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SHEAR STRENGTH OF RECTANGULAR SHORT REINFORCED CONCRETE COLUMNS WITH INTERMEDIATE REINFORCEMENT

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ABSTRACT

The shear tests of 160 specimens are carried out to investigate the effects of intermediate reinforcement on the shear strength of rectangular short reinforced concrete columns. Web reinforcement ratio being small, the diagonal tension failure tends to occur, and their shear strength varies complicatedly by intermediate reinforcement ratio. But, web reinforcement ratio being relatively large, failure type changes into the shear compression before or after the yield of tension reinforcement after a diagonal tension crack. The shear strength in the shear compression failure type before the yield of tension reinforcement is not much influenced by intermediate reinforcement ratio.

Then, the evaluating equations of the shear strength in the shear compression failure type are derived on the basis of the lower bound theorem of limit analysis by using the analytical model consisted of the arch-mechanism, truss-mechanism and intermediate axial reinforcement. Compared with the experimental values of 94 specimens with web reinforcement carried out by auther *et al.*, the calculated values are conservative but in good agreement with the experimental values except those with small web reinforcement ratio in which the diagonal tension failure occurs.

KEYWORDS

Shear Strength; Short Reinforced Concrete Column; Intermediate Reinforcement; Limit Analysis; Lower Bound Theorem.

INTRODUCTION

Reinforced concrete columns generally have intermediate reinforcement which is arranged for bending in another direction besides calculative tension and compression reinforcement. It has been considerd that intermediate reinforcement contributes to bending as the surplus strength but not necessarily to shear. When the horizontal force is applied to the columns, the diagonal tension occurs in the web at an angle of 45 degrees to the member axis in the case of the small axial force. If the columns with intermediate reinforcement have not enough web reinforcement, the anisotropy in the web becomes stronger in the direction of the member axis and the unfavorable diagonal tension failure is likely to occur after a diagonal tension crack. Then the purpose of this study is to investigate the effects of intermediate reinforcement on the shear strength of the rectangular RC columns, especially with short length as compared with their depth. First, the outline of the experiments carried out by auther *et al.* are mentioned, and the failure type being limited to the shear compression failure, the evaluating equations of the shear strength are derived on the basis of the lower bound theorem of limit analysis by using the analytical model consisted of the arch-mechanism, truss-mechanism and intermediate axial reinforcement. The calculated values are compared with the experimental values of 94 specimens with web reinforcement.

OUTLINE OF EXPERIMENTS

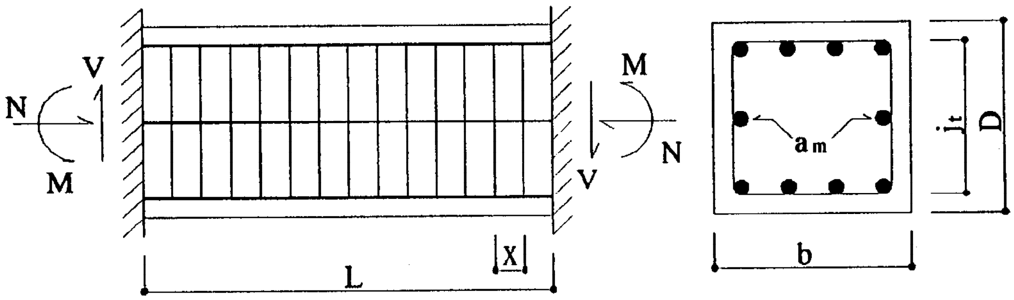
The shear tests of 160 specimens are carried out by auther(Tadehara et al., 1990) and others(Seo *et al.*, 1992; Nimura *et al.*, 1992). The former by auther include those without web reinforcement or axial force. High strength materials are used in the latter 10 specimens by the others. In the former, b = 20cm, D = 20cm(partially b = 28cm, D = 28cm), $L/D = 2, 3, 4$, tension reinforecement ratio $p_t(= a_t/bD) = 0.357\% \sim 1.27\%$, intermediate reinforcement ratio $p_m(= a_m/bD) = 0 \sim 1.91\%$, and web reinforcement ratio $p_w(= a_w/bx) = 0 \sim 1.33\%$, where a_t, a_m and a_w are the area of each reinforcement's section, and b, D and L are the width, depth and length of the member, and x is the spacing of web reinforcement. Concrete compressive strength $\sigma_B = 173 \sim 306\text{kgf/cm}^2$, yield strength of tension and intermediate reinforcement σ_y, $\sigma_{my} = 2902 \sim 3990\text{kgf/cm}^2$, and that of web reinforcement $\sigma_{wy} = 1976 \sim 2633\text{kgf/cm}^2$. The axial force ratio $\eta(= N/\nu bD\sigma_B) = 0.15 \sim 0.53$, when ν is considered($\eta = 0.10 \sim 0.34$, when $\nu = 1$), where N and ν is an axial force and an effectiveness factor of concrete compressive strength(mentioned later). In the latter, b = 20cm, D = 20cm. $L/D = 2$, p_t and p_m = both 1.27%, and $p_w = 0.30\% \sim 1.78\%$ on the basis of 0.60%. $\sigma_B = 582\text{kgf/cm}^2$, σ_y and σ_{my} = both 7350kgf/cm^2, and $\sigma_{wy} = 7350\text{kgf/cm}^2$ and 8630kgf/cm^2. Thus each material has high strength compared with the former. And $\eta = 0 \sim 1.53$ on the basis of 0.77, when ν is considered($\eta = 0 \sim 0.78$ on the basis of 0.39, when $\nu = 1$). The antisymmetric bending moment and the constant axial force are applied to them.

There being intermediate reinforcement, the shear tension or bond failure along tension reinforcement is hard to occur, even if a bending shear crack occurs, and a diagonal tension crack is likely to occur instead. Then, when p_w is small and p_t, p_m are large, this crack leads to the diagonal tension failure without ductility. When p_w is small, by the combination of p_t and p_m, the shear tension failure accompanies the diagonal tension, or the shear compression occurs. As the failure types change complicatedly by p_m like this, shear strength also varies complicatedly. It is noted that there are the cases that shear strength is remakably less than that of the specimen without intermediate reinforcement. But, when p_w is relatively large, the failure type changes into the shear compressionor flexural failure after the diagonal tension crack, and the shear strength of the shear compression failure type before the yield of tension reinforcement tends to become approximately constant in spite of the different p_m. Then, the failure type being limited to the shear compression before or after the flexual tension in the following section, the evaluating equations of the shear strength are derived on the basis of the lower bound theorem.

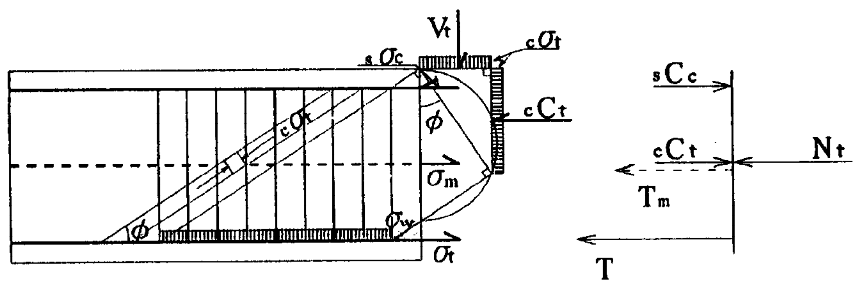
LOWER BOUND SOLUTIONS OF SHEAR STRENGTH

Analytical Model and Assumptions

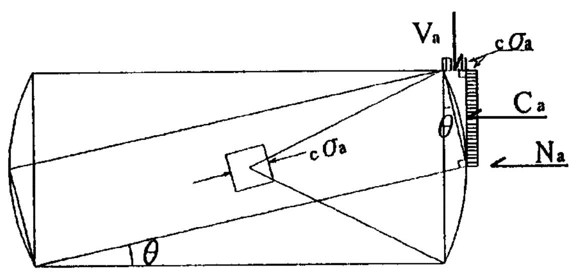
Consider a rectangular column loaded by antisymmetric bending moments M and constant axial forces N(see Figure 1.a). The column is assumed to consist of the truss-mechanism in which shear force is transferred by concrete diagonal struts and web reinforcement(see Fig. 1.b), the arch-mechanism in which only by a concrete arch diagonal member(see Fig. 1.c), and the centered intermediate reinforcement which is loaded with only the part of an axial force(see Fig. 1.b). It is assumed that both their mechanisms have the same width b and that ${}_c\sigma_t + {}_c\sigma_a = \nu\sigma_B$, where $\nu = 0.8 - \sigma_B/2000$(Nielsen, 1984). It is assumed that the difference between the angles ϕ and θ to the member axis of each diagonal stress in both mechanisms is neglected to be safe(AIJ,1990). The angle ϕ is varied with the shear reinforcement degree $\psi_{wy}(=p_w\sigma_{wy}/\nu\sigma_B)$ in principle, but $\cot\phi \leq 2$(AIJ,1990).



(a) Object of analysis



(b) Truss-mechanism and Intermediate reinforcement　　Equilibrium at member's end



(c) Arch-mechanism

Fig. 1 Analytical Model

Shear Force V_t and Axial Force N_t Loaded by Truss-Mechanism

The stress of web reinforcement being assumed to be $\sigma_w(\sigma_w \leq \sigma_{wy})$, the shear force loaded by web reinforcement is $bp_w\sigma_w$ per its unit interval. Therefore, shear force V_t is as follows:

$$V_t = bp_w\sigma_w j_t \cot\phi = V_0 j_{t1}\psi_w \cot\phi \tag{1}$$

where $V_0 = \nu\sigma_B bD$, $\phi_w = p_w\sigma_w/\nu\sigma_B$ and $j_{t1} = j_t/D$. A relation between ${}_c\sigma_t$ and σ_w is found from the equilibrium in the direction perpendicular to member axis as follows:

$$ {}_c\sigma_t = p_w\sigma_w/\sin^2\phi = \nu\sigma_B(1+\cot^2\phi)\psi_w \tag{2}$$

Here, consider the case of only the truss-mechanism. Since ${}_c\sigma_t = \nu\sigma_B$ and $\sigma_w = \sigma_{wy}$, $\cot\phi = \sqrt{1/\psi_{wy} - 1}$ from eq.(2). This $\cot\phi$ being substituted for eq.(1), the shear force without the arch-mechanism is as follows:

$$V = V_0 j_{t1}\sqrt{\psi_{wy}(1-\psi_{wy})} \tag{3}$$

This has the maximum value for $\psi_{wy} = 0.5$. From the assumption of $\cot\phi \leq 2$, eq.(3) is applicable for $0.2 \leq \psi_{wy} \leq 0.5$. For $\psi_{wy} \geq 0.5$, $V = V_0 j_{t1}/2$. By the way, for $\phi_{wy} < 0.2$, it is fixed that $\cot\phi = 2$. Then, ψ_w in eq.(2) being replaced by ψ_{wy}, ${}_c\sigma_t = 5\psi_{wy}\nu\sigma_B$. Since ${}_c\sigma_t < \nu\sigma_B$ in this case, the arch diagonal stress ${}_c\sigma_a$ is generated as the surplus stress($= \nu\sigma_B - {}_c\sigma_t$). Next, consider the axial force loaded by the truss-mechanism N_t. The equilibrium in the direction of member axis at the member's end(see Fig. 1)is as follows:

$$N_t + T + T_m = {}_sC_c + {}_cC_t \tag{4}$$

where $T = a_t\sigma_t$(tension is plus), $T_m = a_m\sigma_m$(tension is plus), ${}_sC_c = a_{ts}\sigma_c$(compression is plus), and ${}_cC_t = bj_t p_w\sigma_w\cot^2\phi$(the component in the direction of member axis of the concrete diagonal; because ${}_cC_t = V_t\cot\phi$ and from eq.(1)). The equilibrium of T, ${}_sC_c$ and the bond force B_0 along the tension reinforecement is as follows:

$$T + {}_sC_c = B_0 \tag{5}$$

$$\text{where} \qquad B_0 = (L - 2d_c\cot\phi)bp_w\sigma_w\cot\phi \tag{6}$$

This relation between B_0 and σ_w is found by considering the following; the equilibrium of the components in the direction perpendicular to members axis of the forces of web reinforcement and concrete diagonalstruts at tension reinforcement, and the equilibrium of the components in the direction of member axis of the bond force and the force of concrete diagonal struts per the interval at tension reinforcement($= L - 2d_c\cot\phi$). And d_c is the thickness of cover concrete. ${}_sC_c$ or T eliminated from eq.(4) and eq.(5), N_t is as follows:

$$N_t = -2a_t\sigma_t - a_m\sigma_m + V_0A_1\psi_w \tag{7}$$

or

$$N_t = 2a_{ts}\sigma_c - a_m\sigma_m + V_0A_2\psi_w \tag{8}$$

where $A_1 = (j_{t1}\cot\phi + \lambda - 2d_{c1}\cot\phi)\cot\phi$, $A_2 = (j_{t1}\cot\phi - \lambda + 2d_{c1}\cot\phi)\cot\phi$, $\lambda = L/D$ and $d_{c1} = d_c/D$.

Shear and Axial Forces Loaded by Arch-Mechanism V_a and N_a

As mensioned above, the arch-mechanism exists for $\psi_{wy} < 0.2$, and $\cot\phi = 2$. As the case in which axial reinforcement yields before web reinforcement yields is also considerd, ψ_w in eq.(2) is left as it is and ${}_c\sigma_a$, V_a and N_a are as follows:

$$ {}_c\sigma_a = \nu\sigma_B - {}_c\sigma_t = \nu\sigma_B(1-k\psi_w) \tag{9}$$

$$V_a = {}_c\sigma_a b(D - L\tan\theta)\cos\theta\sin\theta = V_0(1 - k\psi_w)Bz \quad (10)$$
$$N_a = {}_c\sigma_a b(D - L\tan\theta)\cos^2\theta = V_0(1 - k\psi_w)B \quad (11)$$

where $k = 1 + \cot^2\phi (= 5)$, $z = \tan\theta$, and $B = (1 - \lambda z)/(1 + z^2)$.

Shear Strength in Stress Condition of Each Reinforcement

Consider the case in which web reinforcement yields but axial reinforcement does not yield; that is, $|\sigma_t| < \sigma_y$, $|\sigma_m| < \sigma_{my}$, $|{}_s\sigma_c| < \sigma_y$, $\sigma_w = \sigma_{wy}$. These being substituted for eq.(1) and eq.(10), V and N are as follows:

$$V = V_t + V_a = V_0 j_{t1}\psi_{wy}\cot\phi + V_0 rBz \quad (12)$$
$$N = N_t + N_a = -2a_t\sigma_t - a_m\sigma_m + A_1 b p_w \sigma_{wy} + V_0 rB \quad (13)$$

where $r = 1 - k\psi_{wy}$. For a given N in eq.(13), $(-2a_t\sigma_t - a_m\sigma_m)$ and z have a dependent relation with each other. Then, the former being considered a function of z, V also becomes a function of z. The differential of eq.(12) by z being put into zero, V has the maximum value not related to N, when $z = \sqrt{\lambda^2 + 1} - \lambda (B = 1/2)$, as follows:

$$V = V_0 j_{t1}\psi_{wy}\cot\phi + V_0 r(\sqrt{\lambda^2 + 1} - \lambda)/2 \quad (14)$$
$$N = -2a_t\sigma_t - a_m\sigma_m + V_0 A_1\psi_{wy} + V_0 r/2 \quad (15)$$

The lower limit N_3 of N for which eq.(14) is applicable is found when $\sigma_t = \sigma_y$, $\sigma_m = \sigma_{my}$ in eq.(15), or the upper limit N_4 when ${}_s\sigma_c = \sigma_y$, $\sigma_m = -\sigma_{my}$ in eq.(8) as follows:

$$N_3 = N_0 + V_0 A_1\psi_{wy} + V_0 r/2 \quad (16)$$
$$N_4 = -N_0 + V_0 A_2\psi_{wy} + V_0 r/2 \quad (17)$$

where $N_0 = -2a_t\sigma_y - a_m\sigma_{my}$.

Next, consider the case in which web reinforcement, tension and intermediate reinforcement yield togather; that is, $\sigma_t = \sigma_y$, $\sigma_m = \sigma_{my}$, $\sigma_w = \sigma_{wy}$. These being substituted for eq.(1) and eq.(10), V and N are as follows:

$$V = V_0 j_{t1}\psi_{wy}\cot\phi + V_0 rBz \quad (18)$$
$$N = N_0 + V_0 A_1\phi_{wy} + V_0 rB \quad (19)$$

For a given N in eq.(19), z is decided. This z being substituted for eq.(18), V is as follows:

$$V = V_0 j_{t1}\psi_{wy}\cot\phi + \left(\sqrt{(r\lambda)^2 - 4(N_s/V_0 - r)N_s/V_0} - r\lambda\right)\frac{V_0}{2} \quad (20)$$

where $N_s = N - N_0 - V_0 A_1\psi_{wy}$. Eq.(20) is applicable for $N_2 \leq N \leq N_3$; N_2 is mensioned later.

Next, consider the case in which tension and intermediate reinforcement yield but web reinforcement does not yield; that is, $\sigma_t = \sigma_y$, $\sigma_m = \sigma_{my}$, $0 \leq \sigma_w < \sigma_{wy}$. These being substituted for eq.(1) and eq.(10), V and N are as follows:

$$V = V_0 j_{t1}\psi_w\cot\phi + V_0(1 - k\psi_w)Bz \quad (21)$$

$$N = N_0 + V_0 A_1 \psi_w + V_0(1 - k\psi_w)B \tag{22}$$

For a given N in eq.(22), ψ_w and z have a dependent relation with each other. Then, z being considered a function of ψ_w, V also becomes a function of ψ_w. The differential of eq.(21) by ψ_w being put into zero, V has the maximum value for a given N, when ψ_w is as follows:

$$\psi_w = \frac{N - N_0 - V_0 B_1}{V_0(A_1 - kB_1)} \tag{23}$$

where $B_1 = (1 - \lambda z_1)/(1 + z_1^2)$, $z_1 = (\sqrt{b_1^2 + a_1 c_1} + b_1)/a_1$, $a_1 = j_{t1}\lambda \cot\phi + A_1 + k\lambda^2$, $b_1 = j_{t1}\cot\phi - A_1\lambda + k\lambda$, $c_1 = j_{t1}\lambda\cot\phi + A_1 - k$. These z_1 and ψ_w being substituted for eq.(21), V is as follows:

$$V = \frac{j_{t1}\cot\phi - kB_1 z_1}{A_1 - kB_1}(N - N_0 - \frac{V_0 A_1}{k}) + \frac{V_0 j_{t1}\cot\phi}{k} \tag{24}$$

The lower limit N_1 and the upper N_2 of N for which eq.(24) is applicable is found from the condition of $0 \leq \psi_w < \psi_{wy}$ in eq.(23) as follows:

$$N_1 = N_0 + V_0 B_1 \tag{25}$$

$$N_2 = N_0 + V_0 A_1 \psi_{wy} + V_0 r B_1 \tag{26}$$

The cases except the above stress conditions of each reinforcement are omitted on account of limited space.

Balanced Shear Reinforcement Degree ψ_b

The above section is supposed that web reinforcement certainly yields in the range of the axial force where the shear strength is maximum ($N_3 \leq N \leq N_4$). This premise requires the following condition:

$$N_4 - N_3 \geq 2a_m\sigma_{my}, \tag{27}$$

that is, N_4 and N_3 of eq. (16), eq. (17) being substitued for eq. (27), it is as follows.

$$\psi_{wy} \leq \psi_b \tag{28}$$

$$\textit{where} \qquad \psi_b = \frac{2a_t\sigma_y}{V_0(\lambda - 2d_{c1}\cot\phi)\cot\phi} \tag{29}$$

This shall be called the balanced shear reinforcement degree. If $\psi_w y > \psi_b$, axial reinforcement yields but web reinforcement does not yield. In this case, ψ_{wy} in the boundary axial force N_2, N_3 and N_4 and shear force V in the above section have to be replaced with ψ_b. However the value of $\cot\phi$ has to bedecided by ψ_{wy} beforehand so that the value of ψ_b is found.

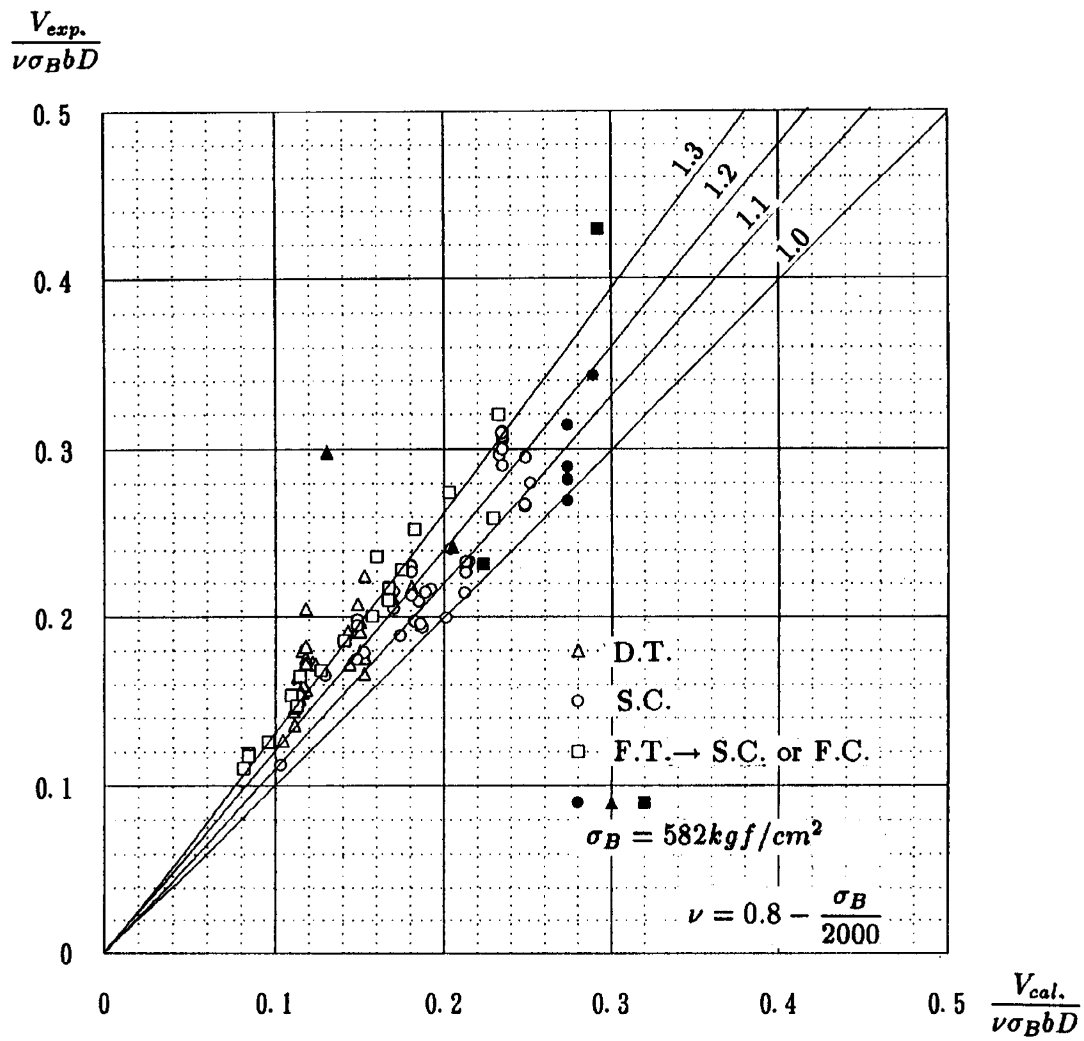


Fig. 2 Comparison between experimental and calculated values

COMPARISON BETWEEN EXPERIMNTAL AND CALCULATED VALUES

A total of 94 specimens with web reinforcement are compared with calculated values. But the failure types include the flexural or diagonal tension failure. Figure. 2. shows the comparison between the calculated value $V_{cal.}$ and the corresponding experimental value $V_{exp.}$. Each value is normalized by V_0. The symbols of ○ and ● in the Figure show the shear compression failure, and □ and ■ show the shear compression or flexural compression failure after the flexural tension. Then △ and ▲ show the diagonal tension failure. These are mainly found when p_w is small. Almost the experimental values are over the calculated values and are in the safety side. When the materials with high strength are used(●, ■, ▲), $V_{exp.}/V_{cal.}$ is between 0.98 and 1.18 in eight specimens out of ten. A symbol ▲ in which $V_{exp.}/V_{cal.} = 2.29$ shows the diagonal tension failare typre at the high axial load($\eta = 1.53$ in which ν is cosidered(η=0.76, when $\nu = 1$)). This experimental value rather corresonds with the value calculated by the principal stress equation which includes the axial stess and concrete tensile strength(Tadehara *et al.*, 1992). So this equation applied, $V_{exp.}/V_{cal.} = 1.02$. On the other hand, in the case of a symbol ■ in which $V_{exp.}/V_{cal.} = 1.47$, p_w is larger than the standard, and ψ_w is limited to ψ_b. In this case, it is considerd that high web reinforcement ratio causes the confining effect which raises the apparent concrete compressive strength.

CONCLUSIONS

The following conclusions can be drawn from this study:
The columns with intermediate reinforcement are likely to become the diagonal tension failure type when they have small web reinforcement ratio. These shear strength is complicatedly influenced by intermediate reinforcement ratio. But, when web reinforcement ratio is comparatively large, their failure types change into the shear compression failure type before or after tensile yield of axial reinforcement. Their shear strength is not much influenced by intermediate reinforcement ratio.

Then the lower bound solutions on shear strength which is decided by concrete compressive strength are derived. Compared with the experimental values, the calculated values are conservative but in good agreement with the experimental values of the specimens in which the shear compression failure occurs before or after the yield of tension reinforcement, except for the diagonal tension failure type in the specimens with small web reinforcement ratio, the same failure type at the high axial load, and the shear compression failure type in the specimen with the high web reinforcement ratio which is considerd to cause the confining effect.

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