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INFLUENCE OF GROUND MOTION CHARACTERISTICS ON THE EFFECTIVENESS OF TUNED MASS DAMPERS

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ABSTRACT

An examination of the effect of ground motion characteristics on the effectiveness of Tuned Mass Dampers (TMD) is made using a nonstationary random vibration formulation. A Kanai-Tajimi spectral density and a modulating function fixed by a single parameter are used to define the input. Effectiveness of the TMD is judged by inspecting the ratio of maximum expected displacement with and without the presence of the damper. The results indicate that the optimum TMD damping is insensitive to the bandwidth and depends on the duration to period ratio only for relatively low values of this quotient (where the optimum decreases with decreasing duration). The results suggest that TMD units may be able to provide notable reductions in spectral ordinates for periods near the dominant motion period when the excitation is narrow band and of long duration. The practical significance of these reductions, however, is lessened by the fact that small inelastic excursions also lead to sharp reductions in response for these conditions.

KEYWORDS

Tuned mass damper; spectral density; random vibration; evolutionary spectral analysis.

INTRODUCTION

A tuned mass damper (TMD) is typically a single-degree-of-freedom oscillator added to a structure with the objective of reducing the response to dynamic excitations. The effectiveness of TMDs as means to reduce vibrations from harmonic loads has been well established since the early work of Den Hartog (1940). Crandall and Mark (1963) studied the effectiveness of TMD in reducing the response of single-degree-of-freedom systems subjected to white base accelerations. These authors characterized effectiveness using the ratio of the stationary variance with and without the damper and found that significant reductions could be obtained using small mass ratios. The results obtained by Crandall and Mark suggested that TMD units may prove effective in reducing response to earthquakes.

Kaynia, Veneziano and Biggs (1980) used a large ensemble of real earthquake motions to asses the effect of TMDs in the maximum elastic response of a number systems. These authors noted that predictions derived with a stationary random vibration formulation consistently overestimated the effectiveness of the damper.

The fundamental causes of the discrepancy were identified as: (1) failure of the response to reach (or approach) stationarity in many of the simulations and (2) the unconservative nature of the ratio of standard deviations (with and without the TMD) as a measure of the ratio of peak response values. In view of the findings by Kaynia et.al. (1980), the examination of the damper presented in this paper is made using a nonstationary formulation. In particular, the formulation presented treats the input as a uniformly modulated random process having a Kanai-Tajimi spectral density function (Kanai 1961) and calculates the response using the evolutionary spectral analysis introduced by Priestley (1965). The maximum response variance is then used to compute the expected value of the maximum response by introducing a peak factors which is computed from the evolution of the first and second moments of the spectral density (Vanmarcke 1977).

Central to the issue of TMD effectiveness is the matter of optimum parameters. Formulas for the parameters that minimize certain objective functions have been derived for various inputs, (Warburton, 1982, Tsai and Lin 1993). One of the objectives of the current study is to examine how the optimum TMD damping for earthquake excitation is affected by the parameters that characterize the motion. Of some interest also is to examine how values from a recent suggestion for selecting the damping in the TMD (Villaverde and Kaoyama 1993) compare to results from the RV formulation. The overall objective of the research reported in this paper was to identify the conditions (if any) where consideration of a TMD as a seismic vibration absorber may be warranted.

The paper begins with a brief review of the theory of evolutionary spectra. The implemented formulation is verified by comparing its predictions with results obtained using time history analysis for an ensemble of 10 artificially generated motions. Numerical results for reductions in peak response as a function of the parameters that govern the input are presented next. The paper concludes with a preliminary examination of the effect of inelastic action in the main structure.

SYSTEM CONSIDERED

Figure 1 illustrates a planar idealization of an n story building with a TMD provided at the roof level. Assuming that a reasonable solution can be obtained using a single shape (ϕ) to describe the displacements of the building, the system can be reduced to one with 2-DOF. Normalizing the assumed shape to 1 at the roof, the equations of motion can be shown to be;

$$\begin{bmatrix} (1+\mu) & \mu \\ \mu & \mu \end{bmatrix} \begin{Bmatrix} \ddot{y} \\ \ddot{\delta} \end{Bmatrix} + 2\omega_e \begin{bmatrix} \xi_e & 0 \\ 0 & \mu f \xi_a \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \dot{\delta} \end{Bmatrix} + \omega_e^2 \begin{bmatrix} 1 & 0 \\ 0 & \mu f^2 \end{bmatrix} \begin{Bmatrix} y \\ \delta \end{Bmatrix} = \begin{Bmatrix} \mu + \Gamma \\ \mu \end{Bmatrix} \tag{1}$$

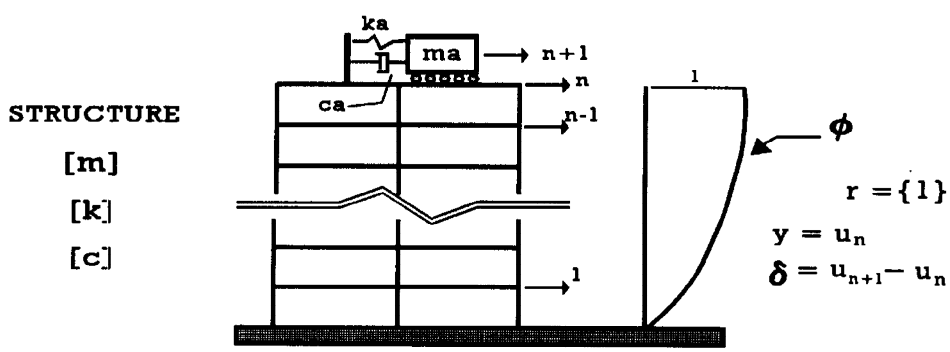

Fig.1 Planar model of multistory building with TMD

where y is the roof displacement and δ is the displacement of the TMD relative to the roof. Inspection of (1) shows that the reduced system is characterized by 6 independent parameters, namely: the mass ratio $\mu = m_a / m_e$ (where $m_e = \phi^t\, m\, \phi$), the participation factor $\Gamma = \phi^t\, m\, r / m_e$, the frequency and damping ratio of the main structure (associated with the shape ϕ) ω_e and ξ_e, the damping ratio of the TMD ξ_a , and the frequency ratio, $f = \omega_a / \omega_e$. The frequency and damping of the main structure ω_e and ξ_e are typically assumed to be those associated with the first mode (i.e ϕ is taken as the first mode shape). As initially noted by Kaynia et.al. (1980), the participation factor can be dropped from the set of characterizing parameters because the ratio of responses with and without the TMD , for realistic values of μ and Γ, is nearly independent of Γ. This contention is evident by noting that since $\Gamma >> \mu$ the right hand side of (1) (and thus the linear response) is nearly proportional to Γ. In the numerical analyses presented here the participation factor has been taken equal to 1.

COMPUTATION OF RESPONSE : RANDOM VIBRATION FORMULATION

The theory of evolutionary spectra introduced by Priestley (1965) with a peak factor formulation from Vanmarcke (1977) are used to obtain the expected maximum response of the system with and without the damper. In particular, the maximum expected response is computed as

$$y_{max} = r_{(p)}\, \sigma_{(m)} \tag{2}$$

where $\sigma_{(m)}$ is the maximum standard deviation and $r_{(p)}$ is the peak factor at probability level p. The time dependent variance is obtained as,

$$\sigma(t) = \int_0^{\infty} SD(t,\omega)\, d\omega \tag{3}$$

where the evolutionary spectral density SD(t,ω) (Howell and Lin, 1971) is given by

$$SD(t,\omega) = |M(t,\omega)|^2\, G(\omega) \tag{4}$$

In (4) G(ω) is the one sided spectral density of the input and M(t,ω) is obtained from

$$M(t,\omega) = \int_0^{t} h(\tau)\, c(t-\tau)\, e^{-i\omega\tau}\, d\tau \tag{5}$$

where h(τ) is the unit impulse response for the DOF of interest and c(t) is a modulating function that accounts for the build-up and decay of the ground motion intensity. As is well known, the foregoing formulation preserves the assumption of stationarity in the frequency content of the input. At the heart of the preceding approach is the calculation of the unit impulse response function h(t). While this function can be built from modal contributions, in this study h(t) is obtained as the Fourier transformation of the frequency response function for the DOF of interest without decoupling the equations. Although a derivation is omitted for brevity, it can be easily shown that the impulse response function for displacement at DOF "j" in any viscously damped linear system subjected to a dynamic load with constant spacial distribution is given by

$$h(t) = \int_{-\infty}^{\infty} N(i\omega)\, e^{i\omega t}\, d\omega \tag{6}$$

where

$$N(i\omega) = A^t B(i\omega)\, g \tag{7}$$

In (7) A is a column vector of zeros with a unit value at the jth location, g is the load distribution vector and B(iω) is the complex matrix given by $[\,(K-\omega^2 M) + \omega\, C\, i\,]^{-1}$ (where M, K and C are the system's mass stiffness and damping matrices). In the case of (1) $g^t = [(\Gamma + \mu) \quad \mu]$ and, to compute the roof response, $A^t = [1 \;\; 0]$. While the expressions used to compute $r_{(p)}$ are not reviewed, the approach is as described by

Vanmarcke (1977), adjusted to account for the modulation in the input motion.

Validation

Figure 2 shows a plot of the reductions in peak response as a function of the damping ratio in the TMD. The parameters that define the spectral density of the input (taken to be of the Kanai-Tajimi form) and the time modulation utilized are shown in the figure. Average results from time history analyses for an ensemble of ten artificial motions compatible with the utilized density and modulation are shown as large dots. As expected, the RV formulation provides a virtually perfect match to the average of the simulations.

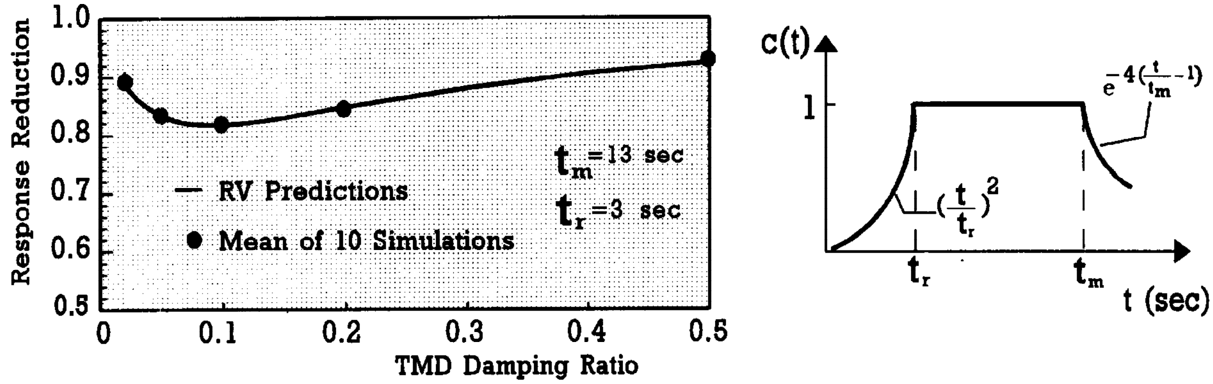

Fig.2. Comparison between RV predictions and simulations ($T = T_g = 1$sec $\xi_g = 0.02$, $\mu = 0.05$, $\xi_e = 0.05$, modulation as shown)

TMD EFFECTIVENESS

Examination of TMD effectiveness is made for the case where the spectral density of the input is of the form proposed by Kanai and Tajimi (1961) multiplied by a frequency modulation (filter) that brings the spectra to zero at zero frequency. Eqs. (8) through (10) define the general form

$$G(\omega) = D(\omega)\, G(w) \tag{8}$$

$$G(\omega) = G_0 \frac{1 + 4\xi_g^2 \eta}{[1-\eta]^2 + 4\xi_g^2 \eta} \tag{9}$$

$$D(\omega) = \frac{64\eta}{\sqrt{(1-64\eta)^2 + 166\eta}} \qquad where \quad \eta = \left(\frac{\omega}{\omega_g}\right)^2 \tag{10}$$

The time modulation used to describe the buildup and duration of the strong motion is as shown in Fig.2. To make the modulation a function of a single parameter the rise time tr is taken as tm/4. For the system in (1), and the ground motion characterization used, the ratio of peak response with and without the damper, y/y_0, is a function of 9 dimensionless parameters, namely: $y/y_0 = f\,[\,\eta, \tau, \xi_g, \xi_a, \xi_a, \mu, f, \Gamma, p\,)$, where η is defined in (10), τ is the normalized duration (tm/T) and p is the probability at which the peak factor calculation is made. Taking $p = 0.5$, $f = 1$ and recalling that the participation factor Γ can be taken as 1 the number of parameters is reduced to 6.

Optimum TMD Damping

In the following, attention is focus on how the motion characteristics affect the optimum TMD damping.

Fig.3 illustrates some typical results for the variation of the reduction as a function of the damper damping and Fig.4 shows plots of the optimum damping as a function of the normalized duration for wide and narrow motion bandwidth. The fundamental observation from these results is that the optimum damper is insensitive to the ground motion bandwidth and is affected by duration only when this parameter is relatively small. A consequence of the preceding observations is the fact that a close approximation to the optimum TMD damping, except for short durations, is given by

$$\xi_a = \sqrt{\frac{\mu(4-\mu)}{8(2-\mu)(1+\mu)}} \tag{11}$$

which holds for stationary response to white base acceleration (Nigam and Narayanan, 1994). It is worth noting that although (11) has been derived on the assumption of an undamped primary structure, numerical results indicate that this result is virtually independent of damping in the main system (Warburton 1982).

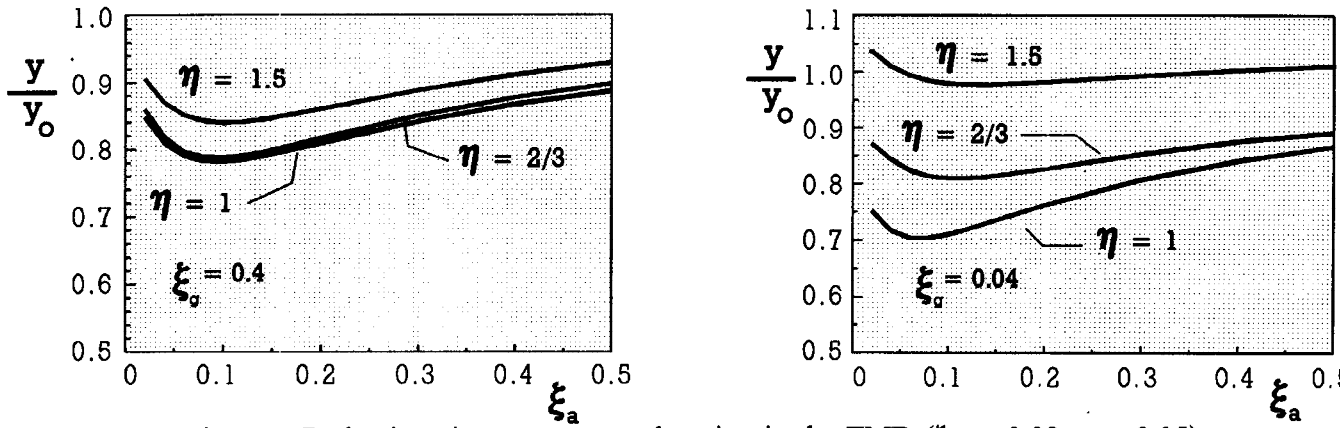

Fig.3 Reductions in response vs. damping in the TMD (ξ_e = 0.02, μ = 0.05).

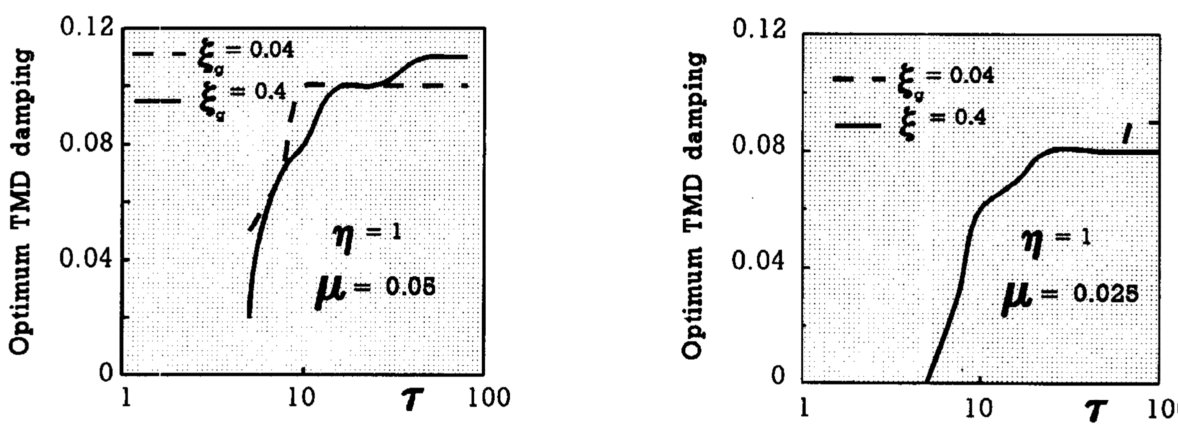

Fig.4. Optimum TMD damping vs normalized duration (ξ_e = 0.02, η = 1).

A plot of the damping in the complex modes of the 2-DOF system in (1) as a function of the TMD damping (for the structural parameters of Fig.2) is illustrated in Fig.5. As can be seen, the damping in one of the modes increases indefinitely as the TMD damping increases while the other reaches a maximum and begins to decrease after a certain threshold - this behavior is typical. It has been recently suggested that optimum TMD performance may be associated with the point where the mode whose damping decreases reaches a maximum (Villaverde and Kaoyama, 1993). This contention, however, appears questionable since it does not explicitly contemplate the dependance of the maximum response on the modal correlation nor the connection between this correlation and the modal damping ratios. As can be seen, the value identified in Fig.5 is not in agreement with the results depicted in Figs.2-4. In any case, it is appropriate to note that the response reductions are actually rather insensitive to increases in the TMD above the optimum value so that higher damping can be used to control the motion of the damper itself without compromising the effectiveness significantly.

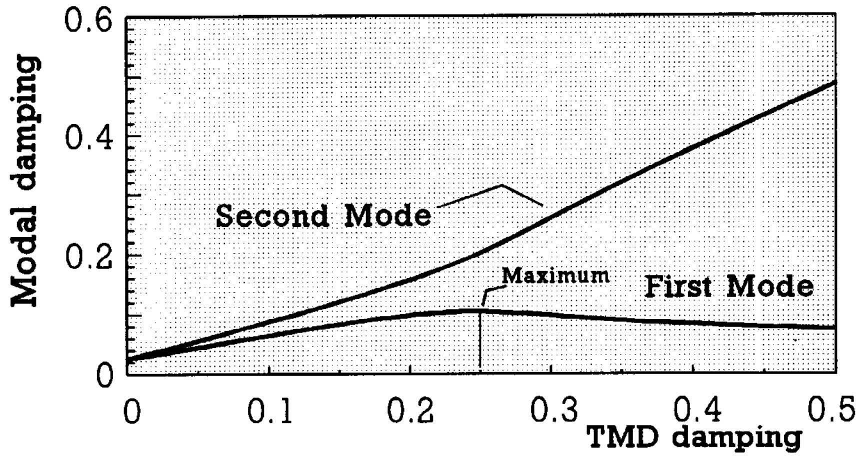

Fig. 5. Damping in complex modes as a function of ξ_a for f = 1 and μ = 0.05.

Some clarifications in connection with the results presented are appropriate. First, the value of ξ_g = 0.04 used to represent the narrow band condition was selected by fitting (9) to the square of the Fourier amplitudes for the strong portion of the SCT record from the 1985 Mexico City Earthquake. This bandwidth is, therefore, probably near the low end of the realistic range. Second, in the narrow band case the results shown for reduction and optimum damping at T/T_g = 1 are actually based on consideration of a band around this point. In particular, the optimum damping and the reductions are obtained by minimizing the response in the range ($0.9 \leq T/T_g \leq 1.1$). If only the ordinate at T/T_g = 1 were contemplated, the reductions in response would be larger and the optimum damping smaller than the values listed. Given that the best information one can expect to have in practice is that the T/T_g ratio is near one, minimization of the spectral ordinate around resonance is reasonable.

Expected Reductions at Optimum Damping

Figure 6 illustrates results for the ratio y/y_0 as a function of the normalized duration for various conditions. Some basic observations can be made from an inspection of this figure. First, it is worth noting that the reductions afforded by the damper in the region around resonance for narrow band motions can be substantial. For example, for a normalized duration τ = 16 and a mass ratio of 0.05 the spectral maximum is reduced approximately 40 % when the damping ratio of the main structure is 0.02 and 30% when it is 0.05. For a mass ratio of 0.025 these reductions decrease to 30 and 20% respectively. The reductions in response for wide band spectra are smaller than the values that apply near resonance in narrow band motions and, as expected, are much less sensitive to the T/T_g ratio. For τ = 16 and ξ_g = 0.4, for example, the average reductions for a mass ratio of 0.05 are 18% if the damping is 5% and 27% for a structure with 2% damping.

Spectra with and without Damper for Actual Records

In the previous discussion the effectiveness of the damper has been assessed from the predictions of RV theory at specific values of the T/T_g ratio. Elastic pseudo-acceleration spectra for the El Centro (1940) and SCT (1985) records computed with and without an added TMD are depicted in Fig.7. As can be seen, the effectiveness of the TMD in the case of the narrow band SCT motion is quite significant. It is interesting to note that the reductions for SCT are somewhat larger than what can be expected on the basis of a Kanai-Tajimi Spectral Density and a reasonable duration. An examination showed that the increased effectiveness derives from the bi-modal shape of the square of the Fourier coefficients for the strong portion of this record.

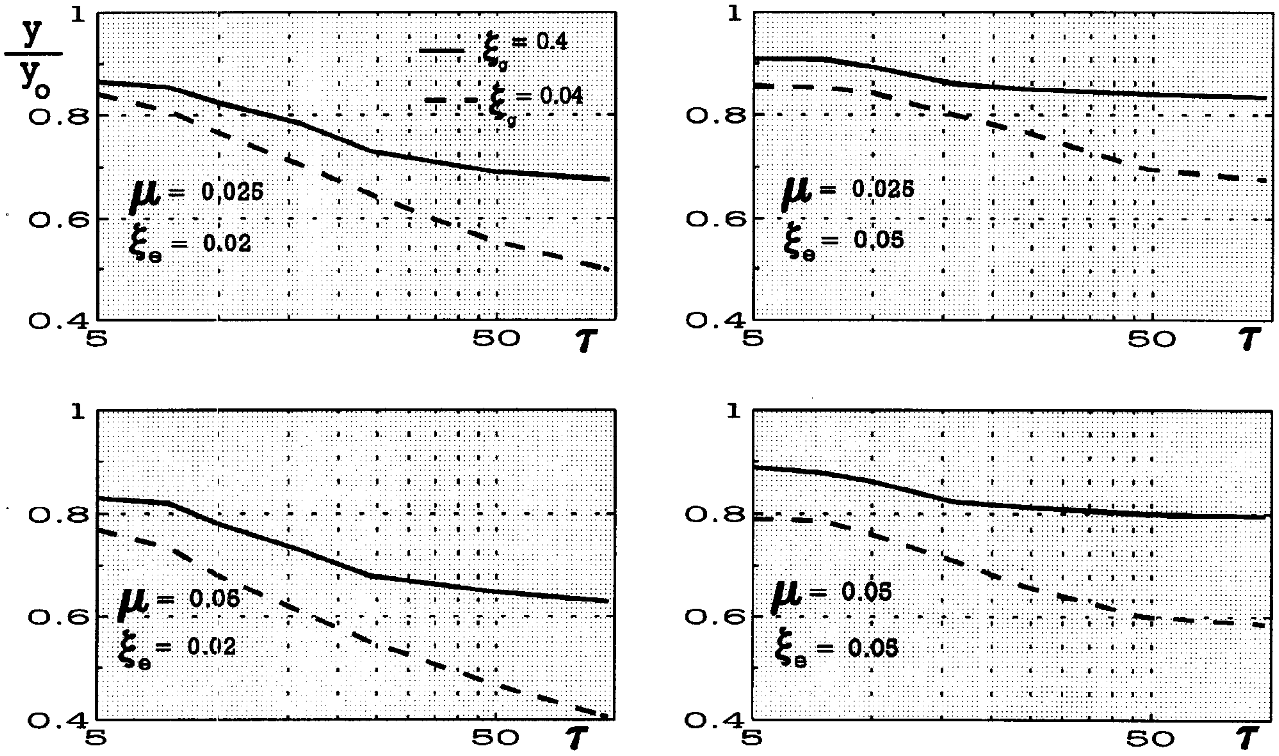

Fig.6 Response reductions vs normalized duration for various conditions (η = 1).

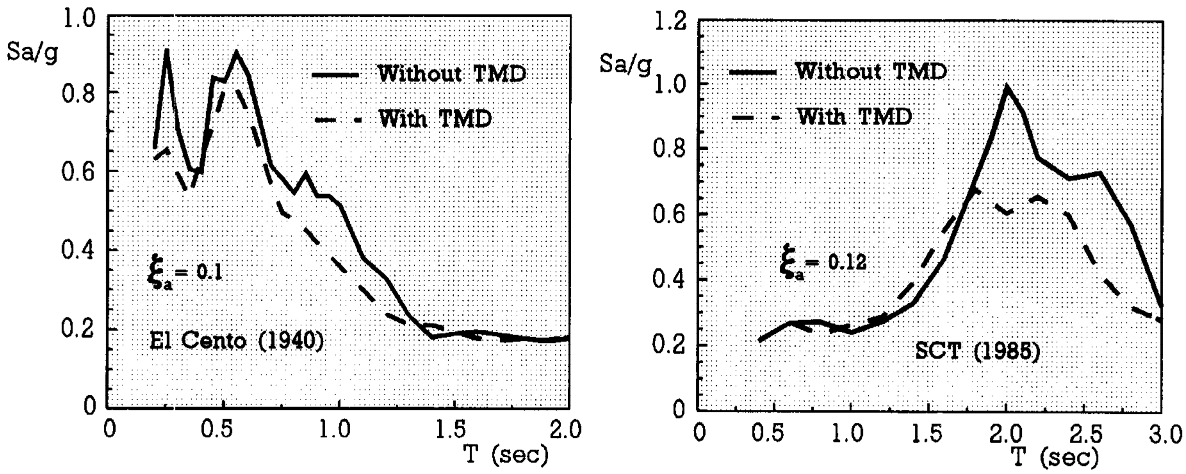

Fig.7 Effect of TMD on elastic response spectra (ξ_e = 0.05 , μ = 0.05)

<u>Exploratory Examination of Inelastic Response</u>

Reductions in response realized by a TMD are not of the order needed to ensure elastic response under severe ground excitation. It is of interest, therefore, to examine the behavior of the response when the motion is strong enough to induce inelastic behavior in the main structure. Clearly, the conditions of most interest are those for which the TMD is effective in reducing the elastic response. Consider a structure with T= 2 sec and 5% damping subjected to the SCT record. A plot of the response ductility vs the yield base shear coefficient is shown in Fig.8 for the cases with and without a TMD with μ = 0.05. As can be seen, the gains derived from the TMD do not carry very far into the inelastic range. This result can be readily rationalized by recognizing that the quasi-resonance which allows the TMD to become effective in the elastic range is lost even with modest inelastic behavior.

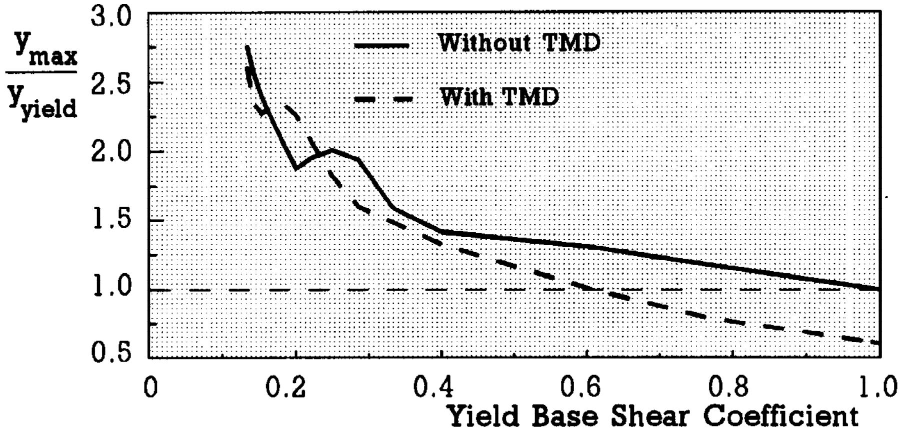

Fig. 8 Effect of inelastic behavior on TMD effectiveness (μ = 0.05, ξ_e = 0.05, ξ_a = 0.12, T = 2 sec).

CONCLUSIONS

The results in this study suggest that the TMD damping at which maximum response reductions are attained is not sensitive to the frequency content of the input motion. This contention is based on the assumption that the damper is designed with the objective of reducing the maximum spectral ordinate in a narrow period band centered around the nominal T/T_g ratio. It is found that the optimum TMD damping increases with normalized duration until it stabilizes near the value that minimizes the stationary response variance for white noise base acceleration. It is noted that the results obtained do not lend support to a recent contention that the optimum TMD damping is that for which the damping in one of the complex modes of the system reaches a maximum. It is found however, in agreement with several previous examinations, that sensitivity of the reductions in response to the TMD damping is typically small. Control of the motion of the damper itself by increasing damping is thus possible without compromising effectiveness significantly. The magnitude of expected reductions from implementation of TMD units appear sufficient to be of practical interest in the case of narrow band motions with long duration. For example, reductions of around 35 to 40% in the maximum spectral ordinate appear feasible with mass ratios of 0.05 for motions resembling the SCT Mexico City record. The practical significance of these reductions, however, is lessened by the fact that small inelastic excursions also lead to sharp reductions in response when quasi-resonant conditions are encountered.

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