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THEORETICAL BASIS AND DYNAMIC DESIGN OF THE SYSTEMS WITH SEMIRIGID CONNECTIONS OF MEMBERS WITH JOINTS

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ABSTRACT

Constructions with not absolutely rigid connections between members, that allow some relative rotation of member ends, are systems with semi-rigid connections in joints. As such system of connections is very often in constructions, particulary in prefabricated ones, it is of interest to analyse them taking in account elasticity of joint connections. Based on the obtained results, it can be concluded that the level of fixing is not to be neglected when the structure is dynamicaly loaded, particulary in the analysis of prefabricated structures.Because of limited space, here it will be given only expressions for design of systems with semi-rigid connections, without deriving.

KEYWORDS

semi - rigid connections, redistribution of effects, static, dynamic, frequency, damping.

STATIC DESIGN

For semi-rigid connected members in joints, expressions for bending moments at the ends as well as conditional equations of slope-deflection method are derived in [3]. If it is introduced designations $\mu_{ik} = \varphi^*_{ik} / \varphi_i ; \mu_{ki} = \varphi^*_{ki} / \varphi_k$ (where φ_i and φ_k are the angles of rotation of the joint "i" and "k" respectively, and angles φ^*_{ik} and φ^*_{ki} of rotations of end cross-sections of the member "ik") and are named fixing degrees of member "ik" in joints "i" and "k", the expressions for bending moments at the ends of such connected members are:

$$M_{ik} = a_{ik}\varphi^*_i + b_{ik}\varphi^*_k - c_{ik}\psi_{ik} + m^{(o)}_{ik} + m^{(\Delta t)}_{ik} \tag{1.a}$$

$$M_{ki} = b_{ik}\varphi^*_i + a_{ki}\varphi^*_k - c_{ki}\psi_{ik} + m^{(o)}_{ki} + m^{(\Delta t)}_{ki} \tag{1.b}$$

or in terms of $\varphi_i, \varphi_k, \psi_{ik}$, in the shape

$$M^*_{ik} = a^*_{ik}\varphi_i + b^*_{ik}\varphi_k - c^*_{IK}\psi_{ik} + m^{(o)*}_{ik} + m^{(\Delta t)*}_{ik} \tag{2.a}$$

$$M^*_{ki} = b^*_{ik}\varphi_i + a^*_{ki}\varphi_k - c^*_{ki}\psi_{ik} + m^{(o)*}_{ki} + m^{(\Delta t)*}_{ki} \tag{2.b}$$

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Constants $a^*_{ik}, b^*_{ik}, c^*_{ik}$, as well as the beging moments of semi-rigidly conected members can be expressed in terms of corresponding values of rigidly connected members and fixing degree, on the following way:

$$a^*_{ik} = \mu_{ik}\left[a_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}b_{ik}\right] \tag{3.a}$$

$$b^*_{ik} = b^*_{ki}\mu_{ik}\mu_{ki} \tag{3.b}$$

$$c^*_{ik} = \mu_{ik}\left[c_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}c_{ki}\right] \tag{3.c}$$

$$c^*_{ki} = \mu_{ki}\left[c_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}c_{ik}\right] \tag{3.d}$$

$$a^*_{ki} = \mu_{ki}\left[a_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}b_{ik}\right] \tag{3.e}$$

$$m^*_{ik} = \mu_{ik}\left[m_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}m_{ki}\right] \tag{4.a}$$

$$m^*_{ki} = \mu_{ki}\left[m_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}m_{ik}\right] \tag{4.b}$$

$$M^*_{ik} = \mu_{ik}\left[M_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}M_{ki}\right] \tag{4.c}$$

$$M^*_{ki} = \mu_{ki}\left[M_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}M_{ik}\right] \tag{4.d}$$

and their phisical meaning is shown in Fig. 1a-1e.

From expressions (3.a)-(3.e) it is easy to notice that by variation of μ_{ik} and μ_{ki} with 1 or 0 can be obtained previously defined types of members : type "k" ($\mu_{ik}=\mu_{ki}=1$) type "g" ($\mu_{ik}=1$; $\mu_{ki}=0$), type "z" ($\mu_{ik}=\mu_{ki}=0$), so that in present analysis all of them can be treated as one type of member.

At the same way as in (5), the final expraessions for M^*_{ik} and M^*_{ki} are obtained in the shape:

$$M^*_{ik} = a^*_{ik}\varphi_i + b^*_{ik}\varphi_k - c^*_{ik}\sum_{j=1}^{n}\psi^{(j)}_{ik}\Delta_j + m^*_{ik} \tag{5.a}$$

$$M^*_{ki} = b^*_{ik}\varphi_i + a^*_{ki}\varphi_k - c^*_{ki}\sum_{j=1}^{n}\psi^{(j)}_{ik}\Delta_j + m^*_{ki} \tag{5.b}$$

Equations of rotation and equations of displacements now look like:

$$\sum_k M^*_{ik} + M_i = 0 \qquad (i=1,2,...,m); \qquad (\varphi_i=1) \tag{6.a}$$

$$\sum_{ik}(M_{ik}^{*}+M_{ki}^{*})\psi_{ik}^{(j)}+R_{j}=0 \qquad (j=1,2,...,n); \qquad (\Delta=1) \tag{6.b}$$

When (10) is introduced in (11), after some transformations, it is obtained:

$$A_{ii}^{*}\varphi_{i}+\sum_{k}A_{ik}^{*}\varphi_{k}+\sum_{j=1}^{n}B_{ij}^{*}\Delta_{j}+A_{i0}=0^{2} \qquad (i=1,2,...,m) \tag{7.a}$$

$$\sum_{i=1}^{m}B_{ji}^{'*}\varphi_{i}+\sum_{l=1}^{n}C_{jl}^{*}\Delta_{l}+C_{j0}^{*}=0 \quad (j=1,2) \tag{7.b}$$

were the following designations are used:

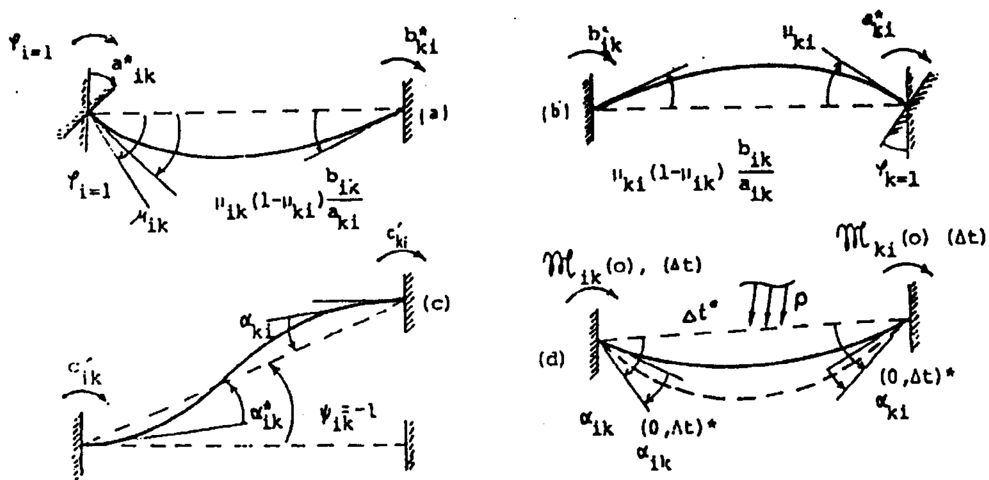
$$A_{ii}^{*}=\sum_{k}a_{ik}^{*}; \qquad A_{ik}^{*}=b_{ik}^{*}; \qquad A_{i0}^{*}=\sum_{k}m_{ik}^{*}+M_{i} \tag{8.a}$$

$$B_{ij}^{*}=-\sum_{k}c_{ik}^{*}\psi_{ik}^{j}=B_{ji}^{'*} \tag{8.b}$$

$$C_{jl}^{*}=\sum_{ik}(c_{ik}^{*}+c_{ki}^{*})\psi_{ik}^{j}\psi_{ik}^{l} \tag{8.c}$$

$$C_{j0}^{*}=-\sum_{ik}(m_{ik}^{*}+m_{ki}^{*})\psi_{ik}^{j}-R_{j} \tag{8.d}$$

After comparing equations (7) with corresponding in [1] it is evident that they are realy the same as well as expessions (5) to corresponding in [1].



$$\alpha_{ik}^{*}=\mu_{ik}-(1-\mu_{ik})\mu_{ki}\frac{b_{ik}}{a_{ik}} \quad \alpha_{ik}^{*(o,\Delta t)}=\mu_{ik}\alpha_{ik}^{(o,\Delta t)}-(1-\mu_{ik})\mu_{ki}\frac{b_{ik}}{a_{ik}}\alpha_{ik}^{(o,\Delta t)}$$

$$\alpha_{ki}^{*}=\mu_{ki}-(1-\mu_{ki})\mu_{ik}\frac{b_{ik}}{a_{ki}} \quad \alpha_{ki}^{*(o,\Delta t)}=\mu_{ki}\alpha_{ki}^{(o,\Delta t)}-(1-\mu_{ik})\mu_{ki}\frac{b_{ik}}{a_{ki}}\alpha_{ki}^{(o,\Delta t)}$$

Fig.1.

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[2] As difference from [1], $\sum_{g}$ and $\sum_{ig}$ do not appear because all of members are included in $\sum_{ik}$.

The case of rigid connections

In the case of dinamic loading it is derived equations of force vibrations in [3] and equatius of forced damped vibratius of frame constructias in [4] for systems with rigid connections of members in joints. As it is known, dynamic design consists of determinig internal forces and displacements caused by dynamic influences with known value and character, or testing the system on resonance at periodicaly changeable loadings with given frequences. For determining of amplitudes of inertial forces, com be also used so called canon equations of Force method:

$$-\frac{I_i}{m_i\Theta^2}+\sum_{k=1}^{n}\delta_{ik}I_k+\delta_{ip}=0 \qquad (i=1,2,...,n) \tag{9}$$

by solving them amplitudes of inertial forces are obtained, while their changing in time is given by expression

$$I_k(t)=I_k\sin\Theta t \tag{10}$$

and displacement values can be calculated from the following formula

$$Y_i(t)=\frac{I_i}{m_i\Theta^2}\sin\Theta t \qquad (i=1,2,...,n) \tag{11}$$

From the expession

$$\mathrm{Y_i(t)}=\sum_{k=1}^{n}(\mathrm{P_k}+\mathrm{I_k})\sin\Theta \mathrm{t} \qquad (i=1,2,...,n) \tag{12}$$

or

$$\mathrm{Y_i(t)}=\mathrm{Y_i}\sin\Theta \mathrm{t} \qquad (i=1,2,...,n) \tag{12'}$$

where it is

$$\mathrm{Y_i}=\mathrm{C_i}=\delta_{ip}+\sum_{k=1}^{n}\delta_{ip}\mathrm{I_k} \qquad (i=1,2,...,n) \tag{13}$$

displacement amplitudes are obtained.

So, amplitudes of dynamic effects are determined as statical effects due to simultaneosly acting of perturbation force amplitude and coresponding to them amplitudes of inertial forces, i.e. due to forces:

$$\mathrm{Z_i}=\mathrm{H_i}+\mathrm{I_i} \qquad (i=1,2,...,n) \tag{14}$$

The case of semi-rigid connections

When it is considered systems with elasticaly conected members, whose masses mi are ateched by dynamical loading Pi(t), in real environment that oposes movement of resistant forces Pi(t), and corespondig inertial forces Ii(t) whose projections on "x" and "y" axis are shown in Fig. 2, taking in account equations (3), (4) and (5) as well as equations (3.99)-(3.115) from [5] can be written equatias of forced damped vibratias of system with finite number of degrees of freedom in the shape:

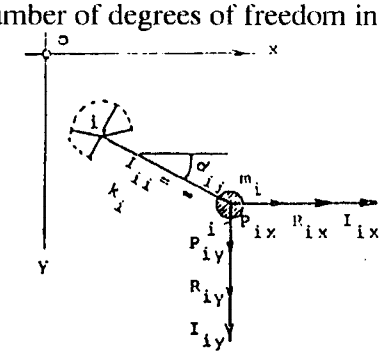


Fig. 2

$$R_{ix}(t)=-\beta_i\dot{u}_i \quad R_{iy}(t)=-\beta_i\dot{v}_i'$$

$$I_{ix}(t)=-m_i\ddot{u}_i \quad I_{iy}(t)=-m_i\ddot{v}_i'$$

$$u_i'=u_i-k_i\varphi_i\sin\alpha_{ii}' \tag{15}$$

$$v_i'=v_i-k_i\varphi_i\cos\alpha_{ii}'$$

$$\begin{bmatrix} \bar{\bar{A}} & \bar{\bar{B}} \\ \bar{\bar{B}}^T & \bar{\bar{C}} \end{bmatrix} \begin{Bmatrix} \ddot{\vec{\varphi}} \\ \ddot{\vec{\Delta}} \end{Bmatrix} + \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{B}^T & \bar{C} \end{bmatrix} \begin{Bmatrix} \dot{\vec{\varphi}} \\ \dot{\vec{\Delta}} \end{Bmatrix} + \begin{bmatrix} A^* & B^* \\ B^{*T} & C^* \end{bmatrix} \begin{Bmatrix} \vec{\varphi} \\ \vec{\Delta} \end{Bmatrix} = - \begin{Bmatrix} \vec{A}_0 \\ \vec{C}_0 \end{Bmatrix} \tag{16}$$

In equation (16) the following submatrix exist:

- diagonal matrix $\bar{\bar{A}}$ and $\bar{A}$ of order "m"

$$\bar{\bar{A}} = m_i k_i^2 \qquad (\bar{\bar{A}}_{ik} = 0, i \neq k)$$

$$\bar{A}_{ii} = \beta_i k_i^2 \qquad (\bar{A}_{ik} = 0, i \neq k) \tag{17.a}$$

-rectangular matrix $\bar{\bar{B}}$ and $\bar{B}$ of order "mxn" and their transposed matrix $\bar{\bar{B}}^T$ and $\bar{B}^T$ of order "nxm"

$$\bar{\bar{B}}_{ij} = \bar{\bar{B}}^T_{ji} = m_i k_i (v_{i,j} \cos\alpha'_{ii} - u_{i,j} \sin'\alpha_{i,i}$$

$$\bar{B}_{ij} = \bar{B}^T_{ji} = \beta_i k_i (v_{i,j} \cos\alpha'_{ii} - u_{i,j} \sin'\alpha_{i,i} \tag{17.b}$$

-square matrix $\bar{\bar{C}}$ and $\bar{C}$ of order "n"

$$\bar{\bar{C}}_{jl} = \bar{\bar{C}}_{lj} = \sum_i i(u_{i,j} u_{i,l} + v_{i,j} v_{i,l}) \tag{18.a}$$

$$\bar{C}_{jl} = \bar{C}_{lj} = \sum_i i(u_{i,j} u_{i,l} + v_{i,j} v_{i,l}) \tag{18.b}$$

-square matrix A of order "m"whose elements are

$$A^*_{ii} = \sum_k \alpha^*_{ik}, \qquad A^*_{ik} = b^*_{ik} \quad (i \neq k) \tag{19}$$

-rectangular matrix B^* of order "mxn" and B^{*T}

$$B^*_{ij} = -\sum_k c^*_{ik} \psi^j_{ik} = B^{*}_{ji} \tag{20}$$

-square matrix C^* of order "n"

$$C^*_{jl} = C^*_{lj} = \sum_{ik} (c^*_{ik} + c^*_{ki}) \psi^j_{ik} \psi^l_{ik} \tag{21}$$

-vector of joint angles of rotation $\vec{\varphi}(t)$ of order "mx1", and its derivatives $\dot{\vec{\varphi}}(t)$ and $\ddot{\vec{\varphi}}(t)$

-vector of displacement parameters $\vec{\Delta}(t)$ of order "nx1" and its derivatives $\dot{\vec{\Delta}}(t)$ and $\ddot{\vec{\Delta}}(t)$-vector $\vec{A}_0$ of order "mx1" and $\vec{C}_0$ of order "nx1".

$$A_{i0} = M_i(P_i) = P_{ix} k_i \sin\alpha'_{ii} - P_{iy} k_i \cos\alpha'_{ii} \tag{22.a}$$

$$C_{j0} = -R_j(\textstyle\sum P_i) \quad \text{work of forces } P_i(t) \text{ at the state } \Delta_j = 1 \tag{22.b}$$

-$u_{i,j}$ and $v_{i,j}$ are displacements of joint "I" in direction X and Y at the state $\Delta_j = 1$, while is coefficient of resistance, and m_i mass in joint "i".

Previously presented is valued for the case where masses are connected by cantilever (length k_i) to the joints of girder. The most often case is when masses are put just in joints (k_j=0). Then, as it is evident from equations (17), (18) and (21), matrix $\bar{\bar{C}}$, $\bar{C}$,$\bar{\bar{B}}$, $\bar{B}$ and $\vec{A}_0$ become null matrix, so that from system (16) after m eliminations, can be eliminated all unknowns φ, and expression (16) get the shape

$$\left[\bar{\bar{C}}\right]\left\{\ddot{\vec{\Delta}}\right\}+\left[\bar{C}\right]\left\{\dot{\vec{\Delta}}\right\}+\left[C^{**}\right]\left\{\vec{\Delta}\right\}=-\left[\vec{C}_0\right] \tag{23}$$

where matrix

$$C^{**} = C^{*(m)} \tag{24}$$

is obtained after m eliminations.

When parameters Δ_j are determined from (23), from the system

$$\left[A^*\right]\left\{\vec{\Phi}\right\}=-\left[B^*\right]\left\{\vec{\Delta}\right\} \tag{25}$$

can be determined values φ_i (i=1,2,...,m) and at last from (2) bending moments at the ends of members and after that other internal forces.

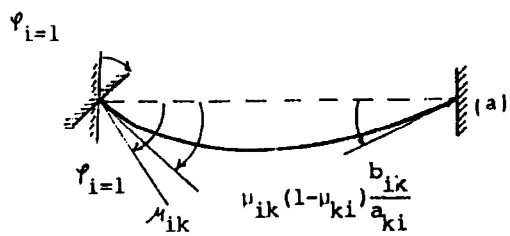


Fig. 3

$$\dot{\varphi}_{ik} = \varphi_i\mu_{ik}$$

$$\Delta\varphi_{ik} = (\varphi_i - \dot{\varphi}_{ik}) = \varphi_i(1-\frac{\dot{\varphi}_{ik}}{\varphi_i}) = \varphi_i(1-\mu_{ik})$$

$$\mu_{ik} = \frac{\dot{\varphi}_{ik}}{\varphi_i}$$

$$\frac{\Delta\varphi_i}{\varphi_i} = 1-\mu_{ik} \Rightarrow \mu_{ik} = 1-\frac{\Delta\varphi_i}{\varphi_i}$$

NUMERICAL EXAMPLE

Proceeding from the new constants of bars, by use of slope-deflection method, for the purpose of illustration, it is worked on an example of a simple frame structure, where fixing coefficients are varied from 0 to 1. (Table 1).

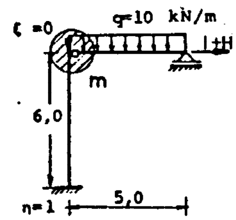
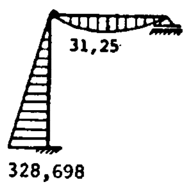
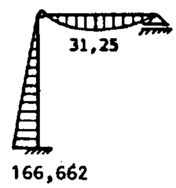
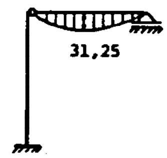
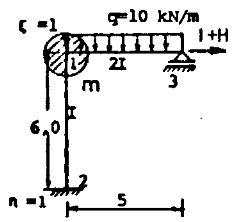
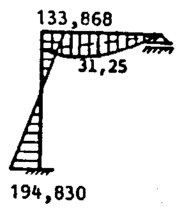
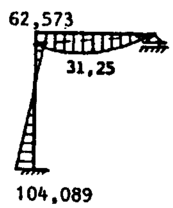
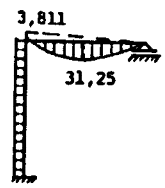
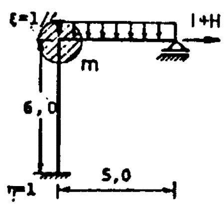
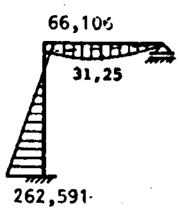
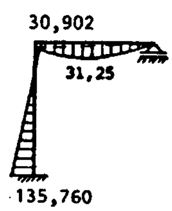
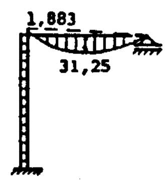
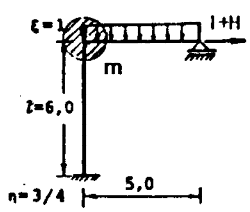
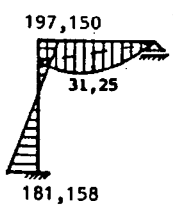
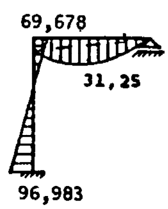
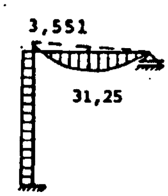
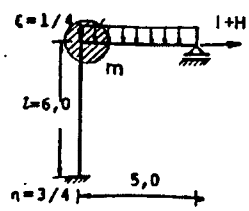
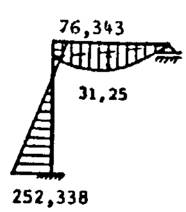
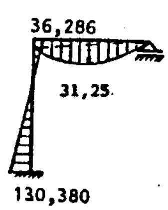
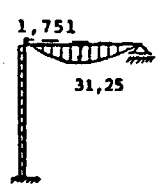
In the case of dynamic load acting, that can be describe by the function $P_i(t) = H_i \sin\theta t$, circle frequences and inertial forces are calculated for $\theta = 0.8\omega$. On the base of obtained results, it can be concluded that circle frequences of the structures shown in the first and fourth drawing are 33% and 71%, respectively, greather than that one of the cantilever column. Inertial forces differs each to other for less than 8%, for the considered example, for different fixing degrees.

After determining maximal values of inertial forces I, amplitudes of bending moments are calculated according to the principle of superposition.

$$M_d = M_i x I + M_{st,H} \tag{26}$$

where Mi, Ii and Mst,H are bending moments of given structure due to inertial forces amplitudes as well as perturbation forces amplitudes.

Table. 1

Structure sheme	Bending moments from the static loading			Dynamic properties and influences caused by perturb. force P_i (t)		
$m = 5\frac{kNs2}{m}$ EI=100000 kNm^2 $\theta = 0{,}8\omega$	M diagram H=20 kN I=34,783 kN	M diagram H=10 kN I=17,777 kN	M diagram H=0 kN I=0 kN	Circ. freq ω $[s^{-1}]$	I_1 from $H_1(t) = 10 \sin \theta t$ [kN]	I_2 from $H_2(t) = 20 \sin \theta t$ [kN]
				5,27	17,778	35,556
				9,154	17,779	37.366
				7,0334	17,776	34,689
				9,000	17,783	35,356
				7,388	17,777	34,783

At the Table T.1, fixing degrees of joint column to foundation is denoted by $\mu_{ik} = \xi$, and column to beam $\mu_{ki} = \eta$. Bending moment diagrams, in the case of dynamic loading are calculated for equaly distributed loading q=10 kN/m and for horizontal force H=10 kN, H=20 kN and H=0. For different fixing degrees and η, it is evident from given diagrams that changes of bending moments are significant.

For example, bending moment at the point of total fixing of column to foundation, with fixing degree column to beam is ξ=o,25 (the third drawing) is 80% of bending moment at the same cross section of centilever column for H=10 kN, i.e, 81,5% for H=20 kN (the first drawing), bending moment in fixing is only 55% for H=10 kN, i.e, 58% for H=20 kN of the moment of cantilever rigidly fixed column (the first drawing).Beside dynamic bending moments due to vibration, there are also computed just static moments due to static loading mg=Q.

Total values of bending moment amplitudes according to superposition principle are:

$$M_t = M_d + M_{st,Q} \quad (27)$$

while changes in time of amplitudes are given by expression according to which amplitudes of perturbation forces (in this case $\sin \Theta t$) are changed too.

CONCLUSIONS

The systems with members elastically fixed in joints, whose masses are exposed to dynamic loading with corresponding inertial forces in the real environment which opposes to the motions by resistant forces, are considered. Assuming that the relations between the actual and absolutely rigid fixing of the ends of members in joints are μ_{ik}, that is μ_{ki}, the formulas for the forces at the ends of the members as well as the conditional equations for the case of the damped forced vibrations of the system with finite number of degrees of freedom are derived by use of slope-deflection method (at first time by the first author).

The expressions that are the object of the work have a particular significance in the earthquake engineering, because just seismic forces cause the joint connections to become slack, and this influences have not been adequately taken in consideration in up-to-date dynamic analysis.

Based on the obtained results, it can be concluded that the level of fixing is not to be neglected when the structure is dynamicaly loaded, particulary in the analysis of prefabricated structures.

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