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RELIABLE CONTROL DESIGN FOR BUILDINGS UNDER SEISMIC EXCITATION*

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ABSTRACT

In this paper we present control design and analysis methods that provide desirable levels of performance and simultaneously account for actuator and sensor reliability (or failure) for buildings under seismic excitations. Performance is defined in terms of disturbance attenuation (H-infinity norm) from disturbances to controlled outputs of the system. The reliability of actuators and sensors refers to the deviation of actual control forces or actual sensor measurements from their ideal levels. Results for a six-story building are used to demonstrate the effectiveness of the control analysis and design method presented.

KEYWORDS

Reliability; robust control; quadratic stability; linear matrix inequalities.

INTRODUCTION

In earthquake engineering applications, the reliability issue of control systems is of great concern. Often the control hardware (e.g., actuator and/or sensor) is embedded in the structure and maybe hard to test and maintain. Also, the actuators are used very infrequently and function at large output levels only during severe earthquake episodes. As a result, during the earthquake episode, feedback signals measured by sensors may deviate from the actual values whereas forces generated by the actuators may differ from the designed control force levels. This form of malfunction of the control system may result in a detrimental effect on the controlled structure. Consequently, a control design methodology that can incorporate the actuator and sensor reliability information is highly desirable.

The framework used in the paper is deterministic and does not require statistical data. It also differs from traditional failure detection techniques in the sense that it results in controllers that immediately perform well in the face of sudden malfunction. Veillette *et al.* (1992), and references therein, have also examined reliability of control systems for complete sensor and actuator outages. While the control techniques presented in this paper could perform this type of analysis and design, the approach used here is meant for systems where the actuator or sensor signals have gain deviations (or variations), particularly during the start-up period, but are not necessarily subject to complete failure. These variations could come from power fluctuations, non-linearities, partial actuator failure for parallel type actuation or any type of variation that causes time varying gain changes in the sensor or actuator signals.

We modify the H-infinity approach to incorporate the reliability concerns discussed above. The analysis method can be used to evaluate the reliable performance of actuators and sensors for a given controller. This aids in identifying the critical hardware components (for service and maintenance schedule). Reliable

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performance implies the system maintains a given level of disturbance attenuation in the presence of the modeled actuator and/or sensor uncertainty. The synthesis problem directly uses data on the hardware limitations to yield the best reliable performance possible, given the reliability (or malfunction) of the actuators and/or sensors. The analysis and state feedback synthesis problems are reduced to a finite dimensional convex programming problem. Under some simplifying assumptions the state feedback synthesis problem reduces to a single Linear Matrix Inequality. This greatly reduces the numerical intensity and allows for systems with many actuators to be analyzed. While the general output feedback case is difficult to solve, our approach attempts to recover the reliable performance of the state feedback controller. Conditions for which this is possible are presented.

Results are presented for a six-story shear-beam-type building. The achievable disturbance attenuation level is computed for a wide range of actuator uncertainty. Significant improvement in the disturbance attenuation level is achieved compared a nominal H-infinity design, showing the benefit of incorporating the uncertainties into the design process. Simulation results also are presented, showing significant reduction in interstory drift do to an earthquake episode even in the presence of significant actuator variations.

ACTUATOR AND SENSOR RELIABILITY ANALYSIS

The basic approach for actuator and sensor reliability analysis is to include actuator and/or sensor uncertainty when analyzing the performance of a system. Here, the actual control force and actual sensor measurement are represented by

$$u_{i,actual}(t) = \left[1+\delta_{u_i}(t)\right]u_i(t) \text{ and } y_{i,actual}(t) = \left[1+\delta_{y_i}(t)\right]y_i(t) \tag{1}$$

where $u_i(t)$ and $y_i(t)$ are the ideal control force and ideal sensor measurement. This representation is depicted by the block diagram in Figure 1. These uncertainties represent changes from the nominal actuator or sensor signal due to variations mentioned above which are not necessarily fixed in time. The corresponding state space equations are given by

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2\left[I + \Delta_u(t)\right]u(t) \tag{2}$$

$$z(t) = C_1 x(t) + D_{11} w(t) + D_{12}\left[I + \Delta_u(t)\right]u(t) \tag{3}$$

$$y(t) = \left[I + \Delta_y(t)\right]C_2 x(t) + \left[I + \Delta_y(t)\right]D_{21} w(t) \tag{4}$$

where

$$\Delta_u(t) \in \underline{\Delta}_u = \left\{diag\left[\delta_{u_1}(t),\, \delta_{u_2}(t),\, \ldots, \delta_{u_m}(t)\right] : \delta_{u_i}(t) \in \left[-\alpha_{u_i}, \alpha_{u_i}\right]\right\} \tag{5}$$

$$\Delta_y(t) \in \underline{\Delta}_y = \left\{diag\left[\delta_{y_1}(t),\, \delta_{y_2}(t),\, \ldots, \delta_{y_l}(t)\right] : \delta_{y_i}(t) \in \left[-\alpha_{y_i}, \alpha_{y_i}\right]\right\} \tag{6}$$

$$\Delta(t) = diag\left[\Delta_u(t), \Delta_y(t)\right] \in \underline{\Delta} = \left\{\begin{matrix} diag\left[\delta_{u_1}(t),\, \delta_{u_2}(t),\, \ldots, \delta_{u_m}(t), \delta_{y_1}(t),\, \delta_{y_2}(t),\, \ldots, \delta_{y_l}(t)\right] : \\ \delta_{u_i}(t) \in \left[-\alpha_{u_i}, \alpha_{u_i}\right],\; \delta_{y_i}(t) \in \left[-\alpha_{y_i}, \alpha_{y_i}\right] \end{matrix}\right\} \tag{7}$$

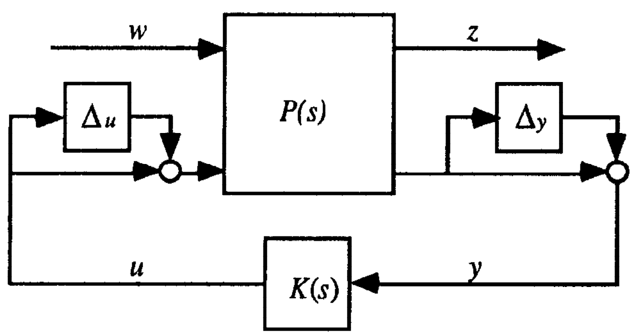

Fig. 1. Actuator and sensor reliability framework

The actuator and sensor uncertainties, $\delta_{u_i}(t)$ and $\delta_{y_i}(t)$, are real time varying, but bounded parameters. With out loss of generality we assume the upper and lower bounds have the same magnitude. This may be accomplished by adjusting the nominal B_2 and C_2 matrices. For future reference we shall denote the vertex sets

$$\underline{\Delta}_{vex} = \left\{diag\left[\delta_{u_1}, \delta_{u_2}, \ldots, \delta_{u_m}, \delta_{y_1}, \delta_{y_2}, \ldots, \delta_{y_l}\right]: \ \delta_{u_i} = \pm\alpha_{u_i}, \delta_{y_i} = \pm\alpha_{y_i}\right\} \tag{8}$$

$$\underline{\Delta}_{u_{vex}} = \left\{diag\left[\delta_{u_1}, \delta_{u_2}, \ldots, \delta_{u_m}\right]: \ \delta_{u_i} = \pm\alpha_{u_i}\right\} \tag{9}$$

$$\underline{\Delta}_{y_{vex}} = \left\{diag\left[\delta_{y_1}, \delta_{y_2}, \ldots, \delta_{y_l}\right]: \ \delta_{y_i} = \pm\alpha_{y_i}\right\} \tag{10}$$

It is easy to see there are 2^{l+m} vertices of $\underline{\Delta}_{vex}$. For a dynamic compensator of the form

$$\begin{aligned} \dot{q}(t) &= A_k q(t) + B_k y(t) \\ u(t) &= C_k q(t) \end{aligned} \tag{11}$$

the closed loop system is given by

$$\begin{bmatrix} \dot{x} \\ \dot{q} \\ z \end{bmatrix} = \begin{bmatrix} A & B_2(I+\Delta_u)C_k & B_1 \\ B_k(I+\Delta_y)C_2 & A_k & B_k(I+\Delta_y)D_{21} \\ C_1 & D_{12}(I+\Delta_u)C_k & D_{11} \end{bmatrix} \begin{bmatrix} x \\ q \\ w \end{bmatrix} \text{ or } \begin{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{q} \end{bmatrix} \\ z \end{bmatrix} = \begin{bmatrix} A_\Delta & B_\Delta \\ C_\Delta & D_{11} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix} \\ w \end{bmatrix} \tag{12}$$

We consider the resulting closed loop system to have reliable performance if the controlled output, $z(t)$, satisfies the following equation in the presence of actuator and/or sensor uncertainties, assuming zero initial conditions:

$$\int_0^\infty z^*(t)z(t)\,dt \le \gamma^2 \int_0^\infty w^*(t)\,w(t)\,dt \tag{13}$$

where γ serves as the measure of performance. Equivalently (13) can be written as $\|z(t)\|_2^2 \le \gamma^2 \|w(t)\|_2^2$ where we have used the standard definition of the L_2 norm of a signal $\|z(t)\|_2^2 = \int_0^\infty z^*(t)z(t)\,dt$. As γ becomes small, the effects of the disturbance, $w(t)$, on $z(t)$ is diminished. For time-invariant systems, the notation in (13) is interchangeable with having the infinity norm of the transfer function from w to z, T_{zw}, be less than γ; i.e. $\|T_{zw}(s)\|_\infty \le \gamma$ where the infinity norm is defined as $\|T_{zw}(s)\|_\infty \equiv \sup_\omega \bar{\sigma}[T_{zw}(j\omega)]$ and $\bar{\sigma}$ denotes the maximum singular value. If (13) holds then we will say the system has achieved a disturbance attenuation of γ.

The closed loop will have reliable performance if it satisfies the strongly robust H_∞ performance criteria of Zhou *et al.* (1995). This is true if and only if $R_1 = \gamma^2 I - D_{11}^* D_{11} > 0$ and there exists an $X = X^* > 0$ such that

$$\Phi = XA_\Delta + A_\Delta^* X + \left(XB_\Delta + C_\Delta^* D_{11}\right) R_1^{-1} \left(XB_\Delta + C_\Delta^* D_{11}\right)^* + C_\Delta^* C_\Delta < 0 \text{ for all } \ \Delta \in \underline{\Delta} \tag{14}$$

Here, $\Phi < 0$ is referred to as an Algebraic Riccati Inequality (ARI). Application of the Schur complement formula to (14) yields

$$Q_\Delta = \begin{bmatrix} A_\Delta^* X + XA_\Delta & XB_\Delta + C_\Delta^* D_{11} & C_\Delta^* \\ B_\Delta^* X + D_{11} C_\Delta & -R_1 & 0 \\ C_\Delta & 0 & -I \end{bmatrix} < 0 \ \text{ for all } \ \Delta \in \underline{\Delta}_{vex} \tag{15}$$

The determination of whether there exists a matrix $X = X^* > 0$ such that (15) is satisfied is a set of Linear Matrix Inequalities (LMI's). Note that only the vertex set needs to be checked since Δ appears linearly in Q_Δ. Determining the feasibility of a set of LMI's is a convex programming problem for which efficient algorithms have been developed.

ACTUATOR RELIABILITY VIA STATE FEEDBACK

In this section we discuss the synthesis of state feedback controllers for the actuator reliability problem. This problem can also be reduced to a convex programming problem. Consider the system

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2[I + \Delta_u(t)]u(t) \tag{16}$$
$$z(t) = C_1 x(t) + D_{11} w(t) + D_{12}[I + \Delta_u(t)]u(t) \tag{17}$$
$$y(t) = x(t) \tag{18}$$

There exists a state feedback controller such that the above closed loop system has reliable actuator performance if and only if $R_1 = \gamma^2 I - D_{11}^* D_{11} > 0$ and there exists matrices W and $Y = Y^* > 0$ such that

$$\tilde{Q}_\Delta = \begin{bmatrix} Q_{11} & Q_{12} & YC_1^* + W^*(I+\Delta_u)D_{12}^* \\ Q_{12}^* & -R_1 & 0 \\ C_1 Y + D_{12}(I+\Delta_u)W & 0 & -I \end{bmatrix} < 0 \quad \text{for all} \quad \Delta_u \in \underline{\Delta}_{uvex} \tag{19}$$

where

$$Q_{11} = YA^* + AY + W^*(I+\Delta_u)B_2^* + B_2(I+\Delta_u)W$$
$$Q_{12} = B_1 + YC_1^* D_{11} + W^*(I+\Delta_u)D_{12}^* D_{11}$$

The state feedback control law is given by $u(t) = WY^{-1}x(t)$. The above result can be simplified greatly if

$$D_{11} = 0, \; D_{12}^* C_1 = 0 \text{ and } D_{12}^* D_{12} = R_u^2 = diag\left[r_{u_1}^2, r_{u_2}^2, \ldots, r_{u_m}^2\right] > 0 \tag{20}$$

This is the case when there is no direct feedthrough term from *w(t)* to *z(t)* and the controlled output vector consists of two variables appended into one vector, where the first is a function of the states and the second is a weighting on the controls. This is usually the case in control system design methodologies. By decreasing the weighting on control, we place more emphasis on the states. The algebraic Riccati inequality of (14) becomes

$$\Phi = XA + A^*X + X\left\{\gamma^{-2}B_1B_1^* + \left[\hat{B}_2 + W^*\left(I+\Delta_u^*\right)\right]\left[\hat{B}_2 + W^*\left(I+\Delta_u^*\right)\right]^* - \hat{B}_2\hat{B}_2^*\right\}X + C_1^*C_1 < 0 \tag{21}$$

where $\hat{B}_2 = B_2 R_u^{-1}$. Here the control law is given by $u(t) = R_u^{-1}WXx(t)$. Selecting the central state feedback solution $W = -\hat{B}_2^*$ we get

$$\Phi = XA + A^*X + X\left[\gamma^{-2}B_1B_1^* - \hat{B}_2\left(I - \Delta_u^2\right)\hat{B}_2^*\right]X + C_1^*C_1 < 0 \text{ for all } \Delta_u \in \underline{\Delta}_u \tag{22}$$

This is true if and only if

$$XA + A^*X + X\left[\gamma^{-2}B_1B_1^* - \hat{B}_2\left(I - \hat{\Delta}_u^2\right)\hat{B}_2^*\right]X + C_1^*C_1 < 0 \tag{23}$$

where

$$\hat{\Delta}_u = diag\left[\alpha_{u_1}, \alpha_{u_2}, \ldots, \alpha_{u_m}\right] \tag{24}$$

Thus the central state feedback case reduces the number of LMIs from 2^m to only 1. In fact a solution can be obtained by solving the algebraic Riccati equation (ARE)

$$XA + A^*X + X\left[\gamma^{-2}B_1B_1^* - \hat{B}_2\left(I - \hat{\Delta}_u^2\right)\hat{B}_2^*\right]X + C_1^*C_1 + \varepsilon I = 0 \tag{25}$$

The solution to (25) satisfies (23) so that standard ARE solvers may be used. This greatly reduces the computational requirements for systems with many actuators for this type of analysis or design. Note that (25) becomes the standard H-infinity ARE when $\hat{\Delta}_u = 0$. This allows the achievable disturbance attenuation level to be computed as a function of actuator uncertainty.

OUTPUT FEEDBACK

The polytopic nature of the actuator reliability problem, makes the output feedback case difficult to solve. The fact that uncertainty enters the B_2 matrix further complicates this problem. However, there are conditions for which reliability performance of the state feedback case may be recovered. If the system

$$\dot{q} = \left(A + \gamma^{-2}B_1B_1^*P\right)q + \begin{bmatrix} B_1 & B_2 \end{bmatrix}\hat{u} \tag{26}$$

$$\hat{z} = C_2q + \begin{bmatrix} D_{21} & 0 \end{bmatrix}\hat{u} \tag{27}$$

is left invertable and minimum phase then there exists an observer that can come arbitrarily close to recovering the reliable performance of the state feedback design.

APPLICATION TO SIX-STORY SHEAR-BEAM-TYPE BUILDING

Consider the building structure shown in Fig. 2, modeled by an n-degrees-of-freedom system

$$M\ddot{\overline{x}}(t) + C\dot{\overline{x}}(t) + K\overline{x}(t) = \overline{B}\left[I + \Delta_u(t)\right]u(t) + \overline{G}w(t) \tag{28}$$

where $\overline{x}(t) \in R^n$ is an n-vector denoting the deformation corresponding to each degree of freedom (e.g., interstory drift). Matrices M, K, and C are n X n mass, stiffness and damping matrices, respectively, and $w(t)$ is the disturbance vector representing the loading due to earthquake ground motion. The m-dimension control vector $u(t)$ corresponds to the actuator forces (generated via active bracing systems (ABS) or an active mass damper (AMD) for example) and $\Delta_u(t)$ represents the actuator uncertainty as described in (2). (4.1) can be put in the form of (19) with the following, where $x(t)$ is the $2n$ state vector:

$$x(t) = \begin{bmatrix} \overline{x}(t) \\ \dot{\overline{x}}(t) \end{bmatrix},\ A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ -M^{-1}\overline{G} \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ -M^{-1}\overline{B} \end{bmatrix} \tag{29}$$

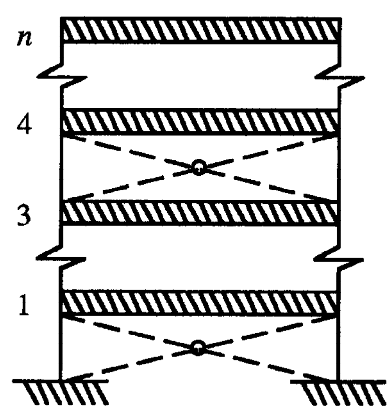

Fig. 2. Model of full-scale building with active bracing system

We consider a six-story full-scale shear-beam-type building with identical floors, for which a lumped-parameter model is used in simulations. There are two actuators, one on the first floor an and another on the third floor, both active bracing systems. The model is similar to the one used in Schmitendorf *et al.* (1993a, b, 1994). The mass of each floor, and the stiffness and damping coefficients of each story unit are; $m_i = 345.6$ metric ton; $k_i = 340{,}400$ kN; $c_i = 2{,}937$ kN-s/m. These values result in a first vibrational mode of 1.2 Hz, with a damping ratio of approximately 3.2%. The total building weight, which is used later for comparison with peak actuator force, is 20,342 kN.

Using $\bar{x}(t)$ as the interstory drift and $w(t)$ as the earthquake ground acceleration, straight forward manipulation results in the following for (29)

$$M^{-1}K = \frac{k_i}{m_i}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}, \quad M^{-1}C = \frac{c_i}{m_i}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$M^{-1}\overline{B} = \frac{1}{m_i}\begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } M^{-1}\overline{G} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We consider a controlled output vector

$$z(t) = \begin{bmatrix} H \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ R_u \end{bmatrix} [I + \Delta_u(t)] u(t) \tag{30}$$

where H is an $r \times n$ matrix and R_u is a 2×2 diagonal matrix. This form satisfies the simplifying assumptions in (20). The weighted control is included in the z-vector to allow penalizing large control input forces. By decreasing R_u we diminish the weighting on the control and place more emphasis on the states.

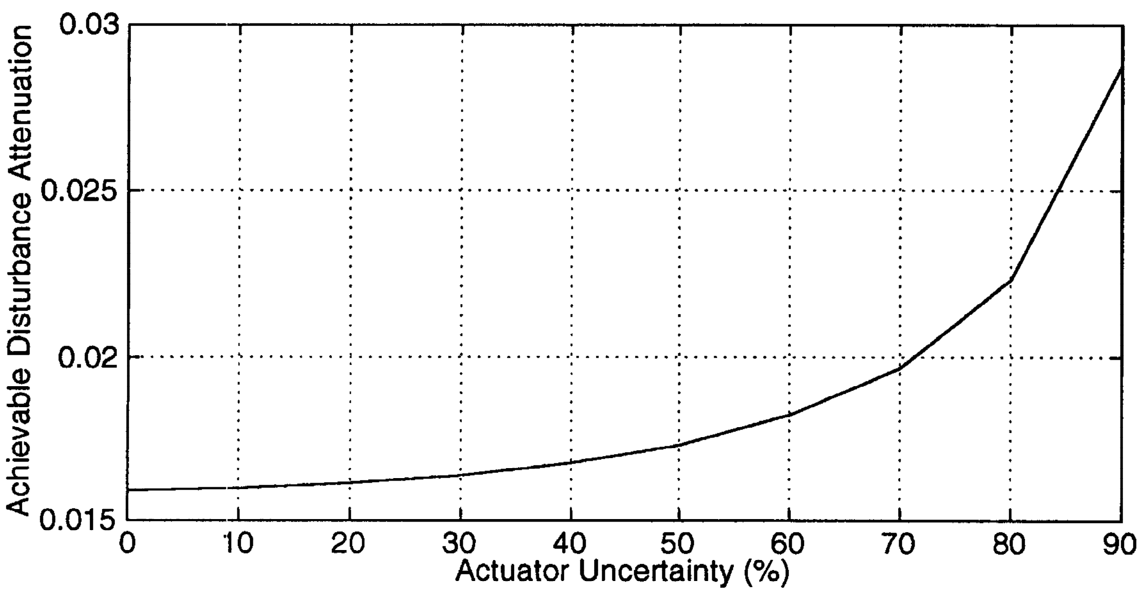

Fig. 3. Achievable disturbance attenuation levels for actuator reliability

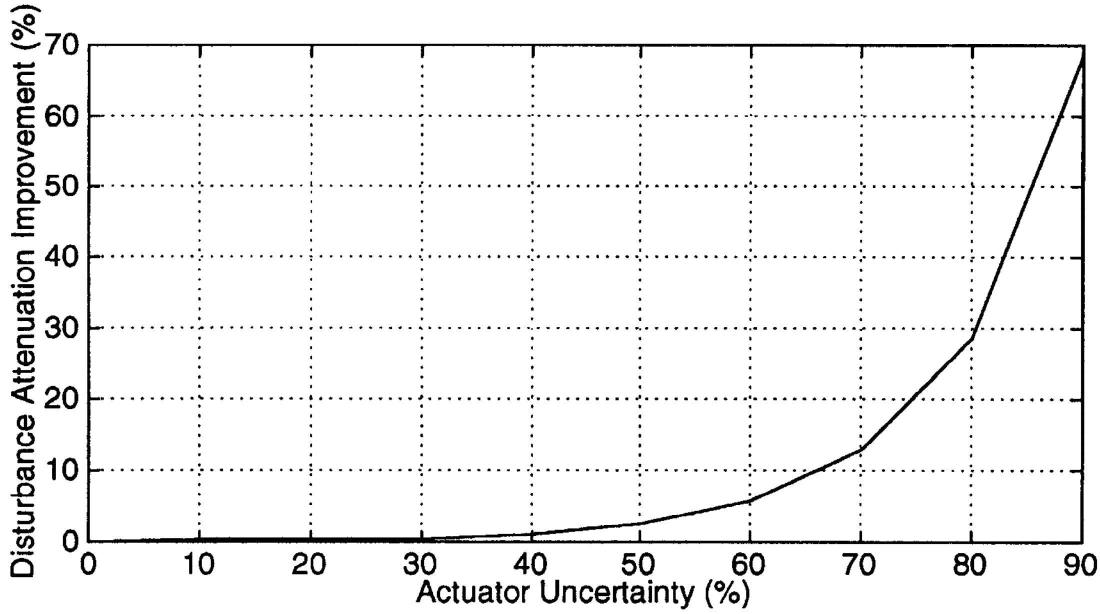

Fig. 4. Improvement in disturbance attenuation level

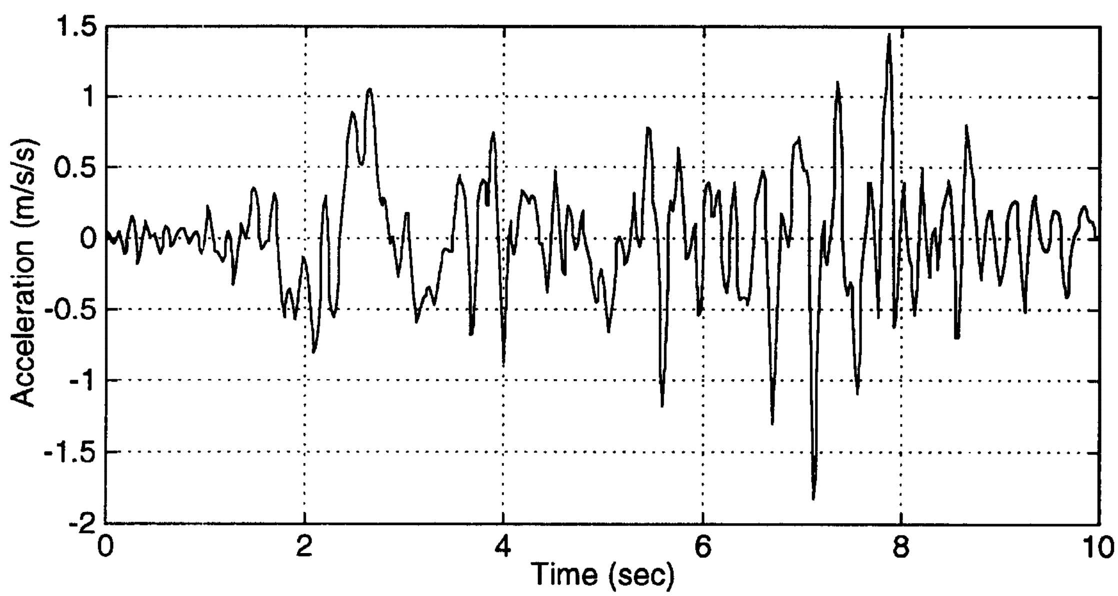

Fig. 5. Pacoima earthquake ground acceleration time history

Under strong earthquakes, one of the main objectives of control is to reduce interstory drift. This leads to selecting H that weights all interstory drifts yielding $H = [I_6 \quad 0_6]$, where I_6 is a 6×6 identity matrix and 0_6 is a 6×6 zero matrix. The control weighting matrix, R_u, was chosen to be $R_u = diag\left(2.5\text{x}10^{-6},\ 2.2\text{x}10^{-6}\right)$. The achievable disturbance attenuation level was computed for actuator uncertainties ranging from 0 to 90% and is shown in Figure 3. The open loop disturbance attenuation level is 0.1475, indicating that we were able to achieve significant disturbance attenuation in the presence of actuator variability. For comparison, the nominal H-infinity controller $\left(\hat{\Delta}_u = 0\right)$ was analyzed by computing the achieved disturbance attenuation level for the same range of actuator uncertainties. Figure 4 shows the improvement in the disturbance attenuation level by incorporating actuator uncertainties in the design process. Significant improvement in the disturbance attenuation level is achieved at the higher range of actuator uncertainties compared to the nominal H-infinity design.

In civil engineering structures, it is also important to reduce the peak response of the interstory drifts of the structure during strong earthquake episodes. To evaluate this simulations were performed. The earthquake ground motion used for the simulation study is the Pacoima earthquake scaled uniformly to a peak ground acceleration of approximately $0.2g$, see Fig. 5. Since the model is linear, the structures response to a stronger or weaker ground motion can be obtained by a simple scaling. For, example, for a moderate earthquake with peak ground acceleration of $0.3g$, the drifts and forces in Table 1 should be increased 50%. Due to the fact that the most intense portion of the earthquake occurred during the first 10 seconds, the simulation results reported below concern this duration only.

Nominal time histories of the response quantities of the structure have been computed for the open-loop system (Case 1), the nominal closed loop system (Case 2) and 4 cases of the closed loop system with actuator variations (Cases 3-6). The design simulated corresponds to a 75% actuator uncertainty. The actuator variations simulated were time-invariant perturbations corresponding to the four corners of the uncertainty parameter space for a 75% actuator uncertainty. The results are given in Table 1. The maximum drifts are typically reduced to 40–50% of the corresponding values for the open-loop case. Since the total building weight is approximately 20,342 kN, the maximum actual control forces are less than 13% of the total weight.

Table 1. Maximum response quantities from simulation results

Case	Interstory Drift (cm)						Actuator Forces (kN)		Actuator Perturbation	
	Floor 1	Floor 2	Floor 3	Floor 4	Floor 5	Floor 6	Act 1	Act 2	δ_{u1}	δ_{u2}
1	1.06	0.85	0.80	0.64	0.47	0.24	n/a	n/a	n/a	n/a
2	0.49	0.63	0.52	0.38	0.26	0.13	2,056	1,467	0	0
3	0.73	0.76	0.63	0.43	0.34	0.18	1,056	861	-0.75	-0.75
4	0.54	0.65	0.53	0.32	0.29	0.19	2,594	684	0.75	-0.75
5	0.64	0.67	0.55	0.57	0.29	0.15	855	2,191	-0.75	0.75
6	0.44	0.60	0.49	0.44	0.24	0.14	2,578	1,876	0.75	0.75

CONCLUSIONS

In this paper we presented a method for the analysis and design of systems to achieve reliable performance. Reliable performance implies the system maintains a given level of disturbance attenuation (H-infinity norm) in the presence of actuator and sensor uncertainties. The analysis case reduced to solving a set of LMIs for which efficient algorithms have been developed. The synthesis of state feedback controllers reduced to solving a single LMI, where a solution could also be obtained by solving an algebraic Riccati equation. This greatly reduces the numerical intensity and allows state feedback controllers to be easily generated for systems with many actuators. Conditions for which the reliable performance of the state feedback controller may be recovered were also presented for the output feedback case. Results were presented for a six-story building to illustrate the effectiveness of the control analysis and design methods.

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