STOCHASTIC PROCESS MODELING AND SIMULATION OF STRONG-GROUND-MOTION ACCELERATIONS

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ABSTRACT
State of the art of stochastic process modeling, synthetics, statistical estimation and simulation of strong ground motion is reported in this paper. In particular, models for seismic source mechanism and earth medium are detailed, which are of paramount importance in synthesizing or simulating a strong earthquake ground motion. Statistical estimation and simulation techniques are also presented so that the nature of spatially-correlated earthquake motion is captured and consequently used in investigating seismic responses of such large scale structures as pipelines and long-span bridges. Numerical examples are shown for illustration.

KEYWORDS
Seismic Source Mechanism, Non Homogeneous Medium, Discrete Wave Number Method, Earthquake Motion Synthetics, Statistical Estimation, Loma Prieta Earthquake

INTRODUCTION
Earthquake ground motion is the result of seismically propagating waves in earth medium, which originate from a seismic source. Therefore, the development of realistic models for the earth medium and source mechanism plays a crucial role in the strong earthquake ground motion synthesis and simulation which provide fundamental information for the aseismic design of structures. The first part of this paper is thus devoted to explore seismologically consistent modeling for earth medium and seismic source mechanism. Uncertainties in both earth medium and seismic source are taken into consideration so that the nature of stochasticity in the earthquake ground motion is captured in the synthetics.

The second part of this paper deals with one of the most important engineering applications of the synthetics. That is to estimate from it such statistics of a strong ground motion as the frequency-wave number (F-K) spectrum and coherence function which characterize the temporal and spatial variations of a strong ground motion. These properties can then be used in simulating spatially-correlated earthquake ground motion, which in turn be used for earthquake-resistant structural design, especially for elongated structures such as pipelines and long-span bridges.
In the third and last part of this paper, applications of stochastic processes, fields and waves in earthquake engineering are described. Particularly highlighted is the simulation of the propagating seismic wave as a nonstationary stochastic vector process that can be made response spectrum-compatible and account for the specified seismic wave characteristics including coherency and nonhomogeneity.

SEISMIC SOURCE MECHANISM

The earth medium, in which the seismic waves propagate, is often assumed as either a half space of linear elastic medium or a layered half space. This assumption is considered adequate to represent the basic characteristics of the real earth medium in pursuit of many engineering applications. As long as the earth medium is thus specified, seismic source mechanism becomes a key in synthesizing the earthquake ground motion.

Barrier Model

Among various kinetic seismic source models, circular crack source mechanism is the best one physically as well as mathematically. Most of seismic faults are usually very large in size and can not be represented by a single circular crack, especially when the near field ground motion is to be investigated. Hence, Papageorgiou and Aki (1983) proposed a specific barrier model, in which a seismic fault is assumed to rupture consecutively in a series of circular sub-faults separated by unbroken barriers. This model has been applied, incorporating with a layered half-space, by Deodatis and Zhang (1993) to synthesize the ground motion. Specifically, a rectangular seismic fault plane is assumed to rupture consecutively in a series of circular sub-faults separated by unbroken barriers, as shown in Fig. 1. The ruptures begin at the center of one circular sub-fault and propagate radially with a rupture speed. When the ruptures reach the circumference of the circular sub-fault (barrier), healing fronts are initiated that propagate from the circumference toward center with a healing velocity. At the instance the healing fronts reach a given point on the circular fault, the slip at that point is stopped, which fixes the final dislocation (slip) at that position. When the ruptures of the circular sub-fault hit its circumference, the second circular sub-fault next to the first one is triggered, which again begins at the center of the circular sub-fault and propagates radially. Such a procedure will continue until the ruptures on the last circular is cracked. Recently, a stochastic barrier model was developed by Deodatis et al. (1996) to capture the nature of randomness of realistic source mechanism. Specifically, a barrier model with irregular boundaries has been utilized to calculate the distribution and the amount of the final slip over the fault plane. The basic idea is that an underlying barrier model with regular circular boundaries is transformed into a corresponding one with irregular boundaries using the spectral representation method (Shinozuka, and Deodatis, 1991) for simulation of stochastic fields. The distribution and amount of final slip over the fault plane using a stochastic barrier model with irregular boundaries is shown in Fig. 2.

Seismologically-Consistent Source Model

While barrier mechanism can be used as a general model for seismic source, a specific earthquake source model can also be established by solving inverse problem with the aid of earthquake ground motion records as well as wave propagation theory.

For the specific earthquake of Loma Prieta of magnitude 7.1 struck at the San Francisco Bay area on October 17, 1989, the seismic source mechanism modeling and/or ground motion prediction have been made. One of the best models was proposed by Zeng et al (1991, 1993) using ray method and near field earthquake records (14 observation sites within 35 km around epicenter), in which the distribution of final slip on the fault plane is predicted. It is noted that only direct arrivals of S waves are taken into account in the ray method used in Zeng's work for the sake of convenience and simplicity.
in computations. However, the results obtained through such an approach are acceptable from the engineering point of view because the direct propagating S waves dominate the seismic travelling wave energy, compared with those carried by P waves and multi-reflected body waves. Again, surface waves are not well developed at the near field. Recently, Zhang and Deodatis (1996) successfully synthesized both near- and far-field Loma Prieta earthquake in a layered half-space with the seismic source model proposed by Zeng et al. (1991, 1993) buried in one of layers. Specifically, the earth model is based on geological profiles of the Santa Cruz mountain area and consists of three layers overlaying a half-space. The seismic source mechanism is assumed to consist of a bilaterally propagating shear slip over a rectangular fault. The fault has a length of 40 km along the strike direction (130° clockwise from the north direction) and a width of 14 km along the dip direction (70° down from the horizontal). The total slip distribution is plotted in Fig. 3. With the aid of the compute code "SEISMO" developed at Princeton University in which the discrete wave number method is used, the ground motion field can then be found. Parenthetically, the synthesized ground motion has been compared with the actual records of the Loma Prieta earthquake with the use of a bandpass filter. The comparison indicates that the synthesized and observed ground motions are basically consistent with respect to magnitude (intensity), wave form (frequency content) and time duration.

SYNTHETICS OF EARTHQUAKE GROUND MOTION

Solution with A Layered Half-Space

Among the techniques applied to analyze earthquake wave propagation, especially in a layered half space, discrete wave number method is perhaps most convenient and applicable, which not only takes into account all kinds of travelling waves, i.e. body waves and surface waves, but also minimises numerical computation problems. In particular, this methodology is developed on the basis of the work of Lamb (1904), Bouchon (1979), Choné (1987), and Dunkin (1965). The discrete wave number technique is used to propagate waves due to the rupture of an extended seismic source through a 3-D layered half-space. The method is deterministic in the description of the wave propagation. It can be probabilistic or deterministic in the description of the rupture of the seismic source. With this method, it is possible to calculate the near-field and the far-field seismic ground motion at any point of a layered viscoelastic half-space, such that the spatial variability of ground motion at distances comparable to the dimensions of engineering structures can be estimated. The extent and magnitude of permanent ground deformation can also be computed, which is very important in the earthquake response of large scale engineering structures with relatively low natural frequencies of vibration, such as long-span bridges and base-isolated structures. In practical computation, the Green function, i.e. the response of ground motion to an effective point source associated with a double couple, may be found first using a propagator-based formalism, in which the wave radiation from the source is then decoupled into P-SV and SH motions and the two problems are solved separately. The earthquake ground motion generated by the aforementioned seismic source mechanism is then synthesized by taking into consideration the evolution of the slip and Green's functions at a certain location on the fault plane.

As an example, Loma Prieta earthquake ground motion was synthesized at a dense grid of observer locations, as shown in Fig. 4. The generation and propagation of different kinds of seismic waves, the spatial variability of ground motion, as well as the development of the permanent (static) ground deformation, can be examined by carefully studying the plots in Fig. 4.

For the scenario earthquake, a portion of the San Andreas fault relative to the San Francisco Bay area would rupture, as shown in Fig. 5. Three different cases of circular rupture patterns are considered. These three cases, displayed graphically also in Fig. 5, differ in the location of the hypocenter. Dis-
placement traces and particle motions are computed at the seven locations indicated in Fig. 6 using a stochastic barrier model depicted in Fig. 2. The first five points correspond to the abutments of two bridges in the area, point 6 is in San Francisco, and point 7 is in Oakland. Figure 7 displays the traces of particle motions on the horizontal plane at points 3, 4 and 5. These results are highly useful for a sensitivity analysis to examine the structural response characteristics.

Effects of Lateral Non-Homogeneities

In investigating the seismic wave propagation and its correspondingly induced ground and/or underground motion responses, the earth is often modelled as a vertically non-homogeneous medium, idealized as a layered half-space with each layer being homogeneous. The lateral non-homogeneities of the earth medium such as irregular surface, irregular interfaces and laterally non-homogeneous layer properties have, however, not considered until recent two decades. The primary reason behind it is perhaps that the introduction of lateral non-homogeneities into the earth medium model of a layered half-space results in the complexity of the problem not only mathematically but also physically.

Specifically, wave scattering occurs in lateral non-homogeneous medium, resulting in the coupling between P-SV and SH waves, which is neither the case of a perfectly layered half-space (without presence of the lateral non-homogeneities) nor the case of a layered media with two-dimensional lateral non-homogeneities. The existence of the lateral non-homogeneities in the earth medium not only affects the seismogram envelopes, response spectra, and power spectra of the ground and/or underground motion to a certain extent, but also is responsible for the generation of coda waves. Because of this, the issues related to the lateral non-homogeneity received increasingly more attention in such fields as earthquake and structural engineering as well as geophysics and seismology recently. Extensive review in this subjected may be seen in Zhang (1994), Zhang and Shinozuka (1996d) and Zhang et al. (1996a,b)

Since P-SV and SH wave motions in the earth medium with 3D non-homogeneities are coupled and no longer analyzed separately, a first order perturbation approach is applied (see, e.g. Zhang and Shinozuka 1996) to solve the 3D wave scattering problem associated with laterally non-homogeneous earth medium. Specifically, the total wave field, generated by a seismic dislocation source buried in the layered half-space, is decomposed into two wave fields. One is a mean wave field, which is a response field in a perfectly layered half-space subjected to a seismic dislocation source. This can be solved using discrete wave number method. The other is a scattered wave field, which is due to the existence of laterally non-homogeneous medium. The effects of the non-homogeneities on the scattered wave field are equivalent to those of fictitious discontinuity sources acting on the perfectly plane boundaries in case of irregular boundaries or those of fictitious distributed body forces that mathematically replace the lateral inhomogeneities. The intensity of the fictitious forces depends on both the mean wave response field and the lateral non-homogeneities. The solution for the scattered wave field is then obtained using the same approach as for the mean wave field.

As an example, two types of non-homogeneous layer properties are investigated and the perturbed parts of S wave speed, which is a function of lateral coordinates, are shown in Fig. 7. The ξ-direction ground accelerations to a point source at selected observation site with and without the presented lateral non-homogeneities are depicted correspondingly in Fig. 7. It can be seen from Fig. 7 that the effects of "smooth" lateral non-homogeneities on the ground responses are primarily on the variation in peak acceleration, while the effects of "rough" lateral inhomogeneities on the ground responses are primarily on the broadening of the seismogram envelopes. These phenomena are consistent with Aki (1975) and Sato (1984).
STATISTICS OF STRONG GROUND MOTION

The spatial variation of earthquake ground motion may have non-negligible effects on the elongated structures such as bridges, underground pipelines etc. While the ground motion is non-stationary in time and non-homogeneous in space, the essential feature of the spatial variation are usually captured by idealizing it as stationary and homogeneous functions of time and space at least within an appropriate time-space window (composed of a space window and a time window), which may be seen schematically in Fig. 8. Only then, it is possible to examine the effects of the spatial variation of the ground motion on the structural responses by taking advantage of such quantities as the frequency-wave number (F-K) spectrum, cross-spectral density function matrix, coherence function and the like. It is noted that these quantities are defined for stationary and homogeneous functions with the physical interpretations pertinent to the structural systems subjected to the ground motion thus idealized. In fact, once the idealization is completed, the F-K spectrum can sufficiently describe the nature of such spatial variation, from which other degenerate quantities such as coherence function can be obtained. Given the F-K spectrum, the corresponding seismic ground motion can be numerically simulated (Shinozuka and Deodatis 1988, 1991 and Deodatis and Shinozuka 1989), which can be used for carrying out the time-space domain structural analysis, particularly non-linear structural analysis.

In many engineering applications, a time-space window can be appropriately selected so that the ground motion may be assumed to be both stationary and homogeneous within this window (e.g. Fig. 8). Further, it is usually assumed that the ground motion in the window is ergodic in both time and space. Then, within the framework of probabilistic mechanics and statistics, the F-K spectrum can be found (see Zhang and Shinozuka, 1996c), which essentially contains all the information of the temporal and spatial statistics of the ground motion in the selected window. Some major features of the ground motion may be seen directly from the F-K spectrum. For instance, the dominant wave type and its propagation direction can be judged on the basis of peak locations in the F-K spectrum plot. The other degenerate statistics of the ground motion such as the cross-spectral density function, the power spectral density function and the frequency-dependent coherence function can be obtained mathematically in terms of the F-K spectrum.

As an example, the statistics of the Loma Prieta earthquake ground motion in a local area (a time-space window) are presently calculated and discussed. The selected window is centered at \((t_0=20 \text{ sec}, x_0=17 \text{ km}, y_0=10 \text{ km})\) with window lengths being \((T_W=15 \text{ sec}, X_W=Y_W=15 \text{ km})\). The coherence functions of the ground acceleration in the selected window at \(\omega=5, 10, 20 \text{ and } 30 \text{ rad/sec}\) are computed, which are displayed in Figs. 9 and 10. As seen in Fig. 9, the maximum value of the F-K spectra is much small at both low \((\omega = 5 \text{ rad/sec})\) and high \((\omega = 30 \text{ rad/sec})\) frequencies, compared with that at \(\omega=10 \text{ and } 20 \text{ rad/sec}\), which indicates the dominant energy carried by the acceleration in the window is around \(\omega=10\) or 20 \text{ rad/sec}. Fig. 10 shows that as the separation distance gets large, the coherence decays both exponentially in an oscillatory fashion at a given frequency and more quickly at the high frequency than at the low frequency as expected. The coherence characteristics observed from the present numerical examples are basically consistent with those obtained using actually earthquake records observed by SMART 1 array, as seen in Loh and Yeh (1988) and Loh, C.H. (1991).

SIMULATION OF STRONG GROUND MOTION

Several methods are currently available to solve a large number of problems in mechanics involving uncertain quantities described by stochastic processes, fields or waves. At this time, however, Monte Carlo simulation appears to be the only universal method that can provide accurate solutions for certain problems in stochastic mechanics involving nonlinearity, system stochasticity, stochastic stability, para-
metric excitations, large variations of uncertain parameters, etc., and that can assess the accuracy of other approximate methods such as perturbation, statistical linearization, closure techniques, stochastic averaging, etc.

One of the most important parts of the Monte Carlo simulation methodology is the generation of sample functions of the stochastic processes, fields or waves involved in the problem. The generated sample functions must accurately describe the probabilistic characteristics of the corresponding stochastic processes, fields or waves that may be either stationary or non-stationary, homogeneous or non-homogeneous, one-dimensional or multi-dimensional, uni-variate or multi-variate, Gaussian or non-Gaussian. Among the various methods that have been developed to generate such sample functions, the spectral representation method is one of the most widely used today. Although the analytical expression existed in a most primitive form with no implication to Monte Carlo simulation applications (Rice 1954), it was Shinozuka (Shinozuka and Jan 1972, Shinozuka 1972) who first applied it for simulation purposes including multi-dimensional, multi-variate and non-stationary cases. For details on the subject of simulation using the spectral representation method, readers are referred to Shinozuka (1987) and Shinozuka and Deodatis (1988, 1991, 1996).

Recently, a spectral-representation-based method was proposed by Deodatis (1996) to simulate non-stationary stochastic vector processes with evolutionary power. If the components of the vector process correspond to different locations in space, then the process can also be non-homogeneous in space (in addition to being non-stationary in time). For important applications of earthquake ground motion simulation, acceleration, velocity, or displacement time histories can be generated at several locations on the ground surface according to a target cross-spectral density matrix. This study also introduced an iterative scheme, based on the proposed simulation algorithm, to generate seismic ground motion time histories at several points on the ground surface that are (a) compatible with prescribed response spectra, (b) are correlated according to a given coherence function, include the wave propagation effect, and (c) have a specified duration of strong ground motion.

In order to demonstrate the unique capabilities of the aforementioned algorithm to simulate non-stationary stochastic vector processes, an numerical example involving simulation of earthquake ground motion is selected. In particular, ground motion time histories are modeled as a uniformly modulated non-stationary stochastic vector process, and sample functions will be generated according to the acceleration response spectra specified by the Uniform Building Code (International Conference of Building Officials 1994). Specifically, the acceleration time histories at three points on the ground surface along the line of main wave propagation are considered to be a tri-variate non-stationary stochastic vector process. The configuration of the three points is shown in Fig. 11 where the arrow indicates the direction of wave propagation. The simulated earthquake ground motion at points 1, 2 and 3 will have the following characteristics:

(i) Points 1, 2 and 3 correspond to different local soil conditions. Specifically, point 1 corresponds to the Uniform Building Code’s Soil Type 1 (rock and stiff soils), point 2 to Soil Type 2 (deep cohesionless or stiff clay soils), and point 3 to Soil Type 3 (soft to medium clays and sands). This is a unique capability of the method, being able to simulate ground motion at neighboring points on the ground surface with different local soil conditions and consequently different frequency contents. Such a case is encountered, for example, at the supports of intermediate to long-span bridges located in areas with abrupt changes in soil conditions along the axis of the bridge.

(ii) In addition to different frequency contents at points 1, 2 and 3, the acceleration time histories at these three points will also be correlated according to a prescribed coherence function and they will reflect the wave propagation effect. This is another unique capability of the method, being able to simulate ground motion time histories that, at the same time, are spatially correlated, include the wave
propagation effect and correspond to different local soil conditions.

(iii) Finally, the acceleration time histories at points 1, 2 and 3 will reflect the non-stationary characteristics of ground motion according to specified modulating functions.

One sample function for the acceleration at points 1, 2 and 3, respectively, is generated after 10 iterations and displayed in Fig. 11. It is possible to detect the following characteristics in the time histories shown in Fig. 11: (i) their mutual correlation according to the coherence functions, (ii) the wave propagation effect according to a velocity of wave propagation, (iii) their amplitude variation as a function of time. Fig. 12 presents comparison between the acceleration response spectra computed using the ground motion time histories shown in Fig. 11 and the target UBC response spectra, showing an excellent match at every frequency.

CONCLUSIONS

Seismologically consistent modeling for earth medium and seismic source mechanism is investigated, in which uncertainties in both earth medium and seismic source are taken into consideration. The statistics of the synthesized strong ground motion, characterizing the temporal and spatial variations of a strong ground motion, are also estimated, which can then be used in simulating spatially-correlated earthquake ground motion. Finally, applications of stochastic processes, fields and waves in earthquake engineering are highlighted with emphasis on the simulation of the propagating seismic wave as a nonstationary stochastic vector process.

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REFERENCES


**Figure 1:** A specific (deterministic) barrier model with regular boundaries.

**Figure 2:** A stochastic barrier model with irregular boundaries.

**Figure 3:** Slip distribution over a rectangular fault in a seismologically-consistent Loma Prieta earthquake source model.
Figure 4: Synthesized displacement fields in the strike direction of Loma Prieta earthquake
Figure 5: Location of rupturing segment of the San Andreas fault and the seven points considered in the study.

Figure 6: Traces of particle motions on the horizontal plane at points 3, 4 and 5.
Figure 7: Effects of lateral non-homogeneities on earthquake ground motion.

Figure 8: Schematic of a time and a space windows selected in a strong ground motion.
Figure 9: F-K spectra of the Loma Prieta earthquake ground acceleration in the strike direction.

Figure 10: Coherence function of the Loma Prieta earthquake ground acceleration in the strike direction.
Figure 11: Generated acceleration time histories at three points.

Figure 12: Acceleration response spectra computed using the generated acceleration time histories versus the UBC acceleration response spectra.