involved by the hysteretic curve (Fig. 2).

The mathematical formulation of the energy dissipation index is:

\[ E_h = \frac{\Delta W}{2\pi F_m D_m} \]  

(1)

\( \Delta W \) is area enclosed by the hysteric curve; \( D_m \) is achieved max. displacement; \( F_m \) is achieved max. force.

The size of this index is equal to the equivalent viscous damping of a linear-elastic system which is capable of dissipating energy amounting to \( \Delta W \), within the time duration of a single cycle of the so called "steady-state" vibration. Using eq. (1), the expressions for the index of energy dissipation for part of the models used in the parametric analysis were obtained.

<table>
<thead>
<tr>
<th>Hysteretic model</th>
<th>Hysteretic energy index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilinear model</td>
<td>( E_h = \frac{2(1-\beta)^2(\mu - \mu^*(1-\beta + \mu \beta))}{\pi \mu (1-\beta + \mu \beta)} )</td>
</tr>
<tr>
<td>Clough model</td>
<td>( E_h = \frac{1}{\pi} \left[ \frac{1}{1 - (1 - \beta + \mu \beta) \mu} \right] )</td>
</tr>
<tr>
<td>Takeda model</td>
<td>( E_h = \frac{1}{\pi} \left[ \frac{1}{1 - \frac{D_y}{D_x} \mu^*(1-\beta + \mu \beta)} \right] )</td>
</tr>
<tr>
<td>Trilinear degrading model</td>
<td>( E_h = \frac{2}{\pi} \left( 1 - \frac{K_p}{K_x} \frac{F_y}{F_x} \right) )</td>
</tr>
</tbody>
</table>

The results obtained from the parametric analyses are presented throughout time histories of hysteretic energy, its maximal values as well as energy indexes. Only selected part of these results are further presented.

**Post-Yielding Stiffness Parameter \( \beta \)**

This parameter is associated with "strain hardening" of steel which allows for the existence of a finite positive stiffness (Ku) after the achieved yielding point. It is mathematically defined as a ratio between stiffnesses Ku/Ky and ranges within 0.05 - 0.2.
Out of all the analyzed parameters, $\beta$ has the strongest influence upon the shape of the hysteretic curves, i.e., the ability for energy dissipation. With the increase in this parameter, they change their shape from "fat" to "thin" (Fig. 3). It is exactly due to the fact that the hysteretic curves are pinched with the increase in $\beta$ (the area that they enclose is made smaller) that the maximal values of hysteretic energy are decreased. The only exception is the Hisada's model that is characterized by stable hystereses. With the increase in $\beta$, the index of hysteretic energy in all the analyzed models (with the exception of the trilinear model with degradation, see Table 2) is decreased (Fig. 4).

![Hysteretic energy](image1)

Fig. 3. Force-displacement relationship and time history of hysteretic energy for different $\beta$ parameters

![Index of hysteretic energy](image2)

Fig. 4. Effect of $\beta$ parameter on energy dissipation and energy index

**Unloading Stiffness Degradation Parameter - USDP**

In the most general case, the parameter USDP is used to model the stiffness degradation at unloading resulting from the occurrence of some of the following phenomena: Baushinger's effects, cracks in the tensile concrete, disturbance of the connection at the concrete - steel contact, sliding or loss of effective anchorage and shear strains. Its determination on the basis of known characteristics of materials and geometry of structures is not possible. For the needs of the analyses, this factor has been varied at interval of $0.0 - 0.5$.

Out of all the analyzed parameters, the USDP factor has the greatest influence upon the nonlinear response of the system. The increase in this parameter leads to a decrease in the hysteretic energy and its index in all the models, with the exception of the trilinear model with degradation. The nonlinear behaviour of the system which is mathematically defined with this model deserves special comment. The very equation for the energy index (Table 2) shows that it does not depend on the ductility but only on the force and displacement values at cracking and yielding points. Taking into account the importance of the hysteretic energy index as an indicator of the different behaviour of the models, its modification has been analyzed for different ductility values at interval of $1.0$ to $6.0$ for some of the characteristic models of interest, (Fig. 5).
The FYRP parameter represents a ratio between yielding force $F_y$ (at random ductility) and yielding force $F_{ys}$ corresponding to the design ductility value (for example, $\mu = 3.0$) i.e. $FYRP = F_y/F_{ys}$.

The parameter has been varied at an interval of 0.6 to 1.5. The effect of the considered parameter upon the energy characteristics of the system is considerable. Its increase causes pinching of the hysteretic curves leading to a decrease in hysteretic energy and its index (Fig. 5).

![Index of hysteretic energy](image)

Fig. 5. Effect of USD and FYRP parameters on energy dissipation and energy index

**Level of the Point of First Crack in Concrete**

The level of force $F_C$ is varied at interval of 0.1 to 0.7 $F_Y$, whereas the initial stiffness is taken twice higher than the yielding stiffness ($K_y$). The effect of the change of the $F_C/F_Y$ ratio is most significant for the trilinear model with degradation (Fig. 6). The increase in this ratio leads to an increase in dissipated hysteretic energy and its index.

**Initial Stiffness**

The initial stiffness is taken 1.5-4.0 times higher than the yielding stiffness. The effect of the change of the $K_C/K_Y$ ratio of the nonlinear response of the system is again most significant for the trilinear model with degradation (Fig. 6). The increase in this ratio leads to an increase in dissipated hysteretic energy and its index.

![Index of hysteretic energy](image)

Fig. 6. Effect of the ratios $F_C/F_Y$ and $K_C/K_Y$ on energy index
Specific Parameters of the Bilinear-Slip and Takeda-Slip model

Two characteristic parameters arising from the very nature of the Bilinear-slip model itself are the B0 parameter (ratio between the sliding stiffness and initial stiffness, i.e., KL/KY) and parameter B1 (ratio between displacement at which the sliding ends and displacement at which sliding starts i.e., DL/DU), Fig. 7. The Takeda-slip model is obtained by modification of the original Takeda model, introducing two new parameters (B2, which defines the "softening" of the stiffness due to the pinching effect and B3, which defines the increase in stiffness after closing of the cracks in the compressed concrete, Fig. 7).

![Graphs showing specific parameters of Bilinear-slip and Takeda-slip models](image)

**Fig. 7.** Specific parameters of Bilinear-slip and Takeda-slip models

With the increase of B0, the angle between the sliding axis and the x-axis is increased leading to a decrease in hysteretic energy and its index, (Fig. 8). With the increase in B1, the hysteretic energy is increased and its index is decreased. The increase in both parameters of Takeda-slip model leads to an energy increase and decrease in dissipation index, (Fig. 8).

![Graphs showing hysteretic energy and index](image)

**Fig. 8.** Effect of specific parameters on energy dissipation and energy index

RESULTS OF THE ANALYSIS OF ACTUAL R/C STRUCTURES

Taking into account the results obtained by the parametric analyses, performed were analytical investigations of several actual RC structures for the purpose of defining the maximal values of dissipated energy and its distribution along the height of the structure.

Analyzed were several different structures designed and constructed on the territory of the city of Skopje. To define the structural response from energy aspects, a dynamic analysis was performed under the effect of
actual earthquakes defined by separate studies of the seismicity of the sites of the structures. The nonlinear behaviour of the structures was modeled by three hysteretic models: bilinear model (which is widely applied due to its simplicity although such a wide application is not so much justified), the Takeda (which is most widely used model for simulating bending behaviour) and Takeda-slip model (representing a modification of the original Takeda model with included shear effect). The results are presented via maximal values of dissipated energy along the height of the structure, Fig. 9.

Fig. 9. Effect of input excitation and selected hysteretic models upon dissipated energy

The response of the structures obtained by using the bilinear model at hight input accelerations is characterized by an unrealistically high energy dissipation, whereas the response obtained by using the Takeda-slip model is characterized by a lower amount of energy in respect to that obtained by the original Takeda's model due to the pinching effect.

CONCLUSIONS

The paper presents part of the performed analytical investigations of the effect of different parameters of hysteretic models on the nonlinear response of structures. The results obtained from the analyses (expressed via time histories and maximal values of dissipated hysteretic energy and its indexes) are considerably different. Hence, a conclusion is drawn that a successful prediction of the response of RC structures exposed to strong earthquake effects requires as realistic as possible definition of the primary curve presenting their strength and deformability characteristics on one hand, whereas on the other hand, it is necessary that a nonlinear dynamic analysis be performed by using hysteretic models that will represent, most appropriately, the characteristics of the analyzed structure. The results obtained from the analyses performed for actual RC structures represent a good basis for further investigations toward definition of the seismic "requirements" expressed through the size of the dissipated energy and its index.

REFERENCES


