A METHOD OF AZIMUTH EXAMINATION FOR SEISMOMETERS
USING OBSERVED EARTHQUAKE

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ABSTRACT

The relative timing and azimuthal error can be often found out in the earthquake observed records. The examination method of the timing and azimuthal error is applied difference of vector of particle motions between two points.

KEYWORDS
Azimuth error; Seismometer; Observed Earthquake; Vector Method; Downhole; Timing error

INTRODUCTION

There are two methods for examination of a azimuth error of seismometers, one is correlation method using coherence or cross-correlation functions, another is the method using orbit circles of two seismogram. The method of this study is close to the method using orbit, because of using the long period components in earthquake records. This proposed method is called Vector method. Azimuthal error with Vector method is obtained by the phase difference of vectors constructed the two horizontal components at x,y (or NS, EW) direction at simultaneous between two locations in downhole. The result by this method is affected by time shift between records. If the time shift in azimuthal error estimation is detected, timing error between two observed points is obtained. This method assume that long period components in earthquake show the same motion at the near locations.

METHOD OF AZIMUTHAL ERROR ESTIMATION

Without time shift between two observed points

Where recorded orthogonal horizontal motions at point A are XA, YA and point B are XB, YB respectively as
shown in figure 1. Relative coordinates of recorded components with point A and B at some time are shown in figure 2. Ground motion at some time is denoted vector in figure 2. Without timing and azimuthal error at the both recorded point A and B, same earthquake records should be obtained. If there is azimuthal error, resultant vector of (XA,YA) and (XB,YB) are same, but YA divided by XA and YB divided by XB are not same. Where the azimuthal angle of A is $\alpha_t$, and B is $\beta_t$ in equation (1).

$$\alpha_t = \tan^{-1} \left( \frac{Y_A}{X_A} \right) \quad \beta_t = \tan^{-1} \left( \frac{Y_B}{X_B} \right)$$

(1)

The difference of azimuthal angle $\theta_t$ at every moment is quantity $\alpha_t$ minus $\beta_t$.

$$\theta_t = \alpha_t - \beta_t \quad \text{..................................................} (2)$$

Accuracy by this method depend on the accuracy of ratio of the two component (XA,YA and XB,YB), does not be affected amplifier of seismometers, but should be taken care of the phase characteristics of seismometers. Another form of the solution of the azimuthal error is given as follows. Seismometer A,B are calibrated correctly, and using the amplitudes of A,B(XA,YA,XB,YB), azimuthal error between based and referenced record is estimated as equation (4). Where based record is A, referenced record B and azimuthal error $\theta_t$, following equation is derived.

$$\begin{bmatrix} \cos \theta_t & -\sin \theta_t \\ \sin \theta_t & \cos \theta_t \end{bmatrix} \begin{bmatrix} X_B \\ Y_B \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \end{bmatrix} \quad \text{..................................................} (3)$$

The solution of azimuthal error $\theta_t$ are

$$\cos \theta_t = \frac{X_A X_B + Y_A Y_B}{X_A^2 + Y_A^2} \quad \text{..................................................} (4)$$

$$\sin \theta_t = \frac{Y_A X_B - X_A Y_B}{X_B^2 + Y_B^2} \quad \text{..................................................} (5)$$
The azimuthal errors calculated by equation(2),(4) and (5) are agree with each other. Therefore, the azimuthal errors can be calculated by three kind equations, in the following studies, equation(2) are used for examination.

With time shift between record points

When observed points A and B are installed with long distant each other than wave length of long period in earthquake motion or having the timing lag between A and B. In the problem without time shift, whose steady-state solution is given by (6) and (7),

\[
\begin{align*}
X_A &= \sin(\omega t) \\
Y_A &= \sin(\omega t + \phi) \\
\end{align*}
\] ............................... (6)

\[
\begin{align*}
X_B &= X_A \cos \theta_t + Y_A \sin \theta_t \\
Y_B &= -X_A \sin \theta_t + Y_A \cos \theta_t \\
\end{align*}
\] ............................... (7)

\[
\begin{align*}
X_B &= \sin[(\omega(t+\Delta t)) \cos \theta_t + \sin[(\omega(t+\Delta t)) + \phi] \sin \theta_t] \\
Y_B &= -\sin[(\omega(t+\Delta t)) \sin \theta_t + \sin[(\omega(t+\Delta t)) + \phi] \cos \theta_t] \\
\end{align*}
\] ............................... (8)

Equation (7) and (8) express the different resultant vectors.

EXAMINATION OF THEORY BY SIMPLE WAVE

Using simplified sinusoidal waves, characteristics of solution are examined. Where period of sinusoidal wave, phase \( \phi \) and azimuth error are assumed as \( 2 \pi \), (30° and 60°) and \( \theta_t = 30^\circ \) respectively. Timing error are adopted the 4 cases that is 0.0, 0.01, 0.02, -0.01 second. The recorded wave form of seismometer A and B are shown in figure 3, where phase \( \phi \), azimuthal error \( \theta_t \), timing error \( \Delta t \) are 30°, 60°, and 0.0 respectively. The solution by equation (2) are shown in figure 4 with variable \( \Delta t \) 0.0, 0.01, 0.02, -0.01 sec. In the case of timing error nothing, correct solution \( \theta_t \) is obtained as a 30° that is upper figure in figure 4. The value \( \theta_t \) increase with timing error \( \Delta t \). The fluctuation of solution increase with a timing error. The minimize of p-p values of solution are obtained by timing shift the record B against record A. The value of minimize timing error agree with time difference of wave traveling between point A and B. This proposed method can be obtained the azimuthal and timing error simultaneously. The azimuthal error solution are shown in figure 5 which phase \( \phi \) is 60° The p-p values of solution is larger than \( \phi 30^\circ \). When timing error \( \Delta t \) is also zero, correct solution \( \theta_t \) is obtained as a 30°. The correct solution can be obtained by convergence calculation while the time shifts are exist between A and B.

![Fig. 3 Simple waves for examination at A and B.](image)
Fig. 4 Azimuthal error at $\phi = 30^\circ$

Fig. 5 Azimuthal error at $\phi = 60^\circ$
Flow Chart of Calculation Procedure

The procedure of examination is shown in figure 6. Concrete procedure of flow chart is explained in the example of calculation of azimuthal error by using observed records.

![Flow Chart of Calculation Procedure](image)

Figure 6 Flow chart of procedure

An Example of Examination Using Observed Records

Horizontal record motions at depth of GL-1.5m are shown as XA, YA in figure 7, that motions are assumed no azimuthal error. XB, YB in figure 7 are referenced motions installing at the depth of GL-75m for azimuthal error examination. The earthquake magnitudes are 6.5. The amplitudes showing in figure 7 are real values in Gal which is calibrated correctly. The Fourier amplitudes in long period domain on XA is shown in figure 8, black circular point is the frequency in regard to the examination. Its frequency is 0.099 Hz. The components of filtered wave is shown in figure 9, and its wave form became to sinusoidal wave through the all duration times. The amplitude and wave form of XA is near to XB and YA near to YB. The solution by equation(2) without caring out the process of time shift are shown in the top of figure 10. The p-p value (Δt=0.0) of the top of figure is larger than another figures. The fluctuation of the solutions decreases in proportion with time shift Δt. When the time shift Δt is -0.06, the fluctuation become to minimize. According this examination method, azimuthal and timing error are obtained as 4.196 degrees and -0.06 second respectively. The solution by orbit is shown in figure 11, and obtained as a +4.5 degrees. This value is near by vector method.
CONCLUSIONS and REMARKS

The azimuthal error by orbit method can not be obtained in the case of $\phi = 90^\circ$, because of orbit traces are reduced to circles. Using proposed this vector method, azimuthal and timing error are obtained simultaneously with high accuracy. For the sake of keeping high accuracy for determination, some items should be considered as follows;

1) The comparatively large scaled amplitude of long period components in Fourier spectrum should be adopted for calculation.

2) The amplitudes of seismometers should be calibrated correctly.

3) The digitized time intervals of records should be fine less than or equal to 0.005 second.

![Graphs of XA, YA, XB, YB](image1)

Fig. 7 Observed records (XA, YA, XB, YB)

![Fourier spectrum graph](image2)

Fig. 8 Fourier amplitudes of XA.
Fig. 9 Filtered waves

Fig. 10 Azimuthal error during timing error.

Fig. 11 Orbit circles of A and B