BOUNDARY ELEMENT MODELING OF SITE EFFECTS

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ABSTRACT

A review of boundary element methods for the numerical solution of seismic responses of surface and subsurface irregularities of soil medium such as canyons, alluvial valleys, sedimentary basins, and faults is presented. An application of a direct three-dimensional boundary element method for evaluating seismic waves in a three-dimensional sedimentary basin for incident seismic waves is described in detail. Some topics are also described about an applicability of the boundary element modeling; some difficulties in numerical analysis, such as numerical integrations of a oscillatory integrand of Green’s functions for an elastic half-space and singularities of the diagonal element of the boundary element matrix equations; the artificial resonance of the solution of the integral equations inherent to the method. We discuss the advantages and disadvantages of the BEM for evaluating seismic response of topographic sites comparing with domain type of numerical methods, such as the finite element method(FEM), and the finite difference method(FDM).

KEYWORDS

Boundary element method; site effects; sedimentary basins; canyons; Green’s functions; three-dimensional model; ground motion; earthquake damage

INTRODUCTION

It has been recognized that the dynamic behavior of a structure during an earthquake is considerably affected by the geologic formation and the dynamic property of the soil medium. In particular, the topographic irregularities in the soil medium seem to have a tremendous effect on the characteristics of the earthquake ground motion, together with the associated structural damage of the building. Many numerical methods, such as the Aki-Larner method(e.g., Aki and Larner, 1970; Bouchon, 1979; Bard and Bouchon, 1980; Horike et al., 1990), the finite element method(FEM) and the finite difference method(FDM), have been developed to evaluate seismic responses of topographic sites.

The boundary element method(BEM, e.g., Beskos, 1987; Malinos and Beskos, 1988), sometimes known as the boundary integral equation method(BIEM), has become increasingly popular in recent years for the solution of dynamic problems in earthquake engineering, and many papers on BEM(Dominguez, 1978; Maeda, 1982; Jiang and Kuribayashi, 1988; Luco et al., 1990; Nowak and Hall, 1993; Hisada et al., 1993; Shinozaki and Yoshida, 1994; Sanchez-Sesma and Luzon, 1995; Shinozaki and Yoshida, 1996), Discrete Wavenumber Boundary Element Method(Campillo and Bouchon, 1985; Kawase and Aki, 1989; Kim and Papageorgiou, 1993), and Hybrid method(FEM + BEM, e.g., Khair et al., 1989; Mossessian and Dravinski, 1992; Fujiwara, 1995) have been devoted to the topics. Since the BEM for evaluating the seismic responses of topographic sites for incident seismic waves is formulated with the aid of the representation theorem(Aki and Richards, 1980), the problems in the domain of interest lead to the boundary value problems just at the surface of the domain.
of interest. Thus, we can reduce the number of dimensionality of the problem by one. As artificial boundaries which are indispensable to domain type of numerical methods such as FEM and FDM are not required for the BEM, there exists no restriction on modeling various types of incident waves such as non-vertically propagating plane seismic waves as well as incident seismic waves generated from point sources and double couple sources.

It is a usual way first to obtain frequency domain responses of the site using the BEM and second to calculate time domain responses with the aid of F-T using those frequency domain responses. Thus the BEM provide a physical insight into the problem of interest, whereas it is difficult in the FDM to obtain directly frequency domain responses of the site. However, it is noted that the BEM has less applicability to model real complex topographic features than the domain type of numerical methods such as the FEM and FDM. The algorithm of the BEM is more complex than those of domain type of numerical methods. It is a formidable task to evaluate numerous matrix elements of the boundary integral equations involving Green’s functions and corresponding tractions for the elastic half-space. As Green’s functions and corresponding tractions for the elastic half-space consist of several terms proportional to the singular functions \( 1/r \) and \( 1/r^2 \), where \( r \) is the distance between a source and receiver, as well as nonsingular terms expressed in terms of the semi-infinite integrals, there exist two main difficulties in evaluating matrix elements, namely, computing efficiently numerical integrations of the semi-infinite integrals which involve an oscillating integrand (Apse
c and Luco, 1983; Yoshida and Kawase, 1988; Hisada, 1994), and computing accurately the diagonal elements which are composed of the singularities of Green’s functions and corresponding tractions (Hayami and Brebbia, 1988; Guiggiani and Gigante, 1990). Since these matrices of the boundary element equations, in general, have dominant diagonal elements, numerical stability of the equations is established for the relatively high frequency range. There exists, however, the non-uniqueness of the solution to the integral equations for certain values of frequency. These so-called fictitious eigenfrequencies are associated with the characteristic wave numbers of the interior problem under homogeneous boundary conditions: e.g., for an exterior traction problem these are the eigenvalues of the homogeneous interior displacement problem associated with the same boundary surfacé (Rizzo et al., 1985; Nowak and Hall, 1993).

In the present study, we illustrate an application of a direct boundary element method to evaluate seismic waves in a 3-D sedimentary basin for both incident plane waves such as SH, P, SV, and Rayleigh waves, and incident waves generated from a point source. (Shinozaki and Yoshida, 1994, 1996).

METHOD OF ANALYSIS

Geometry of the problem is shown in Figure 1. A sedimentary basin is assumed to be a 3-D model (denoted by medium 1), which is characterized by mass density, \( \rho_1 \), P-wave velocity, \( \alpha_1 \), and S-wave velocity, \( \beta_1 \). Medium 1 is surrounded by a half-space medium (denoted by medium 2), which is characterized by mass density, \( \rho_2 \), P-wave velocity, \( \alpha_2 \), and S-wave velocity, \( \beta_2 \). An interface between the media 1 and 2 is denoted by \( S \) and perfect bonding along the interface is understood. The material of them is assumed to be linearly elastic, homogeneous and isotropic.

The total wave field in the surrounding half-space, \( u \) \[ \{ u_x, u_y, u_z \}^T \] is given by

\[ u = u^s + u^{in}, \]  
(1)

where the superscript \( s \) denotes the wave field scattered from the medium interface between the sedimentary basin and surrounding half-space, and the superscript \( in \) denotes the incoming wave field which includes the incident waves and their reflections from the free surface.

To derive the boundary integral representations, we start with Betti’s reciprocity theorem written in the following form:

\[ \int \int \int_{\Omega} (f^A u^A_t - f^B u^B_t) d\Omega = \int \int_{S} (u^B_t p^A_t - u^A_t p^B_t) dS, \]  
(2)

In the above \( u^A_t \) and \( p^A_t \) represent the \( i \)-th components of displacement and surface traction due to body force \( f^A \), while \( u^B_t \) and \( p^B_t \) are the \( i \)-th components of displacement and surface traction due to body force \( f^B \) in region \( \Omega \).

In equation (2) we employ the following sets of functions,

\[ \{ u^A_i (\mathbf{x}), p^A_i (\mathbf{x}) \} = \{ u^A_i (\mathbf{x}), p^A_i (\mathbf{x}) \}, \]
\[ \{ u^B_i (\mathbf{x}), p^B_i (\mathbf{x}) \} = \{ G_{ij}(\mathbf{x}; \mathbf{x}^*), H_{ij}(\mathbf{x}, \mathbf{u}; \mathbf{x}^*) \}, \]  
(3a)

in which \( G_{ij}(\mathbf{x}; \mathbf{x}^*) \) and \( H_{ij}(\mathbf{x}, \mathbf{u}; \mathbf{x}^*) \) are the \( i \)-th components of displacement and traction at \( \mathbf{x} \) due to a point force in the \( j \)-th direction at \( \mathbf{x}^* \), and \( \mathbf{u} \) is a unit outward normal of the boundary \( S \). In this study we
employ as $G_{i j}(x; x^*)$ and $H_{i j}(x, u; x^*)$, the Green’s functions and corresponding traction functions developed by Yoshida and Kawase (1988).

After substitution of equation (3) to equation (2) and exchange of $x$ for $x^*$ as well as of subscript $i_j$ for subscript $i_j$, we get the fundamental equation of boundary integral representation as follows,

$$u_{i}^{*}(x^*) = \int \int_{S} \left\{ G_{i j}(x; x^*) \cdot p_{j}^{*}(x, u) - H_{i j}(x, u; x^*) \cdot u_{j}^{*}(x) \right\} dS. \quad (4)$$

In the direct BEM formulation, we need the expression in which a point $x^*$ lies on the boundary $S$. This can be obtained by considering the limit $x^* \rightarrow x$

$$C \cdot u_{i}^{*}(x^*) = \int \int_{S} \left\{ G_{i j}(x; x^*) \cdot p_{j}^{*}(x, u) - H_{i j}(x, u; x^*) \cdot u_{j}^{*}(x) \right\} dS. \quad (5)$$

where, $C$ is a constant determined by the boundary shape around $x^*$ and equal to $1/2$ in case of a smooth boundary. This equation shows that the boundary values are directly related to each other through the Green’s functions. Note that the integration contains a singularity in $G_{i j}(x; x^*)$ and $H_{i j}(x, u; x^*)$.

On the other hand, the incoming wave field should satisfy the equation

$$(1 - C) \cdot u_{i}^{m}(x^*) = \int \int_{S} \left\{ -G_{i j}(x; x^*) \cdot p_{j}^{m}(x, u) + H_{i j}(x, u; x^*) \cdot u_{j}^{m}(x) \right\} dS. \quad (6)$$

From these two equations and equation (1) we obtain the following BEM equation for the surrounding medium (medium 2).

$$C \cdot u_{i}^{(2)}(x^*) = \int \int_{S} \left\{ G_{i j}^{(2)}(x; x^*) \cdot p_{j}^{(2)}(x, u) - H_{i j}^{(2)}(x, u; x^*) \cdot u_{j}^{(2)}(x) \right\} dS + u_{i}^{m}(x^*). \quad (7)$$

Following the same procedure, we obtain the following BEM equation for the interior region (medium 1).

$$(1 - C) \cdot u_{i}^{(1)}(x^*) = \int \int_{S} \left\{ -G_{i j}^{(1)}(x; x^*) \cdot p_{j}^{(1)}(x, u) + H_{i j}^{(1)}(x, u; x^*) \cdot u_{j}^{(1)}(x) \right\} dS, \quad (8)$$

where, the superscript $^{(m)}$ ($m = 1, 2$) denotes the medium number. Equations (7) and (8) are combined using the boundary conditions prescribed by,

$$u_{j}^{(1)}(x) = u_{j}^{(2)}(x)$$
$$p_{j}^{(1)}(x, u) = p_{j}^{(2)}(x, u). \quad (9)$$

To solve equations (7) and (8), the discretization scheme of both boundary shape and boundary values $u_{j}(x)$ and $p_{j}(x, u)$ should be introduced in the same manner as the finite element method. The simplest boundary element is a constant-value element with fixed slope, which allows to express the integral equation by means of

$$C \cdot u_{i}(n) = \sum_{m=1}^{M} \tilde{G}_{i j}(m; n) \cdot p_{j}(m) - \sum_{m=1}^{M} \tilde{H}_{i j}(m; n) \cdot u_{j}(m), \quad (n = 1, M). \quad (10)$$

In this equation, $M$ is the total number of elements and $\tilde{G}_{i j}(m; n)$ and $\tilde{H}_{i j}(m; n)$ are the element integrations over the surface of $m$-th element $S_m$ expressed by

$$\tilde{G}_{i j}(m; n) = \int \int_{S_m} G_{i j}(x; x_{n})dS_{m} \quad (11a)$$
$$\tilde{H}_{i j}(m; n) = \int \int_{S_m} H_{i j}(x; x_{n})dS_{m}. \quad (11b)$$

Since $G_{i j}(x; x_{n})$ and $H_{i j}(x; x_{n})$ are proportional to the singular functions $1/r$ and $1/r^2$, respectively, where $r$ is the distance between a source and receiver, namely, $r = \|x - x_{n}\|$, when $m$ coincides with $n$, equations (11) should be accurately evaluated with the aid of efficient methods. In this study we dealt with those singularities of Green’s functions and corresponding tractions using the method proposed by Hayami and Brebbia (1988).
By combining equations for different \( n \) we obtain the final simultaneous linear equations to be solved for the unknown boundary values as follows,

\[
\begin{pmatrix}
[H]\^{(2)} & [G]\^{(2)} \\
[H]\^{(1)} & [G]\^{(1)}
\end{pmatrix}
\begin{pmatrix}
[u] \\
[p]
\end{pmatrix}
= \begin{pmatrix}
u^m \\
0
\end{pmatrix},
\]

where \( u \) and \( p \) represent unknown vectors of displacement and traction along the boundary \( S \), respectively, and \( u^m \) is the incoming wave field vector. And \( [G]\^{(j)} \) and \( [H]\^{(j)} \) (\( j = 1, 2 \)) represent coefficient matrices whose elements are evaluated with equations (11). Once \( u \) and \( p \) are numerically solved, then displacements at any point in the medium can be calculated by the discretized form of equation (7) or (8).

It is easily seen that it is a formidable task to evaluate such element integrations as shown in equations (11), since they are composed of Green's functions for a half-space medium, i.e., medium 1 or medium 2, which represent themselves infinite integrals difficult to compute accurately. If full-space Green's functions (Jiang and Kuribayashi, 1988; Sanchez-Sesma and Luzon, 1995) instead of the half-space Green's functions (Shinozaki and Yoshida, 1994, 1996) are employed, a significant amount of computational time can be saved, but the size of computer core memory up to approximately one and a half times as much as the one for the case of the half-space Green's functions is required.

**NUMERICAL RESULTS**

The material properties of the sedimentary basin are normalized with respect to those of the half-space. The normalized frequency \( \eta \) is defined as \( \eta = 2a/\lambda \) where \( \lambda \) is the wavelength of S-wave in the half-space and \( a \) is the radius of the sedimentary basin.

First we show some frequency domain responses of a semi-spherical sedimentary basin due to non-vertically propagating SH waves (Sanchez-Sesma et al., 1989) to illustrate some verification of the present method. Figure 2 shows surface displacement amplitude \( u_y \) versus the normalized frequency \( \eta \), normalized by the amplitude of incident SH waves, for the central site of a semi-spherical sedimentary basin due to an incident plane SH waves whose incident angle is equal to 30°. Material properties for media 1 and 2 are assumed to be the same values as those of Sanchez-Sesma et al. (1989) and be related by \( \beta_1 = 0.45 \beta_2 \) and \( \rho_1 = \rho_2 \) with Poisson's ratios \( \nu_1 = 0.3 \) and \( \nu_2 = 0.25 \), respectively. They are compared with the results of Sanchez-Sesma et al. (1989) indicated by solid circle. Fig. 2 shows very good agreements of the results. We calculated those responses of the semi-spherical sedimentary basin approximating the semi-spherical medium interface with 256 pieces of plane elements. Sanchez-Sesma et al. (1989) calculated the scattering of incident SH waves by a semi-spherical sedimentary basin using wave function expansion in terms of spherical Hankel or Bessel functions. Since each one of these wave functions does not itself satisfy the free-boundary conditions on the half-space surface, their method does not give exact solutions for the scattering of incident surface waves by a 3-D sedimentary basin. It should be noted that the present method is applicable to the scattering of not only incident body waves but also incident surface waves by a 3-D sedimentary basin whose medium interface is arbitrarily shaped, because a boundary element problem is formulated in terms of exact Green's functions which satisfy the free-boundary conditions on the half-space surface.

From frequency domain results, we computed synthetic seismograms using the FFT algorithm. Figs. 3 show some examples of time history of surface displacement \( u_y \) for 79 sites along longitudinal section \( (y = 0) \) of a shallow sedimentary basin due to a point source located at \((-10 \text{ km}, 0, 5 \text{ km})\) excited in the y direction. It is noted that any surface displacement components such as \( u_x \) and \( u_z \) except \( u_y \) vanish along the x direction for the case of this excitation. Source time function is a Ricker wavelet with a characteristic period of 2.5 sec. It is assumed that the diameter and depth of the sedimentary basin are \( a = 10 \text{ km} \) and \( d = 1 \text{ km} \), respectively, and S-wave velocities of inclusion and half-space are \( \beta_R = 0.9 \text{ km/sec} \) and \( \beta_E = 2 \text{ km/sec} \) with Poisson's ratios \( \nu_R = 0.3 \) and \( \nu_E = 0.25 \), respectively. In each left side of the figures, the cross-section of the basin along the x direction is shown. Namely, Fig. 3a and Fig. 3b show the results of sedimentary basin for the case of inclined medium interface being 90° and 45°, respectively. It is seen that body waves, i.e., SH waves, first arrive at the sites and then Love waves generated from the basin edge propagate forward along the longitudinal direction and propagate backward reflected from the other basin edge. It is also seen that the body waves decay with the increase of source-receiver distance, Love waves do not decay because the material property of the basin is assumed to be a perfect elastic medium. It is noted that since we calculated those responses approximating the medium interface with 936 pieces of plane elements, the maximum size of computer core memory used was over 400 MB.

Fig. 4 shows an example of the non-uniqueness of the solution to the integral equations for certain values of frequency (Nowak and Hall, 1993). It shows that solutions for the displacements on the canyon surface reveal an artificial resonance phenomenon when solving the exterior problem in the frequency domain. The use of
an additional source loading in the boundary element method is shown to eliminate these resonances and yield accurate results (Fig. 5).

CONCLUSIONS

From the preceding discussion the following conclusions can be drawn:

1) As the BEM reduces the number of dimensionality by one, it needs less computer core memory than domain type of numerical methods. However, it costs much time, because numerous matrix elements involving Green's functions and corresponding tractions for the elastic half-space should be evaluated.

2) As artificial boundaries which are indispensable to domain type of numerical methods such as FEM and FDM are not required for the BEM, there exists no restriction on modeling various types of incident waves such as non-vertically propagating plane seismic waves as well as incident seismic waves generated from point sources and double couple sources.

3) Though the real features of topographic sites simulated by the BEM are restricted to simple configurations, the method yields accurate solutions which provide benchmarks by which alternate solution methods can be properly judged.

4) As the BEM has been restricted to linear elastic or viscoelastic material behavior and small deformations, the development of effective hybrid schemes which combine the BEM and the FEM is required.

5) The development of innovative computer schemes of the BEM adaptive to a rapid progress of supercomputers and parallel computers is required.

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REFERENCES


![Fig. 1. Sedimentary basin model. The interior medium 1 enclosed by the boundary S has different material properties from the exterior medium 2 surrounding the boundary S.](image)

![Fig. 2. Surface displacement amplitudes $u_\theta$ for central point of a semi-spherical sedimentary basin with radius $a$ versus normalized frequency $\gamma$ due to incident plane SH waves. Incidence of plane SH waves with incidence angle $\gamma = 30^\circ$. They are compared with the results of Sanchez-Sesma et al.(1989) indicated by solid circle. (Shinozaki and Yoshida, 1994)](image)
Fig. 3. Time-domain responses for surface displacement $u_y$ at 79 sites equally spaced along the $x$ axis of the 3-D sedimentary basin due to a point source excited in the $y$ direction located at (-10km, 0, 5km). Source time function is a Ricker wavelet with a characteristic period of 2.5 sec. (Shinozaki and Yoshida, 1996)
Fig. 4. Displacement response of canyon surface showing artificial resonance (Poisson's ratio $\nu = 0.2$). (Nowak and Hall, 1993)

Fig. 5. Displacement response using the additional source (Poisson's ratio $\nu = 0.2$). (Nowak and Hall, 1993)