IMPACT OF ACCIDENTAL ECCENTRICITY IN THE SEISMIC DESIGN OF INDUSTRIAL FACILITIES

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ABSTRACT

This paper examines the asymmetric response of buildings that in general do not incorporate a rigid diaphragm, specifically industrial type frame buildings. To this end, a parametric study was conducted with one-story models representative of industrial buildings, considering a wide range of floor flexibilities and fundamental translational periods. The study uses seismic design spectra representing the translational and torsional component of ground motion. The effects of torsional earthquake input on seismic design forces is assessed by comparing the dynamic response of models including the torsional spectrum to those where only the translational input is considered. The focus is on response quantities related to structural design namely shear forces in both vertical frames and roof members and interstory drifts. It is found that shear forces in roof members can be significantly higher than the forces generated in the vertical frames. The paper also examines to what extent accidental eccentricities can account for the effects of torsional ground motion. While this approach provides conservative estimates of seismic forces in the vertical frames, in general, representative estimates of forces in the roof can be only obtained through a three-dimensional dynamic analysis that accounts for the rotational component of ground motion. Research needs are identified and analysis procedures are proposed that use concepts available in modern computer programs, and provide acceptable means for considering torsional input in the seismic design of structures with flexible floors.

KEYWORDS

Accidental torsion; accidental eccentricity, industrial buildings, torsional ground motion; roof flexibility.

INTRODUCTION AND BACKGROUND

Earthquakes generate torsional response even in ideally symmetric structures due to the torsional components of seismic ground motions. Additionally, torsional moments unaccounted in the seismic analysis are also generated by the variability in the magnitude and location of mass and stiffness properties. Although these factors have been recognized for some time, they are not explicitly evaluated in conventional methods of seismic analysis. Rather, current design practice, as provided in prevalent codes and standards, incorporates the torsional response from these sources by specifying an accidental eccentricity ranging from 5 to 10 percent of the plan dimension perpendicular to the direction of the corresponding translational motion. (ICBO, 1994, DDF, 1993). Calculated forces resulting from this accidental eccentricity are combined with the forces resulting from the translational ground motion in the design evaluation of structural components of the structure's lateral load resisting system. In commercial and residential buildings, where the floors and the roof can be usually considered as rigid diaphragms in their own plane, it is possible to evaluate the effects of
torsion by moving the point of application of the seismic shear forces or the center of masses within the rigid diaphragms, consistent with specified accidental eccentricity.

A typical industrial frame building comprises moment resisting frames connected at the bottom chord elevation by horizontal bracing elements. Typical spacing of the frames is about 25 feet. In nonseismic zones the bottom chord bracing is designed to resist wind loading and other operational loads in the lateral direction. Although it incorporates some in-plane stiffness, the bottom chord bracing is relatively flexible when compared with a floor diaphragm in a commercial building. In the absence of such a rigid diaphragm, individual elements of a three-dimensional lateral load resisting system are not strongly coupled and the structure does not have a clearly defined center of torsion (or shear center). Thus, it is difficult if not impossible, to include potential torsional response simply by moving the position of the center of mass, and it may be appropriate to consider three-dimensional representation capable of explicitly accounting for the distributions of mass and stiffness. Finite element methods can then effectively evaluate the asymmetric response given the appropriate torsional input, reflecting the in-plane stiffness of the members connecting the frames.

This paper studies the impact of the torsional component of ground motion on the response of industrial buildings. The effects of variabilities in masses and stiffness are not considered herein. However, we note that recent work by de la Llra and Chopra (1994a) has shown that the uncertainties in stiffness account for a relatively small part of the torsion due to the accidental eccentricity stipulated by the UBC Code. Additionally, since the magnitude and location of masses and structural members are usually more precisely defined in industrial facilities than in conventional buildings, the effects of uncertainties are significantly smaller, and the torsional earthquake input becomes the primary source of accidental torsion. The paper focuses on long structures with flexible floors and/or roofs, which cannot be considered as rigid diaphragms, and also examines procedures to include torsional ground motions in the seismic analysis.

ANALYTICAL APPROACH

Fig. 1 presents the model used in this study. The model consists of \( n \) masses supported by \( n \) shear springs representing the vertical system resisting lateral forces in a one-story industrial building. These springs are interconnected by \( (n - 1) \) horizontal springs representing the roof flexibility. All springs resist shear forces in the direction of the translational ground motion and are considered to be linearly elastic. The model has \( n \) translational degrees of freedom in the direction of the seismic input and is characterized by the dimension \( b \) perpendicular to the translational ground motion.

We subject the above model to two components of ground motion: a uniform translational motion and a rotational motion about the center of the model. For the purposes of this study, the specific translational motion is rather arbitrarily chosen to be identical with that represented by Newmark's generalized spectrum used in his study on torsion in symmetric structures with rigid diaphragms (Newmark and Rosenblueth, 1971). However, Newmark's corresponding rotational spectrum is reduced by a factor of 3.0 based on de la Llra and Chopra's (1994b) conclusions from their statistical analysis of spectra for California records. We believe that until additional field data is evaluated, this provides appropriate estimates of rotational ground motions.

It is noted that the rotational spectrum is directly proportional to the transit time \( t_b = b/V_s \), where \( V_s \) is the shear wave velocity of the underlying ground. Fig. 2 shows the translational and rotational spectra for a travel time of 0.1 sec selected for use in this study. Both represent the 5 percent damped response to a ground motion with peak displacement of 10 in, peak velocity of 15 in/sec and peak acceleration of 120 in/sec^2. The ordinate of the translational spectrum is the maximum relative acceleration while, following Newmark's normalization, the rotational spectrum provides the maximum value of \( b\phi/2 \), where \( \phi \) is the maximum rotational acceleration.
Fig. 1. Representation of a one-story building with flexible roof members.

Fig. 2. Translational and torsional spectra used in this study.

The equations of motion for the system shown in Fig. 1 subjected to translational and rotational ground motions as defined above are:

\[ M \ddot{u} + C \dot{u} + K u = -M R a(t) \]  

(1)

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices, respectively, and \( u \) is the vector of relative displacements of the masses.
For translational motion, all the elements of vector $\mathbf{R}$ are equal to unity and $a(t)$ is the translational acceleration. For rotational motion, $a(t)$ is the normalized rotational acceleration, and $\mathbf{R} = \mathbf{K}^{-1} \mathbf{K}_e \mathbf{X}$, where $\mathbf{K}_e$ contains the forces generated in the system's degrees of freedom by unit displacements at the support points (Clough and Penzien, 1993). The $\mathbf{X}$ vector represents the translational motion of the supports generated by the ground rotation. Therefore, the $j$-th entry of $\mathbf{X}$ should be the distance, $x_j$, of each vertical spring to the center of rotation. However, since the rotational spectrum includes a normalizing factor $b/2$, the entries of $\mathbf{X}$ become $2 x_j / b$.

The above equations of motion have been solved using the standard response spectrum approach, including all modes of the system in the calculations. The $j$-th mode shape $Z_j$ was normalized such that the generalized modal mass $m_j = Z_j^T \mathbf{M} Z_j$ is equal to unity. As a consequence, the participation factors were calculated as $p_j = Z_j^T \mathbf{M} \mathbf{R}$, using the corresponding $\mathbf{R}$ vector for translational and rotational input ground motion. In view that several modes could have similar frequencies, particularly when the stiffness of "roof" springs is relatively low, the modal results have been combined with the complete quadratic combination rule (Wilson et al., 1981), considering a 5 percent damping ratio in the calculation of the combination coefficients.

Another issue is the combination of peak responses due to the two components of ground motion considered herein, which will not occur simultaneously. It is, therefore, too conservative to add both peak values. Based on stochastic treatment of the problem, Rosenblueth and Contreras (1977) developed an approximate criterion which estimates the combined response as 100 percent of the effects of one component plus 30 percent of the effects of the other. This criteria has been adopted in several recent codes (ICBO, 1994, DDF, 1995). An optional combination criterion, widely used in the nuclear industry (ASCE, 1986), consists in estimating the combined effects as the square root of the sum of squares (SRSS) of the effects of each component.

**NUMERICAL RESULTS**

This study assesses the significance of the response to torsional ground motion by evaluating a large number of symmetric systems. Each system consists of six equidistant frames (vertical springs) of equal shear stiffness $k_1$, connected by horizontal (roof) springs all having a stiffness $k_2$. The total translational stiffness is thus $k_t = 6 k_1$. The six masses supported by the vertical springs were also assumed to have the same value. The systems were characterized by fundamental translational periods, $T$ varying between 0.1 and 10 sec. For each value of $T$, the ratio $k_2/k_1$ was varied from 0.01 to 10. The lower limit represents cases where the roof is very flexible in comparison to the vertical frames and each of these vibrates independently of the others.

A particularly relevant result of the analyses is the ratio, $r$, of the maximum displacement due to the torsional ground motion at the exterior vertical springs of the model, $d_r$, divided by the maximum displacement generated by only the translational component in the same springs, $d_t$. Since the torsional spectrum is a linear function of the transit time, $t_b$, the values of $r$ are also proportional to $t_b$. For this reason, Fig. 3 presents values of $r$ divided by $t_b$. For a specific $t_b$, the ratio $r$ can be obtained multiplying the ordinate in Fig. 3 by this $t_b$.

Fig. 3 presents the variation of $r/t_b$ as a function of the translational period, $T$, for three values of $k_2/k_1$. In general, larger in-plane chord stiffness (i.e., larger $k_2/k_1$) results in smaller torsional response relative to the translational. For very flexible lateral load resisting frames ($T \geq 3$ sec) $r/t_b$ is constant, reflecting the relative shapes of the two input spectra. Typical translational periods for industrial structures range from approximately 0.3 to 1 sec, and typical $k_2/k_1$ is about 0.1 to 1.0. In this range, $r/t_b$ plots linearly on a log-log scale decreasing as $T$ increases. The values of $r/t_b$ in the range of interest vary from 0.6 to 3.0. For a $t_b$ of 0.1 sec, the torsional response is thus 6 to 30 percent of the translational.

Let us consider a somewhat extreme but still realistic case of $t_b = 0.2$ (long structure on a soft soil), with a translational period of 0.2 sec. If the roof is very flexible ($k_2/k_1 = 0.01$), $r$ is approximately $4.2 \times 0.2 = 0.84$. Using the 100-30 percent rule to combine the effects of translational and torsional ground motions, the
translational response would be increased by a factor of $1.0 + 0.3 \times 0.84 = 1.25$. This indicates that the rotational component of the earthquake input generates increases up to 25 percent in the frames response calculated considering only translational input. Application of the SRSS combination criterion also yields a 31 percent increase. However, if, as it is advisable in highly seismic areas, the roof has some in-plane stiffness, say such that $k_2/k_1 = 1$, $r$ is $2.9 \times 0.2 = 0.58$, and the combination rules yield response increases between 16 and 17 percent.

![Graph](image)

Fig. 3. Values of $r/t_b$, where $r = d_\phi/d_1$ and $d_\phi$ = exterior frame displacement due to torsional input and $d_1$ = exterior frame displacement due to translational input.

Another response parameter relevant for design is the shear force in the roof. In our model, only the torsional ground motion produces relative displacements and, consequently, shear forces between the frames. To assess the importance of these forces, Fig. 4 presents, in terms of the period $T$, the ratio $p$, of the shear force $F_{1-2}$, between frames 1 and 2, to the shear force in the exterior frame due to the translational earthquake input, $V_1$. No combination of different components of ground motion was needed to calculate the values of $F_{1-2}$, since it corresponds to only one (rotational) component. $V_1$ was selected as the normalizing factor because, for a prescribed translational period, this frame shear force remains the same independent of $k_2/k_1$. For the case considered in the previous paragraph ($T = 0.2$ sec, $k_2/k_1 = 1$) we have $p = 0.75$, i.e., the diaphragm shear in the exterior bay is 75 percent of the shear force in the exterior frame. If the roof relatively is more rigid ($k_2/k_1 = 10$), $p$ increases to 1.6. This primarily results from the increased chord stiffness, which appears to impact the shear more so than the attendant decrease in relative roof distortion due to torsional input. On the other hand, a very flexible roof ($k_2/k_1 = 0.01$) results in $p \approx 0.02$. Note that in all three cases, a conventional seismic analysis, ignoring accidental torsion, would indicate the frame shear force $V_1$. The response quantity represented by the above values of $p$ illustrates the importance of incorporating the chord stiffness in the seismic analysis of industrial facilities.

It is pertinent to examine to what extent the above results can be accounted for by using the concept of accidental eccentricity. For this purpose, let us consider that such eccentricity is defined as $\alpha \beta$ (typically, $\alpha = 0.05$ or 0.10). Assuming that a center of rotation can be defined, at least in an approximate manner, the accidental torsional moment could be distributed as forces applied to the masses of the structure. Each mass will sustain a force proportional to the mass itself times its distance to such center, say $F_\phi = \beta m_j x_j$. $\beta$ is calculated from the condition that the torsional moment is $V e = \beta \sum m_j x_j^2$, where $V$ is the total shear force.
due to translational ground motion. Assuming that these forces act directly over each frame, for the model studied in this paper, this approach gives \( r = \frac{d_4}{d_1} = 4.26\alpha \). If we consider \( \alpha = 0.1 \), from Fig. 3 we conclude that the accidental eccentricity provides conservative results for all cases analyzed in this paper if \( t_b < 0.1 \) and \( T \geq 0.2 \text{ sec} \). To provide a more accurate estimate, \( \alpha \) should be a function of both the period and the transit time. It should be noted that structural analysis of the entire system is required to calculate the shear forces in roof members, between the vertical frames.

![Graph showing the ratio of shear force between frames 1 and 2 to the shear force due to translational input in the exterior frame](image)

**Fig. 4.** Ratio \( p \) of the shear force between frames 1 and 2 \( (F_{1-2}) \) to the shear force due to translational input in the exterior frame \( (V_1) \).

**CONCLUDING REMARKS**

The results presented herein show that the effects of the torsional component of ground motion on the total seismic forces generated in vertical frames of regular industrial facilities depend strongly on the fundamental translational period of the structure and on the relative stiffness of the horizontal roof members connecting the frames. The effects of torsional input motion are larger for structures with smaller translational periods, independent of the degree of coupling between frames, and the impact is much more significant when the floors are flexible compared to the vertical frames. As the torsional spectrum is in direct proportion of the transit time \( t_b \), its effects in linear elastic systems are also linearly proportional to \( t_b \).

One of the more important observations of this study is that forces in roof members can be, in some instances, significantly higher than the forces generated in the vertical frames. While the use of reasonable accidental eccentricities can provide conservative estimates of seismic forces in the vertical frames, in general, representative estimates of forces in the roof can be only obtained through a three-dimensional dynamic analysis that accounts for the rotational component of ground motion. Present commercial finite element codes can be easily updated to incorporate response spectrum procedures such as the one presented here. This study suggest the need to incorporate into building codes the option of using torsional spectra in lieu of or as a supplement to accidental eccentricities. Research on analytical procedures and on field measurements should continue to better define torsional spectra or incoherent spectra (Der Kiureghian, 1995) for different seismotectonic and site conditions. These type of spectra can be used not only in industrial buildings but also in other long structures such as bridges and piping systems.
The numerical results and observations presented in the previous section are consistent with the structural model used in this study as well as with the relative shapes of the torsional and the translational spectra. Additional work is being presently conducted by the authors using models which include out-of-plane degrees of freedom for the frames and building stiffness in the direction perpendicular to the translational ground motion. This work also considers incoherent motions at each support points.

REFERENCES

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