

# DUCTILITY EVALUATION OF STEEL FRAMES WITH PR CONNECTIONS

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#### ABSTRACT

Nonlinear seismic response analysis of steel frames with fully restrained and partially restrained connections are evaluated in terms of ductility and interstory displacement. At present, there is no engineering definition of ductility in the specifications and codes, although it is always used in the profession. Several definitions of ductility, particularly for multi degree of freedom systems, are suggested in this study. The presence of partially restrained connections in steel frame is considered, and its effect on the ductility evaluation is studied.

### **KEYWORDS**

Ductility; story ductility; global ductility; local ductility; nonlinear analysis, partially restrained connections; strong motion earthquakes

#### INTRODUCTION

The damage suffered by steel structures in a recent strong earthquake forced the profession to reevaluate issues related to the seismic design of steel structures. The evaluation of the maximum inelastic deformation of a structure subjected to a strong motion earthquake is a critical part of this process. A ductility parameter can also be used for this purpose. It is particularly important for steel structures since the considerable beneficial effect of ductility can be utilized to mitigate the seismic hazard. In a recent report (1995), the SAC (Structural Engineers Association of California, Applied Technology Council, and California Universities for Research in Earthquake Engineering) stated, "Ductility is shown in parentheses to emphasize that there is no definition of ductility in our Specification and Codes but is always being used. The metallurgical definition of ductility is the ability of a metal to be stressed beyond its yield strength and into its plastic (inelastic) range, with large elongations before rupturing in a ductile mode. An engineering definition of ductility may be needed, as related to moment-resisting frames design and construction."

Ductility can be conceptually defined as the ratio of the inelastic displacement to the yield displacement. In the context of the seismic analysis of single degree of freedom (SDOF) systems, the yield displacement can be defined as the displacement of the system when it yields for the first time, and the maximum inelastic displacement is the maximum displacement that the system undergoes during the application of the complete

earthquake. However, using this definition of ductility, it is not easy to calculate ductility for seismic loading using experimental results, even for a SDOF system. A typical response time history of a structure will be extremely irregular, similar to earthquake excitation. Thus, the ductility can not be quantified from the response time history alone. When the response time history is plotted, it indicates the presence of some ductility, but can not quantify it. It is very important to appropriately define these two displacements. A sophisticated analytical procedure is necessary to identify the deformation when the first hinge develops in the structure. The task of ductility evaluation for multiple degree of freedom (MDOF) systems is extremely complicated, and there is no unanimity in the profession on how to define it. Therefore, various definitions of ductility are possible, as discussed below.

To calculate the maximum inelastic deflection, design guidelines usually suggest simplified versions of complicated analysis procedures. According to the National Earthquake Hazard Reduction Program (NEHRP), the maximum inelastic deflection can be estimated by calculating design deflecton using elastic analysis and then multiplying it by the deflection amplification factor  $C_d$ . Denoting the response modification factor as R, and without showing the detailed mathematical derivation, the ratio  $C_d/R$  for SDOF systems is expected to be greater than or equal to 1.0 according to Newmark and Hall (1982). Assuming the concept is also applicable to MDOF systems, the ratio for steel framing systems is between 0.5 and 0.9 at present according to NEHRP. The ratio is 1.0 for other similar codes, e.g., Eurocode No. 8 (1988) and Mexico Building Code (1987).

The preceeding definition of ductility is appropriate for story ductility. It is also necessary to properly define local and global ductility and establish appropriate relationships among these parameters. Here, a distinction is made between the ductility of a member such as the rotation at a joint in a flexural member, the ductility of a floor or story, and the overall ductility of a structure. The general definition of ductility used in story ductility is also applicable to the case of local ductility. However, now the maximum inelastic displacement can be the maximum inelastic longitudinal displacement in a tensile member, the rotation at a joint in a flexural member, or the total shearing deformation in a shear wall; while the yield displacement will be the corresponding displacement when the first yield occurs in the member. The global ductility can be some weighted average of the story ductilities. The member ductility can be considerably higher than the story ductility, which in turn may be higher than the global ductility.

In addition to the lack of understanding of the important parameter ductility, there are many misconceptions in the profession regarding the analysis and design of steel frames. The AISC LRFD Specification explicitly recognizes two types of steel construction: fully restrained (FR) and partially restrained (PR) or semi-rigid. However, despite these classifications, almost all steel connections used in practice are in fact essentially PR connections with different rigidities. The FR connection is an assumption made to simplify calculations, and is a major weakness in current analytical procedures. For nonlinear seismic response analysis, proper consideration of the rigidity of connections is essential, no matter how difficult the analysis procedure becomes. The implication is that classical frame analysis procedures may not be applicable to the seismic analysis of steel frames.

In the analysis of steel structures, structural members are usually assumed to be connected by FR connections. However, it has been established in the profession both theoretically and experimentally that these connections are PR connections even when the applied loads are very small. Thus, the inelastic response and the associated ductility of steel frames considering both geometric and material nonlinearities in the presence of PR connections has yet to be taken into proper consideration. The AISC LRFD Specification does not even address the issue. Other model building codes like the Uniform Building Code try to address the issue in a very arbitrary and indirect way by introducing some parameters. Therefore, it is extremely important that the inelastic dynamic behavior of steel frames with PR connection be understood properly, a measure of ductility be introduced explicitly, and design codes be modified so that one of the most desirable properties of steel can be utilized in addressing the safety of steel structures during large earthquakes.

## METHODS AND MATHEMATICAL MODELS

Over the last decade, the PI and his associates developed a very basic yet efficient and robust finite element-based time-domain nonlinear analysis procedure to study the inelastic dynamic response behavior of braced and unbraced steel frames with PR connections (Gao and Haldar, 1995). The procedure estimates nonlinear seismic responses of steel frames with PR connections considering all major sources of energy dissipation, such as the kinetic energy that includes the rigid body translation of the structures, the viscous damping energy, the recoverable elastic strain energy, the irrecoverable hysteretic energy, and the irrecoverable energy dissipation in the flexible connections. For frames with PR connections, the procedure considers as realistically as possible the geometric nonlinearity including the  $P - \Delta$  effect due to drift, as well as the change in member lateral stiffness due to the effect of axial force, the change in member length due to the bowing effect and axial force, and the finite rigid body deformation with small to moderate relative rotation of a member. Material nonlinearity is also considered.

The comprehensive properties of a PR connection are generally represented by a moment-relative rotation (M- $\Theta$ ) curve. Several analytical expressions have been proposed by different researchers to represent the M- $\Theta$  curve. These include the piecewise linear model (Razzag, 1983), the polynomial model (Frye and Morris, 1975), the exponential model (Liu, 1985), the B-spline model (Jones at. al., 1982) and the Richard model (Richard and Abbott, 1975). In this study the four-parameter Richard model is used. This model is expressed as:

$$M = \frac{(K - K_p)\theta}{(1 + \left| \frac{(K - K_p)\theta}{M_0} \right|^N)^{\frac{1}{N}}} + K_p\theta$$
 (1)

where M is the connection moment,  $\Theta$  is the connection rotation, k is the initial or elastic stiffness,  $k_p$  is the plastic stiffness,  $M_0$  is the reference moment, and N is the curve shape parameter. Equation 1 represents the monotonically increasing loading section of the M- $\Theta$  curves. For seismic analysis, the unloading and reloading behavior of the M- $\Theta$  curves is also essential. This subject was addressed recently in the literature (Gao and Haldar, 1995). Using the Masing Rule and the Richard model represented by Eq. 1, the mathematical model for the unloading and reloading behavior of a PR connection used in this study is given by:

$$M = M_a - \frac{(K - K_p)(\theta - \theta_a)}{(1 + \left| \frac{(K - K_p)(\theta - \theta_a)}{2M_0} \right|^N)^{1/N}} - K_p(\theta - \theta_a)$$
 (2)

where  $(M_a, \Theta_a)$  denotes the load reversal point.

The solution strategy for the nonlinear problem under consideration is iterative. In each iteration, the following equation needs to be solved:

$$K \Delta D^{(k)} = F^{(k)} - R^{(k-1)} \tag{3}$$

where  $K^{(k)}$ ,  $\Delta D^{(k)}$  and  $F^{(k)}$  are the tangent stiffness matrix, displacement vector, and external load vector at the kth iteration, respectively; and  $R^{(k-1)}$  is the internal force vector at the (k-1)th iteration. Equation 3 with the tangent stiffness matrix consisting of geometric and material nonlinearities and nonlinearities due to PR connections can be solved using the modified Newton-Raphson method with arc-length procedure.

The dynamic and seismic governing equations of motion for the problem under consideration can be obtained by modifying Eq. 3 by adding terms for the inertia, damping, and applied forces. The equilibrium equations for the nonlinear dynamic and seismic analysis in the incremental form can be expressed as:

$$M^{t+t\Delta t} \dot{D}^{(k)} + {}^{t}C^{t+\Delta t} \dot{D}^{(k)} + {}^{t}K \cdot {}^{t+\Delta t}\Delta D^{(k)}$$

$$= {}^{t+\Delta t}F^{(k)} - {}^{t+\Delta t}R^{(k-1)} - M \dot{D}_{g}^{(k)}$$
(4)

where M, C, and K are the mass, damping and stiffness matrices of the system; 'K is the tangent stiffness matrix of the system at time t;  $^{t+\Delta t}\Delta D$  and  $^{t+t\Delta t}F^{(k)}$  are the incremental displacement vector and the external dynamic load vector of the kth iteration at time  $t+\Delta t$ , respectively;  $^{t+\Delta t}R^{(k-1)}$  is the internal force vector of the (k-1)th iteration at time  $t+\Delta t$ ; and  $\dot{D}_g$  is the seismic ground acceleration vector. The step-by-step direct integration numerical analysis procedure using the Newmark  $\beta$  method is used to solve Eq. 4 (Haldar and Nee 1989). Since actual time histories of earthquakes are used in this study, the inertia and the applied force are available. However, the damping is an important parameter which needs further discussion at this stage. In a realistic seismic analysis of steel frames with PR connections, the amount of damping energy that will be generated will depend on the nonyielding and yielding state of the material, the hysteretic behavior if the material yields, and the moment-rotation behavior of the connections. For mathematical simplicity, the effect of nonyielding energy dissipation is generally represented by equivalent viscous damping varying between 0.1% to 7% of the critical damping. It is observed that the damping is often increased in linear analysis to approximate energy losses due to anticipated inelastic behavior (Leger and Dussault, 1992). Based on an extensive literature review, it can be concluded that the following Rayleightype damping is very commonly used in the profession:

$$C = aM + bK_1 + b_0 K_0 (5)$$

where  $K_t$  is the tangent stiffness matrix;  $K_0$  is the initial stiffness matrix; and a, b, and  $b_0$  are the proportional constants. Various representations of Eq. 5 can be found in the literature. The use of both the elastic stiffness and the tangent stiffness is a very rational approach to estimating the energy dissipated by viscous damping in a nonlinear seismic analysis.

#### **DUCTILITY DEFINITIONS**

As stated earlier, the ratio of the maximum inelastic displacement  $(D_{max})$  and the yield displacement will provide information on ductility. A properly designed steel structure is expected to yield or develop plastic hinges when subjected to strong motion earthquakes. Using the algorithm developed in this study, the ductility for a particular building subjected to a particular earthquake can be calculated. This algorithm identifies the location of plastic hinges as they develop in the structure. Thus, as soon as the first plastic hinge develops, the corresponding displacement will be noted as the yield displacement  $(D_y)$ . After the application of the complete time history of the earthquake, and if at least one plastic hinge is formed in the structure, the maximum inelastic displacement  $(D_{max})$  can also be noted. The ratio of these two displacements will give the ductility.

In order to evaluate the ductility as accurately as possible, the information on the two displacements ( $D_y$  and  $D_{max}$ ) must be available. For a SDOF structure, they can be estimated very accurately by the algorithm discussed earlier and the corresponding ductility can be estimated very easily. For MDOF structures, the evaluation of ductility is more complicated. To study the problem as comprehensively as possible, the following alternative definitions of ductility are considered for MDOF systems:

Definition 1: For each story,  $D_y$  is the interstory lateral displacement when the first plastic hinge forms, and  $D_{max}$  is the maximum interstory lateral displacement after the application of the complete time history of the earthquake. Note that this definition gives a value of story ductility for each story by finding the corresponding ratio  $D_{max}/D_y$ .

Definition 2: For each story,  $D_y$  is the absolute lateral displacement when the first plastic hinge forms, and  $D_{max}$  is the maximum absolute lateral displacement after the application of the complete time history of the earthquake. This definition also give a value of ductility for each story by finding the ratio  $D_{max}/D_y$ .

Definition 3: A single value of  $D_y$  applies to all stories, corresponding to the maximum of the absolute lateral displacement when the first plastic hinge is developed.  $D_{max}$  is defined as in Definition 2.

Definition 4: In this case, D<sub>y</sub> is the absolute lateral displacement of the floor where the first plastic hinge forms. It will be same for all the floors. D<sub>m</sub> is defined as in Definition 2.

Definition 5: Here  $D_v$  is evaluated using Definition 3 and  $D_{max}$  is evaluated using Definition 1.

Definition 6:  $D_y$  is the same as in Definition 4, and  $D_{max}$  is the same as in Definition 1.

It must be emphasized that ductility demand is different from ductility capacity. Ductility demand is the ratio of the maximum inelastic displacement of a structure or member during the application of the seismic loading and the yield displacement when the first yield occurs, while ductility capacity is the ratio of the maximum permissible inelastic displacement to the displacement when the first yield occurs.

Theoretically, ductility capacity should be reached when a collapse mechanism develops in the structure. To obtain this, it needs to be guaranteed that plastic moments are reached at positions of maximum moments before failure due to instability in a member or in a connection occurs. Moreover, local ductilities can not be exceeded; otherwise, the ductility corresponding to the collapse mechanism will not be the ductility capacity. For that reason, some researchers (Osteraas and Krawinkler, 1990) suggest using local ductility as the basis for design because there are numerous laboratory studies on ductility for members. In this regard, some relationships need to be established between local ductilities and global ductility.

### **DESCRIPTION OF STRUCTURES**

In order to evaluate the different definitions of ductility, three steel frame structures are considered in this study. These three frames are shown in Fig. 1. Frame 1 is a 3 story building with a story height of 18 feet for the first floor and 12 feet for the other floors. Frame 2 is identical to Frame 1, except that it is an eight story building (Charles et. al, 1993). Frame 3 is a 15 story building considered by Engelkirk (1994). These three frames are subjected to four strong earthquake motions. One is the N-S component of the El Centro earthquake of 1940, and the other three are from the Northridge earthquake of 1994 recorded at Canoga Park, Edgemont Avenue and Wilshire Blvd. They are denoted hereafter as Earthquakes 1 through 4. These earthquakes are selected to represent many different characteristics of strong motion earthquakes.

#### RESULTS AND OBSERVATIONS

Initially, all three frames are assumed to have FR connections. They are subjected to the four earthquakes and their nonlinear responses are estimated for 2% of critical damping using the algorithm discussed earlier. The corresponding ductility values are calculated using the six definitions discussed above. The results for Frame 1 subjected to the four earthquakes are shown in Figs. 2, 3, 4, and 5, respectively. It is assumed that the ductility capacity of the frame is reached when the local ductility reaches a value of 10. These figures indicate that the story ductility depends on the definition of ductility and the earthquake motions considered. Also, the ductility values may increase or decrease with height depending upon the definition used. The most important observation that can be made here is that the ductility values in terms of absolute displacement are larger than those of the interstory displacements.

Frame 2 subjected to the same four earthquakes is considered next. Fig. 6 shows the results for Earthquake 3 only. The results are similar to those of Frame 1. For Earthquake 3, the ductility values were found to be extremely large according to Definitions 1 and 2. However, "reasonable values" of story ductility are obtained according to Definitions 5 and 6. It is interesting to note that both of these definitions are in terms of relative lateral displacement.

Frame 3 is similarly analyzed, but the results can not be shown here. The observations are similar to those for Frames 1 and 2. The only additional observation is that the ductility values tend to be larger for taller buildings for all the definitions of ductility.

In order to study the effect of PR connections of steel frames on ductility evaluation, the same three frames are considered again. However, the connections are assumed to be PR type represented by Richard curves. Different amounts of connection flexibility are introduced by considering different ratios of  $M/M_{\rm fix}$ , where M is the moment according to the Richard curves and  $M_{\rm fix}$  is the fixed end moment of a beam. Three different ratios of  $M/M_{\rm fix}$ , i.e., 0.95, 0.9, and 0.8, are considered in this study.

Frame 1 with PR connections is again analyzed for the four earthquakes. However, in all cases no plastic hinge formed in the structure. Thus, only the maximum interstory displacements of Frame 1 for different connection rigidities, including when the connections are FR type, are plotted in Figs. 7, 8, 9, and 10 for the four earthquakes. It can be seen from these figures that the maximum interstory displacements of the

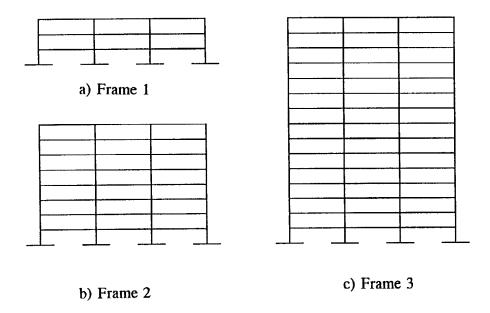


Fig. 1. Three steel frames

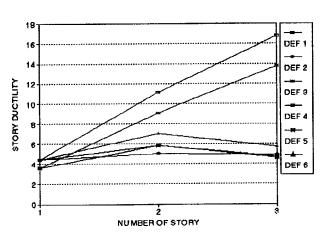


Fig. 2. Frame 1 with FR connections and Earthquake 1

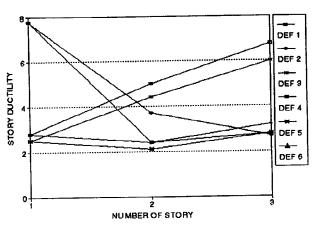


Fig. 3. Frame 1 with FR connections and Earthquake 2

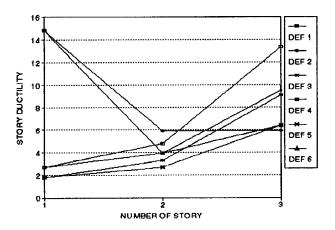


Fig. 4. Frame 1 with FR connections and Earthquake 3

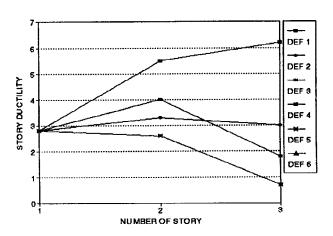


Fig. 5. Frame 1 with FR connections and Earthquake 4

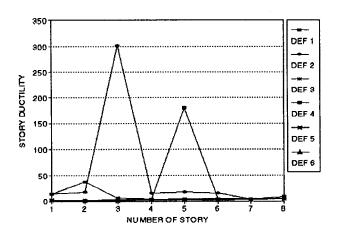


Fig. 6. Frame 2 with FR connections and Earthquake 3

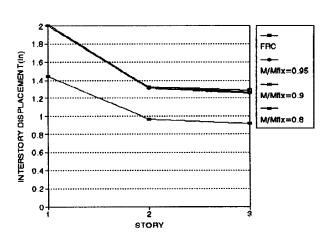


Fig. 7. Frame 1 and Earthquake 1

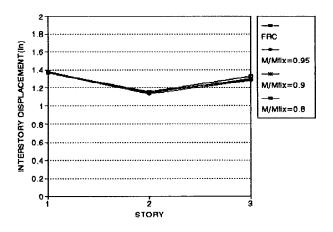


Fig. 8. Frame 1 and Earthquake 2

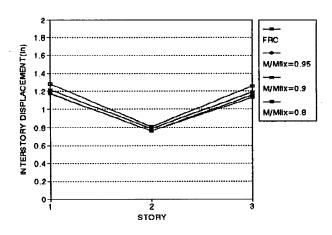


Fig. 9. Frame 1 and Earthquake 3

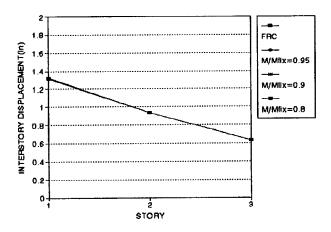


Fig. 10. Frame 1 and Earthquake 4

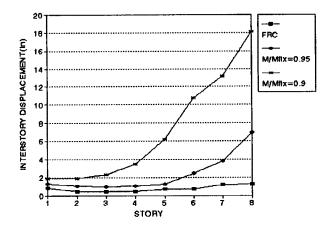


Fig. 11. Frame 2 and Earthquake 3

frame with FR connections are equal to or larger than those of the frame with PR connections.

Frame 2 is then considered with different PR connections. The interstory displacements for Earthquake 3 with different PR rigidities are shown in Fig. 11. In all cases, at least one plastic hinge formed in the frame. It is observed that the interstory displacements are much larger for the frame with PR connections than for the frame with FR connections. Also, as expected, the interstory displacements become larger as the rigidities of the connections decrease. If no plastic hinge forms in the structure, the maximum interstory displacements are virtually the same regardless of the stiffness of the connection. Results similar to Frame 2 were also observed for Frame 3.

#### CONCLUSIONS

The nonlinear seismic response of steel frames with FR and PR connections is evaluated in terms of ductility and interstory displacement. It is pointed out that at present there is no engineering definition of ductility in the specifications and codes, although it is always used. In this study, several definitions of ductility are proposed for MDOF systems. The presence of PR connections in steel frames is considered and its effect on the ductility evaluation is critically studied. A very sophisticated algorithm to evaluate nonlinear seismic response of steel frames already developed by the authors is used to estimate the information required for the ductility and interstory displacement calculations. In all these evaluations, the local ductility is limited to a value of 10.

Three steel frames with different connection rigidities are considered. Four recorded time histories of earthquakes are used in the nonlinear response calculations. It is shown that some definitions of ductility for MDOF systems may not be appropriate, even though they are being used in the profession. Defining ductility in terms of the interstory lateral displacement is more appropriate using the absolute lateral displacements. The results indicate that if no plastic hinge forms in the structure, the maximum interstory displacements are virtually the same regardless of the stiffness of the connections. Also, a ductility evaluation considering the rigidity of the connections gives quite different results than considering them to be FR type. No matter how difficult the problem becomes, a realistic consideration of connection rigidity is essential to estimate the nonlinear seismic response of steel frames.

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