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A COMPOSITE SEISMIC SOURCE MODEL BASED ON FRACTAL DISTRIBUTION OF CIRCULAR CRACKS

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ABSTRACT

Seismic sources present important heterogeneities in both final slip distribution and rupture velocities. Classical dislocation models with constant rise-time on the fault plane do not take into this complexity and therefore do not generate high-frequency. In this study, following the idea proposed by Zeng et al. (1993), we propose a heterogeneous source model based on elementary sources defined as circular shear-cracks. A fractal distribution of the crack size is considered and they are randomly distributed on the fault plane allowing overlap. It is shown that the final slip distribution generated is in agreement with the k-square model. As a consequence, the proposed model radiates the classical omega-square model. The model is defined by 8 parameters namely : the fractal dimension of the size distribution, the maximal and minimal size of cracks, the stress-drop, the length and the width of the fault plane, the rupture velocity of each elementary crack and the rupture front velocity that propagates on the fault plane triggering the rupture of each crack. One major interest of the models is that it allows to perform easily parametric studies by varying these different parameters. As an exemple, we study the effect of the size of the breaking zone (which depends of the ratio between the crack rupture velocity and the rupture front velocity) on ground motion in directive direction. It is shown that the first spectral hole of a directive accelerograms corresponds to a frequency directly related to the size of the breaking zone.

KEYWORDS : composite source model, circular crack, fractal distribution.

INTRODUCTION

Complexity of actual accelerograms recorded at close distances from faults clearly reflects the complexity of the rupture process. Many studies, based on inversion procedures, have shown that sources present important heterogeneities in both final slip distribution and rupture velocities.

Several methods have been proposed for synthetizing realistic accelerograms for engineering purposes. A first category of methods directly inverts in time a spectral function with the adequate shape and amplitude and associated with a stochastic phase (Boore, 1983). A second category of method is based on the idea that the self-similarity and the spectral law of the seismic radiation should be described by some self-similar rupture process (Hanks, 1979; Frankel, 1991). More recently, Herrero and Bernard (1994) defined a kinematic model of rupture presenting a self-similar distribution of rupture parameters (final slip and rupture time) and based on reasonable physical constraints that fits a given spectral law of the far-field displacement. Finally, a third category of method invokes kinematic models of finite faults, with some random process, coupled with empirical or theoretical empirical Green's functions techniques for propagating the field to the site (Hartzell, 1978; Papageorgiou and Aki, 1983, Irikura, 1983, 1986; Hutchings and Wu, 1990; Zeng et al., 1993 among others).

In this study, we propose a new kinematic source model belonging to this latest category. This model is composed of an aggregate of circular cracks with a fractal distribution of their size.

METHOD OF CALCULATION AND GEOMETRY OF THE MODEL

The basic element of the faulting process is an expanding circular crack. The dynamic characteristics of this model derived from fracture physics have been obtained by Madariaga (1976). The fault-slip history inferred from this study can be described approximately by analytical expressions. Such representations have been used to model the radiation from "crack models" of the earthquake source (Campillo, 1983, Campillo et al., 1989; Gariel et al., 1990).

In this model, rupture is initiated at the center of the circular fault and propagates toward the edge of the fault with a velocity v_r. At each instant the rupture front is represented by a circle. The slip D at each point of the fault is described by :

$$\frac{D(\vec{r},t)}{D_0} = 0 \qquad t < t_{0(r)} \qquad (1)$$

$$\frac{D(\vec{r},t)}{D_0} = \sqrt{v_r^{\,2} t^2 - r^2}, \quad t_0(r) < t < t_1 \qquad (2)$$

$$\frac{D(\vec{r},t)}{D_0} = \sqrt{v^2\, t_1^2 - r^2}, \quad t > t_1 \qquad (3)$$

with

$$t_0 = \frac{r}{v_r} \qquad t_1 = \frac{R}{v_r} \qquad v_r = \frac{v_r}{2} \qquad (4)$$

and where D_0 is the final displacement at the center of the crack and R denotes the final crack radius.

To calculate synthetic accelerations, we use the method of Bouchon (1981) which consists of a discretization of the wavefield in terms of horizontal wavenumbers. Following Campillo (1983), a circular shear-crack is represented by an array of point-sources and the superposition of the elastic field radiated by all elementary sources is done in the frequency horizontal wavenumber domain. The interval between elementary sources is chosen to be smaller than one-fifth of the shortest wavelength considered.

In all the calculations presented in this paper, the fault plane is a vertical pure strike-slip 20 km long and 14 km width. The top of the fault is at a depth of 2 km. The hypocenter is located on one vertical edge of the fault plane at a depth of 10 km. The accelerations are computed at three receivers (see fig. 1) located at 25 km from the epicenter in directive, anti-directive and non-directive directions. The propagation medium is an infinite half-space.

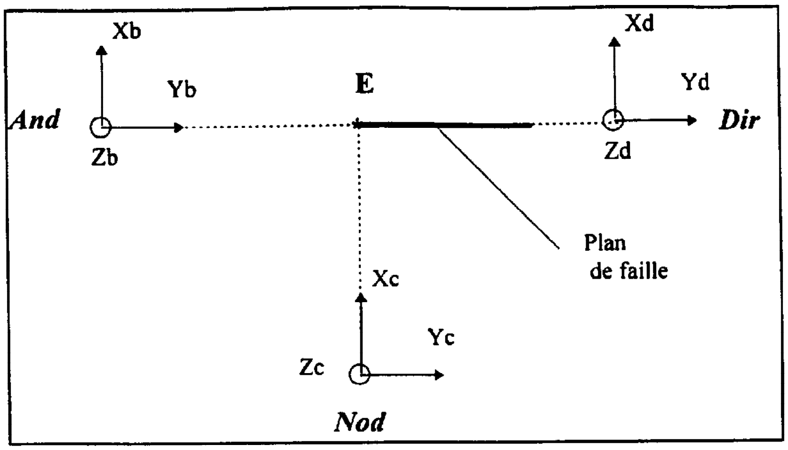

Figure 1 : Geometry of the model used in the calculations (map view).

FRACTAL COMPOSITE SOURCE MODEL

This composite source model, with a fractal distribution of the size of the sub-events, has been first proposed by Frankel (1991), then modified by Zeng et al. (1993). The initial assumption in their model is that the source slip-function can be simulated, in a kinematic sense, by randomly distributed subevents on the fault plane. The earthquake is made up of a hierarchical set of smaller earthquake. As proposed by Zeng et al. (1993), the number of circular subevent (here, defined as circular shear-cracks) with radius R is specified by the following formulae :

$$\frac{dN}{d(\ln R)} = pR^{-D} \qquad (5)$$

where D is the fractal dimension, N is the number of subevents, and p is a constant of proportionality. Following Zeng et al. (1993), equation (5) can be integrated and therefore gives the number of subevents with radii larger than R :

$$N(R) = \frac{p}{D}(R^{-D} - R_{\max}^{-D}) \qquad (6)$$

In (6), we considered that $R_{\max}$ is the radius of the largest crack that fits inside the fault plane. For a circular crack, Keilis-Borok (1959) has shown that the stress-drop $\Delta\sigma$ is related to the radius R and to the seismic moment M_0 by the relation :

$$M_0(R) = \frac{16}{7} R^3 \Delta\sigma \qquad (7)$$

Tumarkin et al. (1994) have shown that, because of the principle of energy conservation, it is necessary that the total area of the subevents to be greater than the area of the fault plane. Therefore, overlap of circular cracks is allowed in our model. To evaluate the total seismic moment of a series of sub-events with a distribution given by (6), we write :

$$n(R) = -\frac{dN}{dR} = pR^{-D-1} \qquad (8)$$

and :

$$M_0^E = \int_{R_{min}}^{R_{max}} n(R)\, M_0(R)\, dR \qquad (9)$$

This constraint leads to the value of p:

$$if \quad D \neq 3 \qquad p = \frac{7M_0^E}{16\Delta\sigma} \frac{3-D}{(R_{max}^{3-D} - R_{min}^{3-D})} \qquad (10)$$

$$if \quad D = 3 \qquad p = \frac{7M_0^E}{16\Delta\sigma} \frac{1}{\ln(R_{max}/R_{min})} \qquad (11)$$

The size distribution given by (6) is obtained in generating N random real numbers, N_i, which are uniformly distributed from 0 to N. The size of the corresponding event is :

$$R_i = \left(\frac{DN_i}{p} + R_{max}^{-D} \right) \qquad (12)$$

Our model is different from that of Zeng et al. (1993) in the sense that they assumed that the radiation from each subevent, considered as a point-source, takes the shape of the Brune (1970) pulse. In this study, each subevent is a circular crack characterized by several physical parameters : radius, rupture velocity and stress-drop. Seismic radiation of each crack is related to the stopping phases of the rupture at the edge of the crack. This model is a scale-dependent one because source-time function at a given point is a function of the size of the subevent.

APPLICATION OF THE MODEL

For our application, we fixed arbitrarily the total number of cracks N to 500. This other parameters have the following values :

$$D = 2$$
$$R_{max} = 7\ km$$
$$R_{min} = 0.5\ km$$
$$\Delta\sigma = 50\, bars$$
$$M_0^E = 7.25x10^{18}\ N.m \qquad M_W = 6.5$$

The fractal distribution of the size of the subevents provides cracks with radius ranging from 0.45 to 4.6 km. In order to save computing time, we considered 9 different classes of size from 500 to 4500 meters. The distribution is indicated in table 1.

Class (m)	500	1000	1500	2000	2500	3000	3500	4000	4500
Number of cracks	404	64	12	5	6	3	3	2	1

Table 1 : fractal distribution of the size of the elementary circular cracks.

All the circular cracks are then distributed randomly on the fault plane. Fig. 2 shows the distribution of the final slip on the fault plane. The average final slip of the model is 0.87 m, whereas the maximum final slip is 3.34 m. Our model allows to consider two different rupture velocities. One associated with the rupture of each elementary crack (v_r) and another one (V_r) associated with the propagation of the rupture front that triggers the rupture of each crack. The ratio V_r/v_r will determine the size of the "breaking zone". If v_r is close from V_r the breaking zone will be very small while if v_r is smaller than V_r, the size of the "breaking zone" will increase. In this application, we considered $Vr = v_r = 2.7$ km/s.

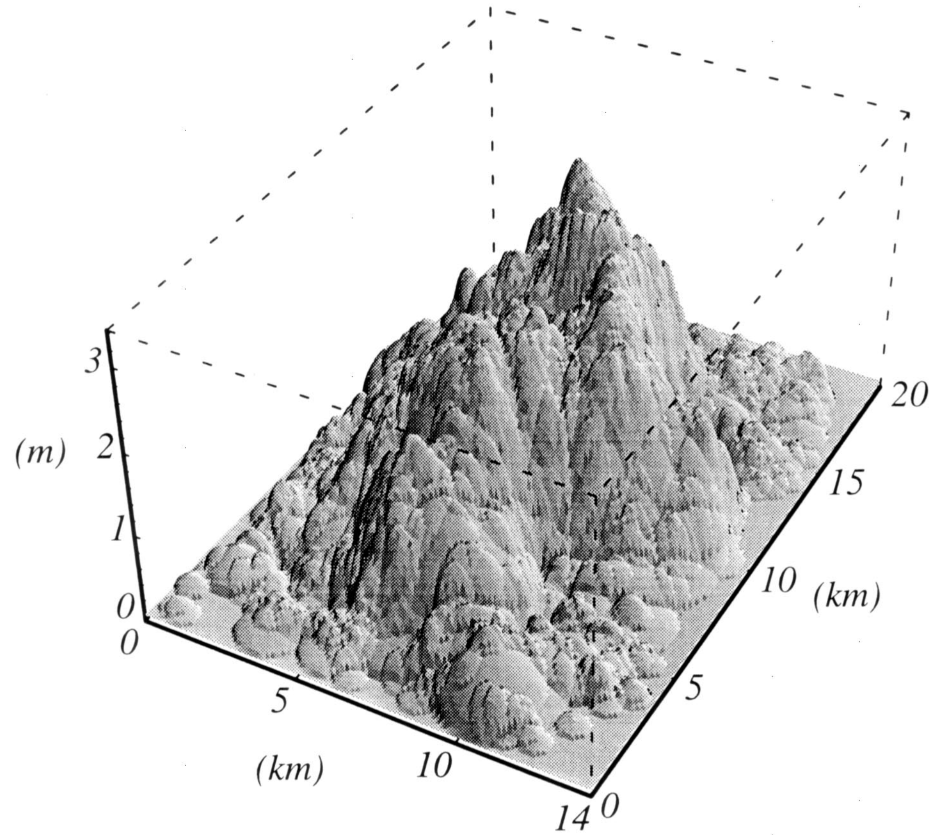

Fig. 2 : 3-D representation of the final slip on the fault plane. Radius of cracks range from 0.5 to 4.5 km.

Synthetic accelerograms at the three receivers and their associated Fourier spectra are shown on fig. 3. Accelerograms show high-frequencies that are related to the stopping phases emitted at the edge of each crack when rupture stops. Peaks acceleration are quite realistic and maximal values are 0.2, 0.3 and 0.6 g for respectively anti-directive, non-directive and directive stations. As could be expected, directivity are well reproduced in the time domain and a longer duration is observed at the anti-directive station whereas a smaller one characterizes the directive station. In spectral domain, synthetic spectra are in agreement with the omega-square model. This is due to the coseismic slip distribution on the fault plane. As was suggested by Herrero and Bernard (1994), a self-similar distribution of slip led to define a k-square model. They showed that the k-square model, combined with the assumption of a constant rupture velocity and of a scale-dependent rise-time, results in a kinematic model radiating the classical omega-square model. In the case of our model, we verified that the spatial Fourier transform of the final slip distribution was in agreement with the k-square model (fig. 4).

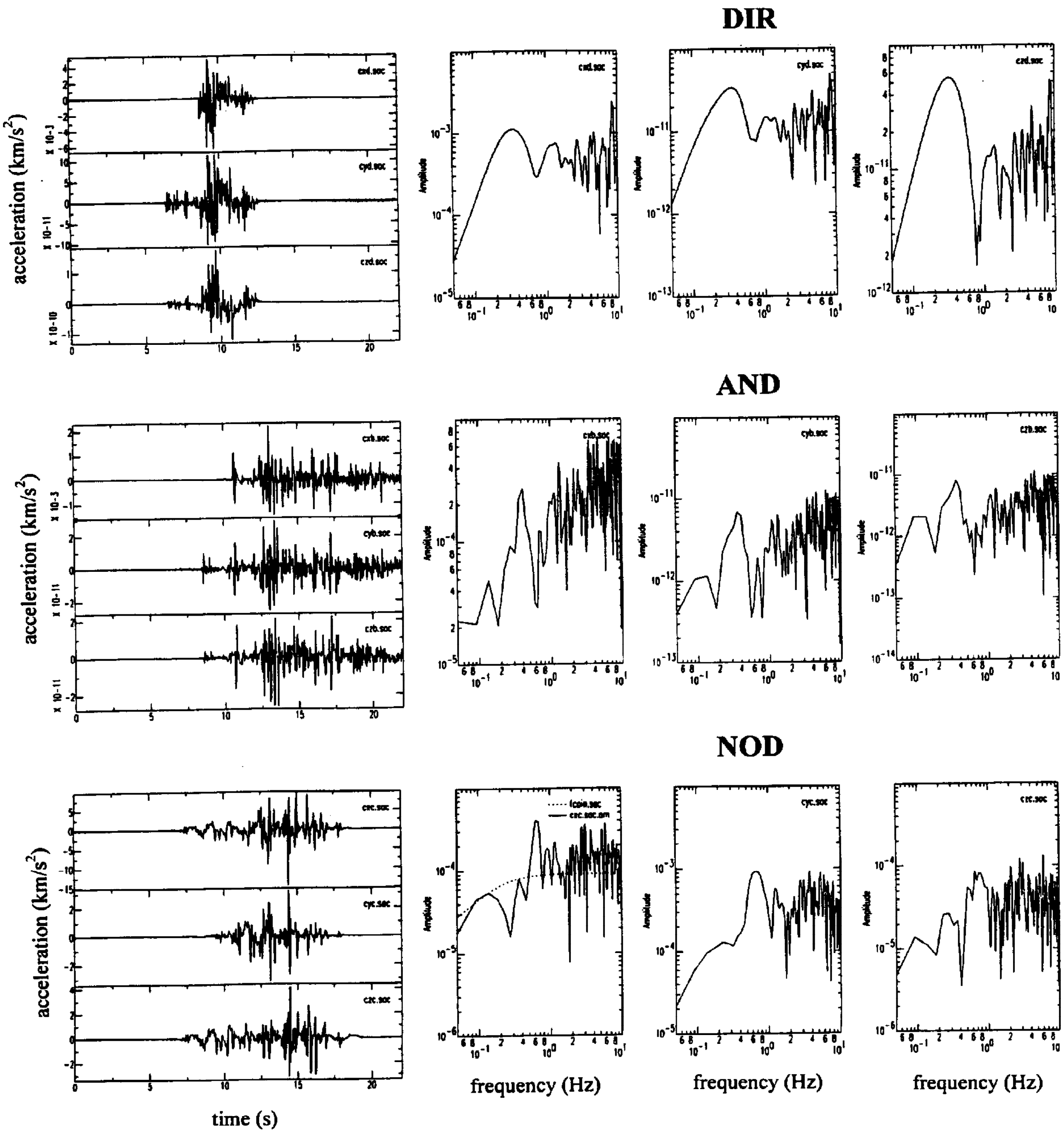

Fig.3 Synthetic accelerograms and corresponding Fourier spectra calculated at stations DIR (top), AND (middle) and NOD (bottom).

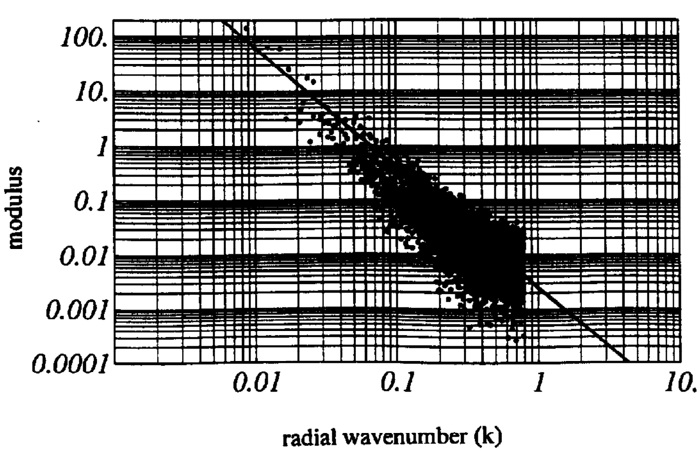

Fig. 4 : modulus of the Fourier transform of the final slip distribution as a function of radial wavenumber k.

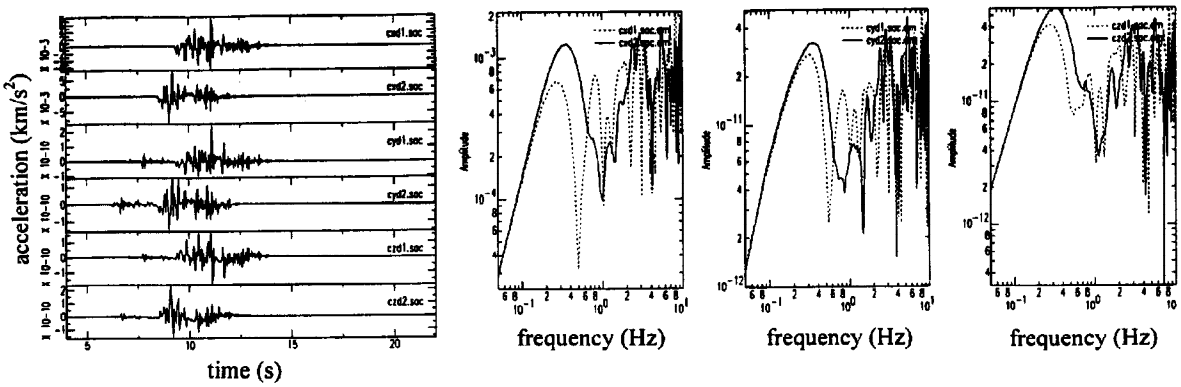

Fig. 5 : left : synthetic accelerograms obtained at station DIR (directive) for two different rupture velocities of elementary cracks (2.25 km/s = suffix 1 and 2.7 km/s = suffix 2). Right : Fourier spectra of the corresponding spectra (2.25 km/s = dotted line; 2.7 km/s = solid line).

Finally, one major advantage of the model proposed here is that parametric studies can easily be handled. As an example, we studied the effect of the size of the breaking zone on seismic motions. As mentionned above, the size of the breaking zone is governed by the ratio V_r/v_r. To study this effect, using the model previously described and considering a crack rupture velocity v_r of 2.25 km/s, we performed two calculations : one with V_r=3 km/s and another with V_r=2.25 km/s. Resulting accelerograms and their associated Fourier spectra at the directive stations are shown on fig. 5. One may notice a clear shift of the first spectral hole towards the high-frequency when V_r increases (the size of the breaking zone increases). This is in agreement with the study by Herrero and Bernard (1994) who showed that the first spectral hole of a directive record corresponds to a frequency f_p, directly related to L_0 by the relation :

$$f_p = \frac{V_r}{L_0} = \frac{1}{\tau_{\max}} \qquad (13)$$

where $\tau_{\max}$ is the total duration of slip. Therefore, this shows that it is possible to retrieve the size of the breaking zone from the study of directive (or anti-directive) records.

CONCLUSION

In this paper, following the idea of Zeng et al. (1993), we present a composite seismic source model based on a fractal distribution of circular shear-cracks. This model is described by 8 parameters namely : the fractal dimension D, the maximal and minimal radius of cracks ($R_{\max}$, $R_{\min}$), the stress-drop $\Delta\sigma$, the length and the width of the fault plane, the rupture velocity of elementary cracks v_r and the rupture front velocity V_r. A first advantage of this model is that elementary sources (cracks) correspond to a physical model derived from fracture physics. Another point is that the distribution of cracks with varying sizes results in an heterogeneous distribution of rise time on the fault plane. This is a clear difference with models assuming a propagating dislocation with a rise time t distributed homogeneously in space and which are limited to frequencies below 1/t. Finally, the major interest of the model proposed here is that it will allow to perform easily parametric studies.

REFERENCES

Boore D.M. (1983). Stochastic simulation of high-frequency ground motion based on seismological models of the radiated spectra, *Bull. Seism. Soc. Am.*, **73**, 660-682.

Bouchon M. (1981). A simple method to calculate Green's functions for elastic layered media, *Bull. Seism. Soc. Am.*, **71**, 959-971.

Brune J.N. (1970). Tectonic stress and spectra of seismic shear waves from earthquakes, *J. Geophys. Res.*, **75**, 4997-5009.

Campillo M. (1983). Numerical evaluation of near-field, high-frequency radiation from quasi-dynamic circular faults, *Bull. Seism. Soc. Am.*, **73**, 723-734.

Campillo M., J.C. Gariel, K. Aki and F.J. Sanchez-Sesma (1989). Destructive strong-ground motion in Mexico City: source, path and site effects during great 1985 Michoacan earthquake, *Bull. Seism. Soc. Am.*, **79**, 1718-1735.

Frankel A. (1991). High-frequency spectral falloff of earthquakes, fractal dimension of complex rupture, b value, and the scaling of strength on fault, *J. Geophys. Res.*, **96**, 6291-6302.

Gariel J.C., R.J. Archuleta and M. Bouchon (1990). Rupture process of an earthquake with kilometric size fault inferred from the modeling of near-source records, *Bull. Seism. Soc. Am.*, **80**, 870-888.

Hanks T.C. (1979). b values and ω^{γ} seismic source models: implications for tectonic stress variation along active crustal fault zones and the estimation of high-frequency strong ground motion, *J. Geophys. Res.*, **84**, 2235-2242.

Hartzell S.H. (1978). Earthquake aftershocks as Green's function, Geophys. Res. Lett., **5**, 1-4.

Herrero A. and P. Bernard (1994). A kinematic self-similar rupture process for earthquakes, *Bull. Seism. Soc. Am.*, **84**, 1216-1228.

Hutchings L. and F. Wu (1990). Empirical Green's functions from small earthquakes: a waveform study of locally recorded aftershocks of the 1971 San Fernando earthquake, *J. Geophys. Res.*, **95**, 1187-1214.

Irikura K. (1983). Semi-empirical estimation of strong motions during large earthquakes, *Bull. Dis. Prev. Res. Inst.*, Kyoto Univ., **33**, part 2, n° 298, 63-104.

Irikura K. (1986). Prediction of strong acceleration motions using Empirical Green's Function; *Proceedings of the Seventh Japan Earthquake Engineering Symposium*, 151-156.

Keilis-Borok V. (1959). On estimation of the displacement in an earthquake source and of source dimension, *Ann. Geofis.* (Rome), **12**, 205-214.

Madariaga R. (1976). Dynamics of an expanding circular fault, *Bull. Seism. Soc. Am.*, **66**, 639-666.

Papageorgiou A. and K. Aki (1983). A specific barrier model for the quantitative description of inhomogeneous faulting and the prediction of strong ground motion. Part I: Description of the model, *Bull. Seism. Soc. Am.*, **73**, 693-722.

Tumarkin A.G., R.J. Archuleta and R. Madariaga (1994). Scaling relations for composite earthquake models, *Bull. Seism. Soc. Am.*, **84**, 1279-1283.

Zeng Y, J.G. Anderson and G. Yu (1993). A composite source model for computing realistic synthetic strong ground motions, *Proceedings of the International Workshop on Strong Motion Data*, **vol. 2**, Menlo Park, California.