WHAT IS MISSING FOR USING IN PRACTICE AVAILABLE RESULTS ON FOUNDATION IMPEDANCES

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ABSTRACT

In the recent literature, there are very interesting papers in geotechnical engineering based on powerful numerical methods. Generally, authors illustrate their methods with the help of one example and/or a comparison with results obtained by other authors. Consequently, the practicing engineer has only the results for few cases. Of course, it is obviously possible that the engineer has powerful softwares, but he has not necessarily the indispensable long experience for the use of these softwares, especially in dynamics problems. It appears necessary to discuss about the interest to continue to publish complete results (tables or charts) as it was the practice in the sixties and in the seventies. For example, if no mistake, no results are available concerning the impedance functions of a rectangular foundation on a layer on a rigid inclined substratum, but the softwares able to calculate these functions exist. In the background of this discussion is also the question - very important for all teachers - of the contents of the educational program of our students.

KEYWORDS

Soil-structure interaction, rectangular surface footing, dynamic, static stiffness, coupled movements

Concerning the dynamic soil-structure interaction, a lot of papers is published each year. The reading of these papers shows clearly an evolution of their contains : they are less and less numerical results or empirical relations obtained from systematic parametric calculations. Looked at from this point of view, the most important contributions were published by Dominguez and Roesset (1978) for rectangular footings and by Gazetas (1983) who has written a very complete State of the Art. The author (1992) himself completed this work in giving all known impedance functions in a practical form. Wolf (1994) proposed recently a new very detailed approach to calculate impedance functions. Especially during the ten last years, the publications describe new theoretical approaches or improve existing numerical methods. These new methods are generally illustrated by only one or two examples. This statement brings to put the question if an engineer can practically use these informations to solve his problem.

The expansion of computers is certainly the major reason of this evolution which can be noted also in other scientific domains. It is possible that the majority of authors thinks each engineer possess his own necessary hardware and software - what it is true - and is able to develop himself his softwares from the indications given in the publications - what it is not so easy. By virtue of this principle, everybody should be able to calculate himself the necessary values to solve his problem. Looked at from this point of view, publication of systematic results is not necessary. Of course, this way must be used to solve complex problems concerning constructions as earthdams or nuclear power installations which present a risk for population and environment : simplified solutions with large approximations are not acceptable in these cases. But often, the engineer needs immediately results to justify the feasibility of his project. There is every reason to suppose he will prefer use published values instead to do again complicated calculations which demands a very good knowledge of the software, and takes time.
If the engineer does not find directly the response of his problem in the literature, he will try to solve his problem by comparison to typical well-known cases. The typical example concerns the rectangular foundation replaced by an equivalent circular footing. Due to its axial symmetry, the theoretical solution of circular foundation is easier to obtain than that of a rectangular foundation which needs a three-dimensional calculation. Therefore, there is so many results on this type of foundation, which are not often realised in practice.

![Diagram showing rectangular and circular footings](image)

**Fig. 1. Equivalent method**

To show the error induced by using the equivalent circular footing method, we compare the results obtained directly by Dominguez et al (1978) for a square foundation, and those obtained by using an equivalent circular footing. The radius is calculated by writing that it has respectively the same area for translation movements and same moment of inertia for rotation movements. Formulas for each mode are given in table 1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Equivalent circular footing</th>
<th>Square footing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>$4.51 \frac{GB}{1-v} \left[ 1 + 0.44 \frac{D}{B} \right]$</td>
<td>$4.75 \frac{GB}{1-v} \left[ 1 + 0.47 \frac{D}{B} - 0.05 \left( \frac{D}{B} \right)^2 \right]$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>$9.03 \frac{GB}{2-v} \left[ 1 + 0.59 \frac{D}{B} \right]$</td>
<td>$9.41 \frac{GB}{2-v} \left[ 1 + 1.13 \frac{D}{B} - 0.16 \left( \frac{D}{B} \right)^2 \right]$</td>
</tr>
<tr>
<td>Rocking</td>
<td>$3.97 \frac{GB^3}{2-v} \left[ 1 + 1.75 \frac{D}{B} \right]$</td>
<td>$4.38 \frac{GB^3}{2-v} \left[ 1 + 0.98 \frac{D}{B} + 1.13 \left( \frac{D}{B} \right)^2 \right]$</td>
</tr>
<tr>
<td>Coupled Horiz. - Rock.</td>
<td>$0.40 K_H D$</td>
<td>$0.35 K_H D \left[ 1 + 0.19 \frac{D}{B} \right]$</td>
</tr>
<tr>
<td>Torsion</td>
<td>$7.93 GB^3 \left[ 1 + 2.34 \frac{D}{B} \right]$</td>
<td>$8.71 GB^3 \left[ 1 + 2.80 \frac{D}{B} - 0.19 \left( \frac{D}{B} \right)^2 \right]$</td>
</tr>
</tbody>
</table>

**Table 1. Static stiffnesses for square footing**

Relative errors are given in table 2. B is the half width of the foundation, D the embedment length, G the shear modulus of the soil, and v its Poisson's ratio.

<table>
<thead>
<tr>
<th>Embedment D/B</th>
<th>Mode</th>
<th>Vertical</th>
<th>Horizontal</th>
<th>Rocking</th>
<th>Horiz.-Rock.</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>-5.05</td>
<td>-4.04</td>
<td>-9.36</td>
<td>0</td>
<td>-8.92</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>-5.34</td>
<td>-18.54</td>
<td>-4.12</td>
<td>-14.98</td>
<td>-16.01</td>
</tr>
<tr>
<td>1.5</td>
<td></td>
<td>-0.93</td>
<td>-22.53</td>
<td>-34.44</td>
<td>-31.10</td>
<td>-13.98</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2.54</td>
<td>-20.12</td>
<td>-45.45</td>
<td>-33.85</td>
<td>-11.46</td>
</tr>
</tbody>
</table>

**Table 2. Relative errors (%) - static stiffnesses**

This table shows clearly the limits of this equivalent method, especially for horizontal and rocking movements which are induced by wind, earthquakes, and so on. However, one has at one's disposal only few results concerning rectangular footings on a half-space soil foundation. For embedded footings, Dominguez gives only
results for two ratios length / width = 1 and 2. For surface footings without embedment depth, one can find more results.

Consequently, we can easily see the interest to complete the systematic studies of embedded rectangular footing:
- with various lateral welding lengths,
- on a half-space or on a layer resting on an horizontal or inclined substratum.

To encourage researchers to publish in this way, we give new results concerning static stiffnesses of non embedded strip foundations on a layer resting on a rigid inclined substratum (fig.2).

![fig. 2. Strip foundation on a layer limited by inclined substratum](image)

The calculations are performed with the software SSI 2D developed by Schmid (1988). Table 3 gives the results concerning the static stiffnesses in an approximate manner.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Static Stiffness</th>
<th>Range of Validity</th>
</tr>
</thead>
</table>
| Vertical   | \[
\frac{0.6 G}{1-1.45\nu} \left[ 1 + 7 \frac{B}{h} \right]
\] | \(\tan \alpha < 1\) \(\nu \leq 0.4 : 1 \leq h/B \leq 12\) \(\nu > 0.4 : 2 \leq h/B \leq 12\) |
| Horizontal | \[
\frac{1.82 G}{2-\nu} \left[ 1 + 2.3 \frac{B}{h} \right]
\] | \(\tan \alpha < 1\) \(0.25 \leq \nu \leq 0.3\) \(1 \leq h/B \leq 12\) |
|            | \[
1 + \left[ 0.3 \frac{B}{h} \left( 1 + 33 \frac{3}{\nu} + 0.22 \right) \right] \tan^2 \alpha
\] | \(\tan \alpha < 1\) \(0.3 < \nu \leq 0.45\) \(1 \leq h/B \leq 12\) |
| Rocking    | \[
\frac{\pi GB^2}{2(1-\nu)} \left[ 0.93 + 0.68 \frac{B}{h} \right]
\] | \(\tan \alpha < 1\) \(\nu \leq 0.4 : 1 \leq h/B \leq 12\) \(\nu > 0.4 : 2 \leq h/B \leq 12\) |

Table 3. Static stiffnesses - strip foundation on a layer limited by inclined substratum

We see that static stiffness is not influenced by slope of substratum for vertical and rocking movements. Concerning horizontal movement, static stiffness is sensitive to substratum slope only for Poisson's ratio greater than 0.3.

On the other hand, it is also necessary to give all terms of the impedance matrix. Often are given only the terms of the main diagonal which concern the uncoupled movements. It is well know that horizontal translation and rocking movements are ever coupled. Of course, the values taken by the coupling terms are minor for a non embedded surface footing. But in practice, a footing has ever an embedment depth corresponding to freezing depth (between 0.6 and 1 meter depending on geographical situation and climatic conditions). In this case, it is not possible to neglect coupled horizontal displacement - rocking term, all the more as footing are generally loaded by inclined and eccentric forces.
To illustrate this purpose, we come back to the example of the square footing with an embedment ratio D/B = 1/2, considering here also only static aspects of the problem (fig. 3).

![Diagram of footing loaded by eccentric inclined force]

Fig. 3. Footing loaded by eccentric inclined force

Equations (1) give the transformation of the eccentric inclined load \( P \) to vertical load \( P_V \), horizontal load \( P_H \) and moment \( M \) applied on the center of the base of the foundation.

\[
\begin{align*}
P_V &= P \cos \alpha \\
P_V &= P \sin \alpha \\
M &= -e \ P \sin \alpha
\end{align*}
\]  

(1)

Vertical \( u_V \) and horizontal \( u_H \) displacements, and rotation \( \theta \) can be calculated by using equations (2).

\[
\begin{align*}
u_V &= \frac{P_V}{K_V} \\
u_H &= \frac{K_R P_H - K_{HR} M}{K_H K_R - K_{HR}^2} \\
\theta &= \frac{K_H M - K_{HR} P_H}{K_H K_R - K_{HR}^2}
\end{align*}
\]  

(2)

Figures 4 show that the error induced by neglecting coupling can be very important. For \( \alpha = -30^\circ \) and \( e/B = 0.1 \), rotation is positive with coupling and negative without coupling!

It is also very important that authors give all results concerning the impedance matrix.

![Graph of error vs. horizontal displacement]

Fig. 4a. Equivalent method - relative error on horizontal displacement
Fig. 4b. Equivalent method - relative error on rocking movement

In the background of this discussion the question - very important for all teachers - is also about contents of the education of our students. Dynamic soil-structure interaction can be taught:
- in giving practical methods sufficient for solve classical simple cases,
- in developing theoretical methods as Boundary Elements Method and in using performing software to have a through knowledge of these subjects. In this way, there is a risk that the education will emphasize on Mathematics or Computers than on Civil Engineering materials.

REFERENCES


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