SEISMIC BEARING CAPACITY OF SHALLOW FOUNDATIONS

A. PECKER

Géodynamique et Structure
157, Rue des Blains, 92220 Bagneux, FRANCE

ABSTRACT

The evaluation of the seismic bearing capacity of shallow foundations resting on cohesive or dry cohesionless soils is reviewed. Recent results obtained within the framework of limit analyses methods are presented in the form of analytical formulae defining the bounding surface for the pseudo-static bearing capacity. Methods of computation of earthquake induced permanent displacements are presented which compare favorably with observed behavior.

KEYWORDS

Seismic bearing capacity; shallow foundations; earthquake induced permanent displacements; cohesive soils; dry cohesionless soils.

1. INTRODUCTION

The evaluation of the seismic bearing capacity of foundations has not received much attention from the earthquake engineering community. If the cyclic behavior of foundations has been extensively studied, with the development of impedance analyses in the last decade, their behavior at failure did not initiate much research. The major reason probably lies in the few observations of foundations failures during earthquakes (Auvinet and Mendoza, 1986; Romo and Auvinet, 1992; Pecker, 1994). This situation changed with the 1985 Michoacan Guerero earthquake and significant improvements in the foundation design have been obtained ever since. The object of this paper is to review the recent developments in the evaluation of the seismic bearing capacity of shallow foundations for which rational methods of analyses seem to emerge in the last few years.

2. FACTORS AFFECTING THE BEARING CAPACITY OF SHALLOW FOUNDATIONS

2.1 Pre-earthquake Conditions

Observations of foundation behavior during the 1985 Michoacan-Guerero earthquake clearly evidenced that the initial static pressure and load eccentricity have pronounced effects on the seismic behavior of foundations. Foundations with low static safety factors (high contact pressures) or with significant load eccentricities behaved poorly whereas well-designed foundations appear not to have suffered significant damages.
2.2 Seismic Loads from the Superstructure

In the general case, the dynamic forces have six components:
- a vertical force which, in most cases, can be neglected since its magnitude is small with respect to the static one;
- two shear forces (T) inducing an inclination of the resultant force;
- two overturning moments (M) inducing an eccentricity of the resultant force,
- a torsional moment.

For strip foundations, these forces reduce to the vertical one, the shear force and the overturning moment. It will become apparent later that generally, only the maximum value of these forces are needed but, in some instances, when permanent displacements are to be computed, their variations in time are also required.

2.3 Soil Strength

Even for static conditions, the correct evaluation of the soil strength is a challenge for the geotechnical engineers. It depends on many factors such as the soil fabric, stress and strain history, stress path, drainage conditions, anisotropy, ... Additional factors must be considered for seismic conditions, depending on the soil type which will be broadly characterized as cohesionless soils or cohesive soils:

- the rate of loading may significantly affect the value of the strength measured in conventional laboratory tests usually carried out at slower rates than those anticipated in the field during an earthquake. The strength of cohesionless soils is not affected by the rate of loading but plastic cohesive soils may exhibit undrained strengths 30% to 60% higher than the conventional strength. Figure 1 presents a typical example of such an increase measured on a clay sample from Mexico city (Romo, 1995);

![Fig. 1. Dynamic Stress Plastic Deformation Relationship](image)

- degradation under cyclic loading. The repetition of alternate cycles of loading may cause a cyclic degradation of the material and a subsequent decrease in its strength. However, not all materials are subject to degradation: cohesionless soils are insensitive to it and cohesive soils will only experience degradation when they are strained beyond a given strain threshold which is material dependent. Figure 2 shows, for instance, that permanent cyclic deformations start to accumulate for Mexico city clays only when the cyclic strain is higher than approximately 3% (Romo, 1994);

- pore pressure build-up and drainage conditions. Saturated cohesionless soils usually experience an increase in their pore water pressure due to cyclic loading under undrained conditions, which may lead to a liquefaction condition unless the drainage conditions allow for a rapid dissipation of these pressures. Whether the soil is initially in a dense or in a loose state will lead to completely different situations: a loose sand loses its whole strength and gives rise to large deformations and catastrophic failures; dense sands, owing to their dilatant behavior, can mobilize very large undrained strengths, provided drainage is prevented, and cannot develop dramatic failures.
2.4 Inertia Forces in the Soil Mass

During an earthquake, the passage of seismic waves gives rise to inertia forces within the soil volume, which are equilibrated by dynamic stresses (mainly horizontal shear stresses). These stresses mobilize a fraction of the available soil strength and consequently, the strength available to balance the inertia forces arising from the superstructure is not necessarily the full soil strength. As an extreme case, when the ground acceleration at a given depth is too large, the induced seismic stresses may cause failure: this phenomenon called fluidization (Richards et al, 1990), has been noticed in numerous theoretical studies (Pecker - Salençon, 1991; Budhu and Al Karni, 1993; Richards and Shi, 1994).

However, it must be noticed that the inertia forces within the soil mass and the forces acting on the foundation, arising from the inertia forces of the superstructure, both vary in time and there is no obvious reason why they should be in phase and maximum, with the same direction, at the same instant. Moreover, inertia forces within the soil profile decrease with depth due to the attenuation of accelerations, which is not reflected in the aforementioned studies. To properly account for the influence of the soil inertia forces, they must be treated as an independent parameter (Pecker - Salençon, 1991). To consider the same seismic coefficient $k_I$ for the soil inertia force ($F_X = \rho g k_I$) and for the structural inertia forces ($T = k_I m g$) leads to erroneous conclusions with respect to the influence of the soil inertia forces; the major reduction in the foundation bearing capacity outcomes from the load inclination and eccentricity on the foundation and not from the soil inertia forces (Dormieux - Pecker, 1995).

3. GENERAL FRAMEWORK FOR THE EVALUATION OF THE FOUNDATION BEARING CAPACITY

3.1 Pseudo-Static Evaluations

Up to very recently, the seismic bearing capacity of shallow foundations was checked using classical bearing capacity formulae in which the seismic action is regarded as an equivalent static force and load eccentricity and inclination are treated as correction factors ($S$ and $i$) to the $N_h$, $N_c$ and $N_q$ bearing capacity factors. The ultimate bearing capacity writes:

$$ q_u = \frac{1}{2} \gamma B S_y i_y N_y + C S_c i_c N_c + q S_q i_q N_q $$

A more elegant solution to the problem is provided by the concept of bounding surface in which the loading parameters are treated as independent parameters.
To this end, the yield design theory provides a rigorous treatment of the problem (Salençon, 1983, 1990). This theory belongs to the category of limit analysis methods. Unlike any limit analysis method, the derivation of upper bound and lower bound solutions allows to bracket the exact solution and possibly to determine it exactly when both bounds coincide. A proper application of the yield design theory requires the knowledge of:

- the problem geometry; in the following, unless otherwise stated, the foundation is assumed to be a strip footing resting on the surface of an homogeneous halfspace;

- the materials strengths; they refer to the soil strengths which are represented by a Tresca strength criterion (cohesive soil) or a Mohr-Coulomb strength criterion (cohesionless soil) and to the soil-foundation interface which is assumed without tensile strength to allow for uplift between the soil and the foundation;

- the loading parameters; six independent loading parameters are considered in the derivation of the bounding surface: the normal force \( N \), the horizontal shear force \( T \), the overturning moment \( M \) and the soil inertia forces \( F_x \) and \( F_y \) in the vertical and horizontal direction and the overburden pressure \( Q \) at the foundation level.

The set of admissible loads is located within a surface, defined in the loading parameters space, called the bounding surface.

\[
\phi (N, T, M, F_x, Q) \leq 0
\]  

(2)

In the case where \( F_x = 0 \), experimental evidence of equation (2) has been given by Butterfield - Gottardi (1994) and Kitazume - Terashi (1994).

Most of the recent solutions to the bearing capacity problems are based on limit equilibrium methods (Sarma - Iossefili, 1990; Budhu - Al Karni, 1993; Richards et al, 1993) but make only reference to upper bound solutions obtained with the assumption of a given, predetermined, failure mechanism; moreover, they do treat the soil inertia forces as an independent parameter and assume that the horizontal acceleration in the soil and in the structure are equal. As mentioned previously, this constitutes a severe limitation to the usefulness of the results. Other solutions (Pecker - Salençon, 1991; Salençon - Pecker, 1995a and b; Pecker et al, 1995; Faccioti et al, 1996) which do not suffer such limitations are presented in this paper.

3.2 Dynamic Approach

As noted previously, the forces acting on the foundation or within the soil mass vary with time. They can exceed the available resistance of the foundation soil-system for short periods without leading to a general failure of the foundation. This is an essential difference between static, permanent loading and dynamic, time-varying loading. In the first instance, an excessive load generates a general failure, whereas the second situation induces permanent, irreversible displacements which superimpose to the cyclic displacements. Failure can therefore be no longer defined as a situation in which the safety factor drops below 1.0. It must rather be defined as excessive permanent displacements which impede the proper functioning of the structure. This definition, first introduced by Newmark (1965) has been successfully applied to the design of dams, gravity retaining walls assimilating the potentially unstable soil mass to a rigid sliding block. It has also been used for the bearing capacity of foundations (Sarma - Iossefili, 1990; Richards et al, 1993).

This method can be further extended, relaxing the condition of rigid soil blocks and considering a more realistic deformable body, as it is actually assumed in the computed failure mechanisms. The soil foundation system is assumed to behave as an elastic perfectly plastic system, in which the bounding surface defined previously, is adopted as the boundary for the apparition of plastic deformations. Using the kinetic energy theorem, the angular velocity of the foundation and its permanent displacement can be computed (Pecker - Salençon, 1991). Under the assumptions spelled above, the method permits a rigorous definition of failure in terms of unacceptable permanent displacements.
4. **COHESIVE SOILS**

4.1 **Strip Footing on a Homogeneous Isotropic Halfspace**

Solutions to the bearing capacity of shallow strip footings resting on the surface of a cohesive halfspace have been obtained by Pecker - Salençon (1991), Salençon - Pecker (1995 a and b) for a soil with or without tensile strength. These solutions were derived from the static and the kinematic approaches of the theory and it was shown that both, the lower bound and the upper bound solutions, were very close to each other, giving therefore an almost exact solution to the problem. The most prominent kinematic mechanisms used are presented in figure 3 in two situations: without uplift of the foundation and with uplift. The first situation is prevailing for small load eccentricities or inclinations whereas the second one governs when these two parameters become significant. These mechanisms depend upon three geometric parameters for which the optimum values, which minimize the maximum resisting work, are numerically determined.

![Fig. 3. Skeletal View of the Bounding Surface](image)

The results can be expressed in terms of the adimensional parameters:

\[
\bar{N} = \frac{N}{CB}, \quad \bar{T} = \frac{T}{CB}, \quad \bar{M} = \frac{M}{CB^2}, \quad \bar{F} = \frac{F_x}{C}, \quad \bar{Q} = \frac{Q}{C}
\]

where \(C\) is the soil undrained shear strength, \(B\) the foundation width and \(N, T, M, F_x, Q\) the five independent loading parameters.

In the case where \(\bar{F}\) and \(\bar{Q}\) are nil and for a soil without tensile strength, the bounding surface is presented in figure 4; only the upper part of the surface corresponding to \(\bar{M} \geq 0\) is presented in figure 4. The equation of the bounding surface can be written:

\[
\frac{(\beta \bar{T})^2}{(\alpha \bar{N})^a [1 - \alpha \bar{N}]^b} + \frac{(\gamma \bar{M})^2}{(\alpha \bar{N})^c [1 - \alpha \bar{N}]^d} - 1 = 0
\]

with

\[
\bar{N} = \frac{N}{CB}, \quad \bar{T} = \frac{T}{CB}, \quad \bar{M} = \frac{M}{CB^2}
\]

\(a = 0.70, b = 1.29, c = 2.14, d = 1.81, \quad \alpha = \frac{1}{\pi + 2}, \quad \beta = 0.5, \quad \gamma = 0.36\)

under the constraints \(0 < \alpha \bar{N} \leq 1, |\bar{T}| \leq 1\).
Fig. 4. Example of Kinematic Mechanisms

When the soil inertia forces are taken into account ($ \overline{F} \neq 0 $), the following conclusions were derived in Pecker - Salaçon (1991) and further confirmed by Pecker et al (1995):

- for normally encountered ground accelerations, characterized by a value $ \overline{F} \leq 2 $, and for foundations for which $ \overline{N} \leq 2.5 $, i.e. for foundations with a safety factor higher than 2.0 under a vertical centered load, the effect of the soil seismic forces can be neglected without loss of accuracy. For foundations with lower safety factors, the soil seismic forces induce a dramatic reduction in the bearing capacity. This is illustrated in figure 5 which presents cross-sections of the bounding surface for various values of $ \overline{F} $.

Fig. 5. Bounding Surfaces

The methodology for the computations of the permanent displacements, as described in paragraph 3.2, has been successfully applied for buildings which suffered severe failures in Mexico city during the 1985 Michoacan-Guero earthquake (Pecker et al, 1995). The angular velocity of the foundation around point $ \Omega $ of figure 3 is given by:

$$ \omega(t) = \frac{K}{\rho B^3} T^+ \int_{t_1}^{t_2} \left[ \frac{T(\tau)}{T^+} - 1 \right] d\tau $$  \hspace{1cm} (4)

where $ K $ is a factor related to the geometry of the optimum mechanism, $ \rho $ is the soil mass density, $ B $ the foundation width, $ T^+ $ the maximum admissible load computed from (3) and $ T(\tau) $ the time history of the applied force, computed from an independent soil-structure interaction analysis. In (4), it is assumed that $ M $ is related to $ T $ by $ M = T \cdot h $ where $ h $ is the elevation of the building center of gravity. Integrating (4) between $ t = t_0 $ such that $ T(t_0) = T^+ $, and $ t = t_1 $ such that $ \omega(t_1) = 0 $ gives the permanent rotation of the foundation. Figure 6 presents the results of the computations for one particular building and shows that a very good agreement is achieved between prediction and observation.
Fig. 6. Computed Foundation Tilt

Interesting enough in figure 6 is the fact that rotations increase very rapidly when the soil strength drops below 30 kPa; it turns out that this value is close to the value (33 kPa) for which the pseudo-static safety factor is equal to 1.0. Similar results were obtained for other buildings indicating that, although the pseudo-static safety factor could theoretically be smaller than one, induced permanent displacements can very quickly become unacceptable for the structure. It therefore seems appropriate, at least in seismic building codes, to require that the pseudo-static safety factor be greater or equal to 1.0.

4.2 Three-Dimensional Footings

The preceding results which are valid for a 2D plane strain problem can be extended to account for a rectangular shape of the footing (Faccioli et al., 1996); the modification, expressed as a correction factor to the 2D ultimate vertical load (equation 1), is written:

\[
Sc = 1 + 0.32 \left( 1 - \frac{2 e}{B} \right) \frac{B}{L}
\]  \hspace{1cm} (5)

In most practical situations, the correction is small.

4.3 Anisotropic Soil

The preceding results have been extended to the case of an anisotropic cohesive soil, with or without tensile strength (Pecker et al., 1995). The undrained strength \( C \) obeys to Bishop's law:

\[
C(\psi) = C_b \left( 1 - a \cos^2 \psi \right) \left( 1 - b \sin^2 2 \psi \right)
\]  \hspace{1cm} (6)

where \( \psi \) is the angle between the vertical direction and the direction of the major principal stress \( \sigma_1 - C_b \). \( a \) and \( b \) are material constants related to the undrained strength measured along various stress paths.

Figure 7 presents cross-sections of the San Francisco bay mud bounding surface for isotropic and anisotropic situations (soil without tensile strength). This example corresponds to a soft normally consolidated clay, as Mexico city clay, for which bearing capacity problems are more likely to occur. It appears that the difference between the isotropic and anisotropic material is small, and the difference becomes almost negligible for practical seismic situations where the normalized vertical force \( \overline{N} \) is smaller than 2.5 and the load eccentricity becomes significant.

Different conclusions were reached for overconsolidated materials where the soil anisotropy gives higher bearing capacity values.
5. DRY COHESIONLESS SOILS

Solutions for the cohesionless soil, obeying Coulomb’s strength criterion with or without tensile strength, have been obtained within the framework of the yield design theory (Salençon - Josseron, 1994). The kinematic mechanisms of the upper bound approach are similar to those presented in figure 3, provided the arc of circles are replaced by log spirals. The same conclusions as for the cohesive soils were reached (Faccioli et al, 1996): in a range of reasonable values of the pseudo-static seismic coefficient ($k_H \leq 0.3$) the bearing capacity reduction due to the effects of the inertia forces in the soil is small and does not exceed 15-20%.

Based on the results of Salençon-Josseron (1994), and on additional results using the same theory (Paolucci, 1995), a simplified formula has been developed for the equation of the bounding surface (Faccioli et al, 1996):

$$\bar{T} = 0.85 \bar{N} \left[1 - 3 \sqrt{\frac{\bar{N}}{\eta \chi}}\right]$$  \hspace{1cm} (7)

where the quantities $\bar{T}$, $\bar{N}$ are normalized with respect to $N_{\text{max}}$, the maximum, vertical, centered force.

$$\bar{T} = \frac{T}{N_{\text{max}}} \quad \bar{N} = \frac{N}{N_{\text{max}}} \quad \bar{M} = \frac{M}{M_{\text{max}}}$$  \hspace{1cm} (8)

and

$$\eta = \left(1 - \frac{2 M}{B N}\right)^{1.8} \quad \chi = \left(1 - \frac{k_H}{\tan \phi}\right)^{0.35}$$  \hspace{1cm} (9)

As opposed to the case of cohesive soils, the computations of the permanent displacements following the methodology of 3.2 poses a serious difficulty for the interpretation of the results. The results are only meaningful for an associated flow rule which is known to overestimate the volumetric strains in cohesionless soils. However, this shortcoming is not a limitation since most shallow foundations on cohesionless soils are designed, under static loads, for the settlements; they therefore present a large safety factor with respect to the ultimate load under static conditions and it is very unlikely that the bearing capacity be exceeded during an earthquake. This is consistent with the fact that, aside the case of liquefaction, foundation failures on cohesionless soils are seldom reported after an earthquake. More insights on the computations of earthquake induced permanent displacements under a footing resting on a dry cohesionless soil are given in Faccioli et al.

6. CONCLUSIONS

The state of the art in the evaluation of the seismic bearing capacity of shallow foundations has been reviewed. Much progress has been achieved in the last five years leading to a rational, rigorous, approach.
For cohesive soils, the results are fairly well developed and permit the incorporation of soil inertia forces, soil anisotropy, independent consideration of the loading parameters in the bearing capacity evaluations. Earthquake induced permanent displacements can be computed and they compare favorably with observed behaviors.

The pseudo-static bearing capacity of footing on dry cohesionless soils is also well-developed. However, the computations of earthquake induced permanent displacements cannot be rationally achieved, except under very restrictive assumptions; as mentioned previously, this is not a factor of primary importance.

Much work remains to be done in the case of footing resting on saturated cohesionless soils where the effect of the pore pressure increase on the bearing capacity needs to be assessed.

REFERENCES


