

# A MORE ACCURATE METHOD FOR DETERMINING INELASTIC MOMENT DEMANDS ON SEISMIC MOMENT FRAME COLUMNS

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#### ABSTRACT

Inelastic time history analyses typically indicate that the traditional sub-assembly "capacity" approach required of most model building codes for use in the design of ductile moment frames grossly underestimates the maximum moments experienced by the columns during a maximum credible earthquake. In addition, these analyses predict that the maximum column demand moments often occur at approximately mid-height of concrete structures, whereas a conventional elastic analysis predicts maxima at the lowest levels of these structures. The reason for the discrepancy is that the global column displacements and corresponding demands generated during the earthquake are ignored when the only column demands are assumed to be generated by the restraint of the attached beams. The effects of the actual column displacements are amplified during an inelastic analysis because the columns will resist all additional displacement demands after the beams at the joint are rotating in a plastic state, therefore generating demands at an accelerated rate. This phenomenon can be demonstrated by incrementally displacing the structure into various critical deformed shapes while accounting for all inelastic beam activity, then determining the maximum column moments experienced. For original design purposes, incremental displacement analyses using modal properties and displacements predicted by a maximum credible response spectrum can be used to reliably predict the maximum anticipated column demand moments.

#### **KEYWORDS**

Inelastic moment demand, capacity design, ductile moment frame, sub-assembly mechanism, column design.

#### INTRODUCTION

The design of ductile moment frames of both concrete and steel currently strives for a "capacity" design, or a design that attempts to maintain column strength while promoting beam plastic hinging. The adjoining column is designed to remain strong during beam plastic hinging by employing a sub-mechanism "capacity" design approach, typically by assuming mid-story inflection points.

Unfortunately, non-linear inelastic time history analyses demonstrate that the sub-mechanism approach may underestimate the maximum column moment demands during a maximum credible earthquake by more than 100% in both steel and concrete structures<sup>[1][4][5]</sup>. The increase in column moment demand is described as "dynamic magnification," and texts such as Paulay and Priestley's<sup>[5]</sup> attempt to quantify this increase with the use of a magnification factor. The factor suggested is intended to be applied to the elastic frame moment distribution, and the factors were developed for concrete frames. While this may be practical for new

construction of concrete moment frames, designers need to understand that this is not unique to reinforced concrete. Engineers evaluating existing structures may require a more specific analysis technique in order to provide the most economical retrofit solution.

Consider an example ten story one-bay reinforced concrete moment frame (Figure 1) subjected to the 1994 Northridge Earthquake Newhall - L.A. County Fire Station 90 Degree Component<sup>[6]</sup>.

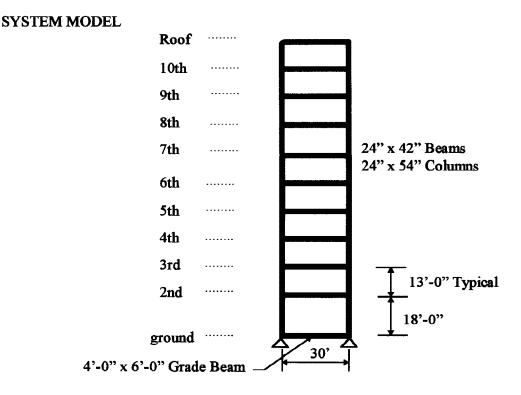


Figure 1. Example frame

The seismic weight proportioned to the frame is 500 kips at each level. The plastic capacity of the beams was determined by using the Equivalent Static Force Procedure of the 1994 Uniform Building Code<sup>[3]</sup> (UBC), increased by 1.4 (the load factor), by 1.25 (strain hardening, etc.), and divided by 0.9 (the capacity reduction factor). The structure is assumed to exist in seismic zone 4 with a site factor of 1.2. The same beam design was used for two levels before adjusting the reinforcement. The columns were modeled with infinite yield capacities in order to identify all of the potentially over-stressed columns. An inelastic non-linear analysis was performed on the frame using the program DRAIN-2DX and the acceleration ground record shown in Figure 2. Gravity loads on the frame members are assumed negligible.

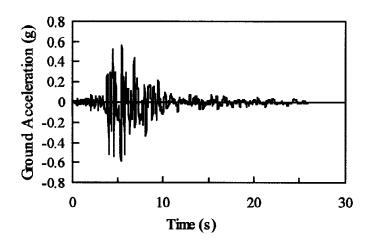


Figure 2. 1994 Northridge Earthquake, Newhall 90 degree component

When the frame is subjected to the earthquake, there are three significant time steps that together envelope all of the maximum moment response values. These steps occur at times 5.54, 8.49, and 10.47 seconds, and the corresponding deflected shapes of the structure are shown in Figure 3.

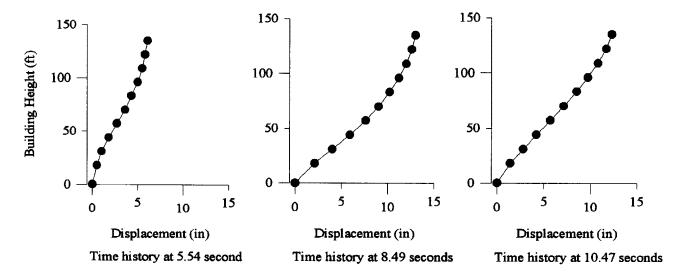


Figure 3. Critical deflected shapes during the time history analysis

The column moment diagrams corresponding to these time steps are shown in Figure 4. The maximum value experienced at each level is shown at the corresponding time in the analysis.

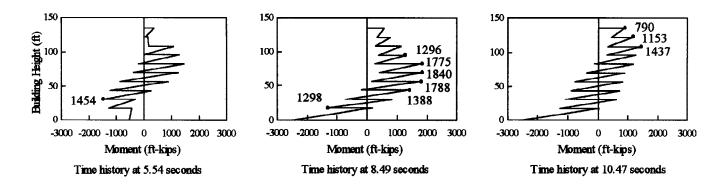


Figure 4. Column moments at critical times

## PREDICTING MAXIMUM COLUMN MOMENTS

To demonstrate the capability to predict the maximum column moments experienced during the time history analysis, the 6th level will be analyzed. This level corresponds to the largest moment experienced above the base, which is 1840 ft-kips at time 8.49 seconds. Referring to Figure 5, the shape at this time is not a clear mode shape but it definitely represents a significant first mode response. At this level the beam is yielding (along with most of the other beams), and its plastic moment capacity is 1635 ft-kips. A typical "sub-assembly mechanism" analysis assuming mid-story inflection points and the beam plastic capacity would estimate that the maximum column moment at the joint face would be 703 ft-kips (1635 ft-kips divided by 2 and reduced to the joint face). The difference results in an error of 162%, which clearly defines a need for a more accurate method of determining the maximum column moments.

The critical issue that is implicitly ignored by building codes is that the elastic column (assuming that the column does remain elastic) will have moment demands independent of the moments that the beams deliver. The column will continue to displace and change the demand even after the beam at that level has reached its plastic capacity. The best way demonstrate this is to place the column only, unrestrained by any beams, in the same displaced shape as the structure. The moments produced become the underlying "basic" moment diagram to which the moments generated from beam plastic activity (the traditional sub-assembly mechanism moments) are added. The deflected shape of the example frame at time 8.49 seconds is shown in Figure 5, and the resulting "bare" column moments in Figure 6. The column properties used in this analysis were consistent with those used in the time history analysis.

When this approach is applied to the 6th level the maximum demand reduced to the joint face is estimated as follows:

$$M_{max} = M_{(basic column)} + M_{(from plastic beam)}$$
  
= 1137 ft-kips + 703 ft-kips = 1841 ft-kips,

which is virtually identical to the time history moment of 1840 ft-kips at this time (Figure 4).

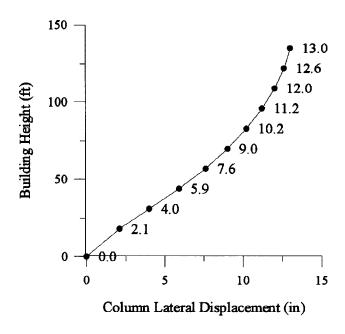


Figure 5. Story lateral displacements at time 8.49 seconds

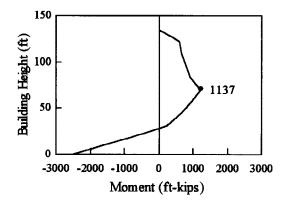


Figure 6. "Bare" column moment diagram at displacement of time 8.49 seconds

While the previous approach works well when the maximum displaced shapes of the structure are known, it does not lend itself well for use in design. A more design oriented approach is to perform incremental displacement analyses using the maximum displacement anticipated by an elastic modal and response spectrum analysis for each mode. It is well documented that inelastic and elastic maximum displacements will be similar for periods longer than the period associated with the peak acceleration response<sup>[2]</sup> therefore they are also assumed to be the same in this analysis. The frame or structure is "pushed" out to the maximum absolute displacement at the corresponding level determined from the elastic modal and response spectrum analysis while accounting for all inelastic behavior. The shape that the frame or structure takes on will be that of the "inelastic mode shape", which will typically be a smoother version of the elastic mode shape. Inelastic mode shapes depend upon the maximum modal displacement and the corresponding inelastic beam action at that displacement, so they cannot truly be considered mode shapes.

#### THE INCREMENTAL DISPLACEMENT ANALYSES

Incremental displacement analyses were performed for the first, second, and third modes. These modes represent the three translational modes whose periods lie at or to the right of the peak acceleration response value, which typically are the modes that may produce noticeable non-linear effects. The critical column moment diagrams determined from the time history analysis from Figure 4 are re-plotted next to the maximum moments generated by the individual modal displacement analyses (Figure 7). Notice that there is dominant 1st, 2nd, and 3rd mode response at times 8.49, 10.47, and 5.54 seconds, respectively. However, there is also significant first mode response in each of the critical time history plots. This helps explain why the 1st mode displacement analysis closely predicts most of the moment demands from the time history analysis.

It is also important to realize that the greatest column moments at the lowest levels of the structure may not occur concurrently with the maximum roof displacement during the first mode displacement analysis. When the beams at the lowest levels of the frame begin to yield, the beam moments are distributed significantly toward the upper column. But as the frame continues to displace the column moment diagram at yielded joints will tend to "slide" in the direction of the displacement. This sliding of the moment diagram is occurring because the elastic column is continuing to resist further displacement, and the moment demands are reacting accordingly. Therefore, it is important to document the maximum column moments throughout the displacement analyses, not simply the demands at the conclusion of the analysis.

The column envelope plots of Figure 8 show the maximum column moment demands predicted by the UBC, the inelastic time history analysis, and the displacement analyses. The time history and displacement analyses agree very well at most levels. However, the code estimated demand significantly underestimates the time history demands, particularly at the middle levels of the structure. The discrepancy in anticipated moment demand is accounted for by the moments produced due to the displacements of the elastic column.

Although excellent correlation exists between the time history analysis and the incremental displacement analyses, some demands are underestimated by the displacement analyses. A conservative solution to enveloping the maximum column demands would be to use a modal combination technique, such as the square-root-sum-squares approach. This method would conservatively estimate the maximum 3rd level column moment to be 1742 ft-kips compared to the 1048 ft-kips estimated by the individual modal displacement analysis and the 1298 ft-kips estimated by the time history analysis.

The January 17, 1995 Kobe Earthquake demonstrated the vulnerability of mid-rise structures to middle level collapse. This is often attributed to the apparent change in construction material type at these levels. While this is most likely a significant factor, the corresponding demands relative to capacities should also be verified before any conclusions are drawn. There is reason to believe that the mid-level demand on these structures was at least that of the lowest levels, and the capacity significantly less.

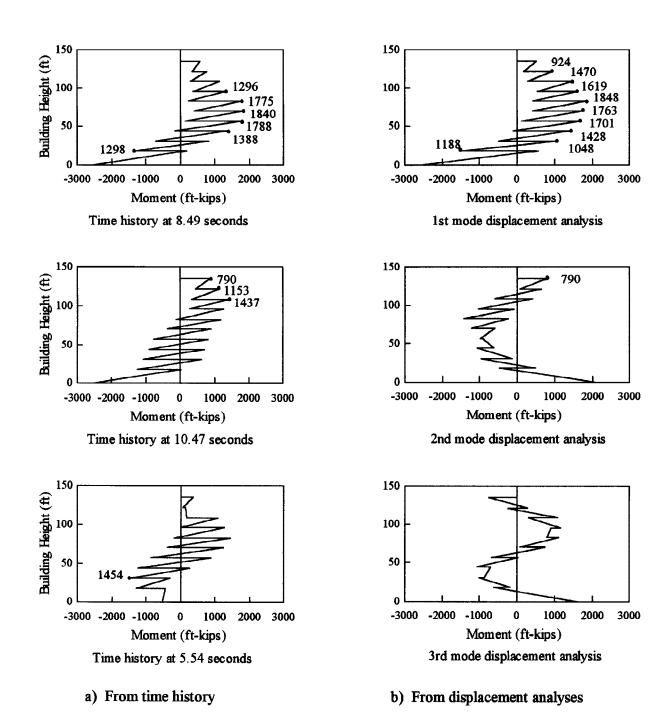


Figure 7. Column moment demand comparison

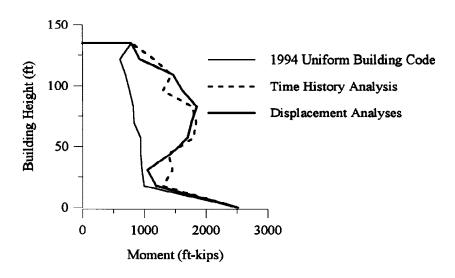


Figure 8. Column moment demand envelopes

## CONCLUSIONS

The state-of-practice "capacity" design of flexible moment frames has been shown by inelastic non-linear dynamic analyses and possibly by actual earthquakes to significantly underestimate the moment demands on the columns. The philosophy of "capacity design" must be revised to include the significant effects of the elastic column deformations in addition to the plastic action of the attached beams. Accurate estimates of the maximum column moment demands can be made for moment frame structures by incrementally displacing the structure to the maximum displacements predicted by an elastic modal analysis combined with a maximum credible response spectrum. The envelope moments found from the displacement analyses can be used confidently to estimate the true maximum anticipated moments during a maximum credible earthquake.

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