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CALIBRATION OF THE EQUIVALENT LINEARIZATION METHOD FOR THE SEISMIC NONSTATIONARY RESPONSE OF SDOF SYSTEMS

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ABSTRACT

The equivalent-linearization technique for hysteretic systems under random excitation is applied to study the influence of the level of non-linearity on the response of several systems. The cases considered include both stationary and nonstationary narrow band excitations. The results are presented in the form of response spectra. The accuracy is calibrated with the results of Monte Carlo simulation. The calibration is made in terms of the values of the standard deviation of displacement in mass-spring-damper systems. Some recommendations are made about the use of equivalent linearization in single degree of freedom systems.

KEYWORDS

Equivalent Linearization, Nonstationary and stationary processes, Monte Carlo simulation, Random vibration, Hysteretic behavior, Probabilistic analysis.

INTRODUCTION

The solution of the equations of motion for non-linear structures subjected to random vibrations is a difficult problem. Several techniques have been developed for calculating the statistical response of nonlinear systems. The purpose of this paper is to calibrate the equivalent linearization technique. Previous studies, i.e. Park (1992) have show that the efficiency and accuracy of the method is about 20% in practical applications. However, Beaman (1980) reports near to 100% for some non-linear systems.

This paper deals only with the influence of the level of non-linearity on the response. Both stationary and nonstationary excitation are considered. The results are presented in the form of spectra and the accuracy of these results is calibrated with the results of Monte Carlo simulation. The calibration is made in terms of the standard deviation of displacement of mass-spring-damper systems. Some recommendations are made about the use of Equivalent Linearization in single degree of freedom systems.

FORMULATION OF THE METHOD

For the purpose of illustration, consider the stochastic differential equation of a single degree of freedom system (*SDOF*) with non-linear behavior

$$g(X(t)) = f(t) \tag{1}$$

where $X(t) = [x(t), \dot{x}(t), \ddot{x}(t)]^T$, $x(t)$ is the displacement response, $\dot{x}(t)$ the velocity and $\ddot{x}(t)$ the acceleration. $g(X(t))$ is a non-linear function of the vector $X(t)$ and $f(t)$ is the excitation. $X(t)$ and $f(t)$ are modeled as stochastic process. The Equivalent Linearization Approach (*ELA*) consists in replacing eq. (1) by the following equivalent form

$$m_e \ddot{x}(t) + c_e \dot{x}(t) + k_e x(t) = f(t) \tag{2}$$

where m_e, c_e y k_e are time-dependent if the statistical properties of $f(t)$ are time-dependent. The error ε is the difference between the first members of eqs. (1) and (2). The next stage is to minimize the square value of that error ε with respect to the parameters m_e, c_e y k_e. This process will yield to a set of equations involving the average values of several functions of $X(t)$. For the evaluation of these expected values, it is necessary to know the probability functions associated to the process $X(t)$. In general, $X(t)$ is assumed as a Gaussian process. Under this assumption we can formulate a system of equations for m_e, c_e and k_e.

MODELING AND LINEARIZATION OF HYSTERETIC RESTORING FORCES

Suppose a nonlinear *SDOF* system with hysteretic behavior, governed by the following equation

$$m\ddot{x}(t) + Q(\dot{x}, x, t) = f(t) \tag{3}$$

where $Q(\dot{x}, x, t)$ is a non-linear function that represents the restoring force. For a nearly elasto-plastic system that force can be modeled by

$$\begin{aligned} Q(\dot{x}, x, t) &= c\dot{x} + \alpha_2 kx + (1-\alpha_2)kz \\ \dot{z} &= G(\dot{x}, z) \end{aligned} \tag{4}$$

Here, c is the linear damping coefficient, k the initial or pre-yielding stiffness, α_2 the ratio of post-yielding to pre-yielding stiffness, z the hysteretic component with units of displacement and $G(\dot{x}, z)$ a first-order differential equation that models the hysteresis loops. Some authors have proposed different expressions for $G(\dot{x}, z)$ for bilinear systems and for systems for which the transition between the elastic and the inelastic ranges is smooth. In this work we used the model proposed by Bouc (1967) and Wen (1980) which is capable of representing several forms of the hysteretic cycles. This model is expressed by the following first order differential equation

$$\dot{z} = \alpha_3 \dot{x} - \alpha_4 |\dot{x}| z |z|^{\alpha_6 - 1} - \alpha_5 \dot{x} |z|^{\alpha_6} \tag{5}$$

Here, α_3, α_4, α_5 y α_6 are parameters that control the amplitude of the hysteresis loop, their shape and the smoothness of the transition from the elastic to the inelastic range.

Equation (5) can be linearized in the form: $\dot{z} = C\dot{x} + Hz$. In particular, when $\dot{x}$ and z are assumed to be jointly Gaussian with zero mean, the minimization of the error equation gives place to expressions for C and K (Atalik and Utku, 1976). Wen (1980) have obtained some expressions for C and K. These are used in this study.

BASE EXCITATION MODELS

In this paper two classes of filtered *white noise* processes are considered: stationary and nonstationary.

Stationary Filtered White Noise.

The systems studied here are excited by a process with similar characteristics to the component E-W of the motion recorded at the parking lot of the Ministry of Communications and Transportation in Mexico City during the September 1985 earthquake (*SCT-85*). This motion is a narrow band process with characteristic of the soft soil of Mexico City. From its Fourier spectrum, the following values were found for the *Clough-Penzien* filter parameter's (Clough and Penzien, 1975): ξ_g =0.025, ω_g=3.14 rad/s, ξ_f =0.045, ω_f=2.48 rad/s and S_0=31/(4π) cm^2/s^3.

It is necessary to add two second order differential equations to the system of equations, in order to take into account the *Clough-Penzien* filter.

Nonstationary Filtered White Noise.

In order to obtain an even more representative process for strong ground motions, the nonstationary character of actual accelerograms are considered through the following scheme:

$$A(t) = c(t)B(t)$$

defined as an *oscillatory stochastic process* (Bolotin,1960). $c(t)$ is a slowly varying deterministic function of the time t, which modulates the variance of $A(t)$; $B(t)$ is a real-valued zero mean stationary process in the wide-sense with power spectral density $S_B(\omega)$, and $A(t)$ is a nonstationary process with power spectral density $S_A(\omega)=c^2(t)\, S_B(\omega)$. In this paper, $B(t)$ represents a *white noise* process ($n(t)$). The following value of $c^2(t)$ was used here (Grigoriu *et al*, 1988)

$$c^2(t) = 0.8238\exp\left(\frac{-(t-58)^2}{25}\right) + 0.1834\exp\left(\frac{-(t-54)^2}{400}\right)$$

METHODOLOGY

The differential equations mentioned before can be expressed as

$$\frac{d}{dt}\boldsymbol{Y} = \boldsymbol{L}\boldsymbol{Y} + \boldsymbol{F}$$

where $\boldsymbol{Y}=[x, x_f, x_g, z, y, y_f, y_g]^T$, $y=\dot{x}$, $y_f=\dot{x}_f$, $y_g=\dot{x}_g$ and $\boldsymbol{F}=[0,0,0,0,0,0, c(t)n(t)]^T$. $\boldsymbol{L}$ is the structural matrix. By using the classical random vibration theory the following differential equation is obtained

$$\frac{d}{dt}\Sigma_Y = \boldsymbol{L}\Sigma_Y + \Sigma_Y \boldsymbol{L}^T + \Omega$$

where $\Sigma_Y=E[YY^T]$. All the elements of Ω are null except Ω_{77} which is equal to $2\pi c^2(t)G_o$, and G_o is the two sided power spectral density of the *white noise* process ($G_o=2\, S_o$). For the stationary case ($d\Sigma_Y/dt=\boldsymbol{0}$) Bartels and Stewart (1972) algorithm was used. The nonstationary case was solved by using the *DGEAR* subroutine of the *IMSL* library.

MONTE CARLO SIMULATION METHOD

In order to calibrate the *ELA*, the results of several SDOF systems are compared. These correspond to this technique and to the Monte Carlo method. In this paper 10 accelerograms are used, based on the *SCT-85* record (Grigoriu *et al.*, 1988). All the accelerograms were scaled so that they have the same Arias intensity. The step-by-step response was obtained by integrating the equations of motion, where the excitation is a simulated motion. Also for the simulation analysis the *DGEAR* subroutine was used.

SYSTEMS STUDIED

Sixty *SDOF* systems with vibration periods T between 0.1 s and 6.0 s are analyzed. Their hysteretic behavior is determined by the following parameters $\alpha_1 = K$; $\alpha_2=\xi=0.05$, $\alpha_3=1.0$ and $\alpha_4=\alpha_5$. The latter depends on the yield force F_y. Figure 1 shows the variation of the mean value of F_y for different periods T and ductility demands μ. These forces and ductility demands correspond to systems with elastoplastic behavior (α_6 too large).

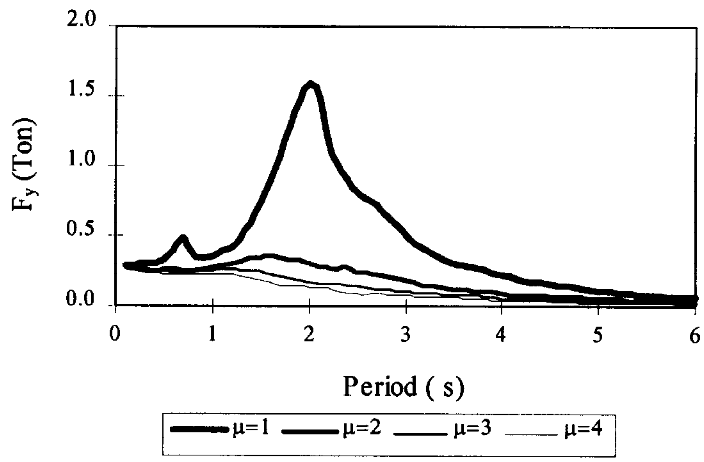

Fig. 1. Average of yield forces for the systems studied

The influence of the α_6 value on the maximum displacement was evaluated. The results are shown in Fig. 2. This presents results of systems with $T=2.1$ s, with two different yield forces, F_{y1}=1.534 Ton and F_{y4}=0.164 Ton, associated to μ=1 and 4, respectively. In this case the *SCT-85* record was used. From Fig. 2 it can be seen that the value of α_6 has a higher influence in the response of systems with small ductility demands than in those with high ductility demands. In what follows α_6=1.0 is adopted.

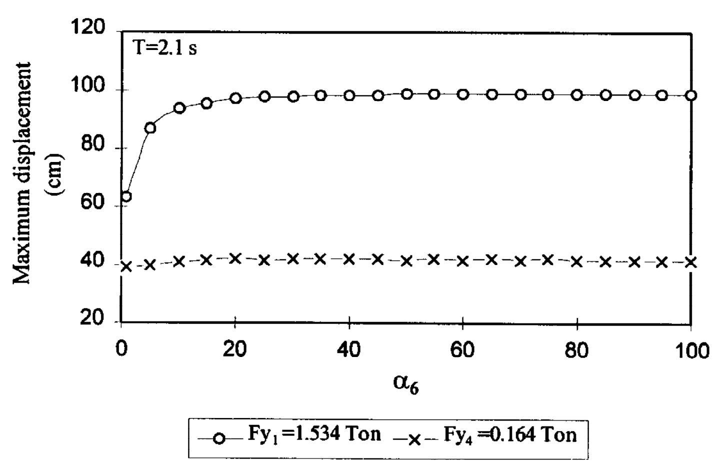

Fig. 2. Influence of α_6 in the maximum displacement

Accuracy

In this section it is analyzed the influence of nonstationarity on the peak standard deviation (*PSD*) of systems with different vibration periods and ductility demands. Such influence is show in Fig. 3. The results associated to the stationary motions are higher than those to the non-stationarity assumption, as expected.

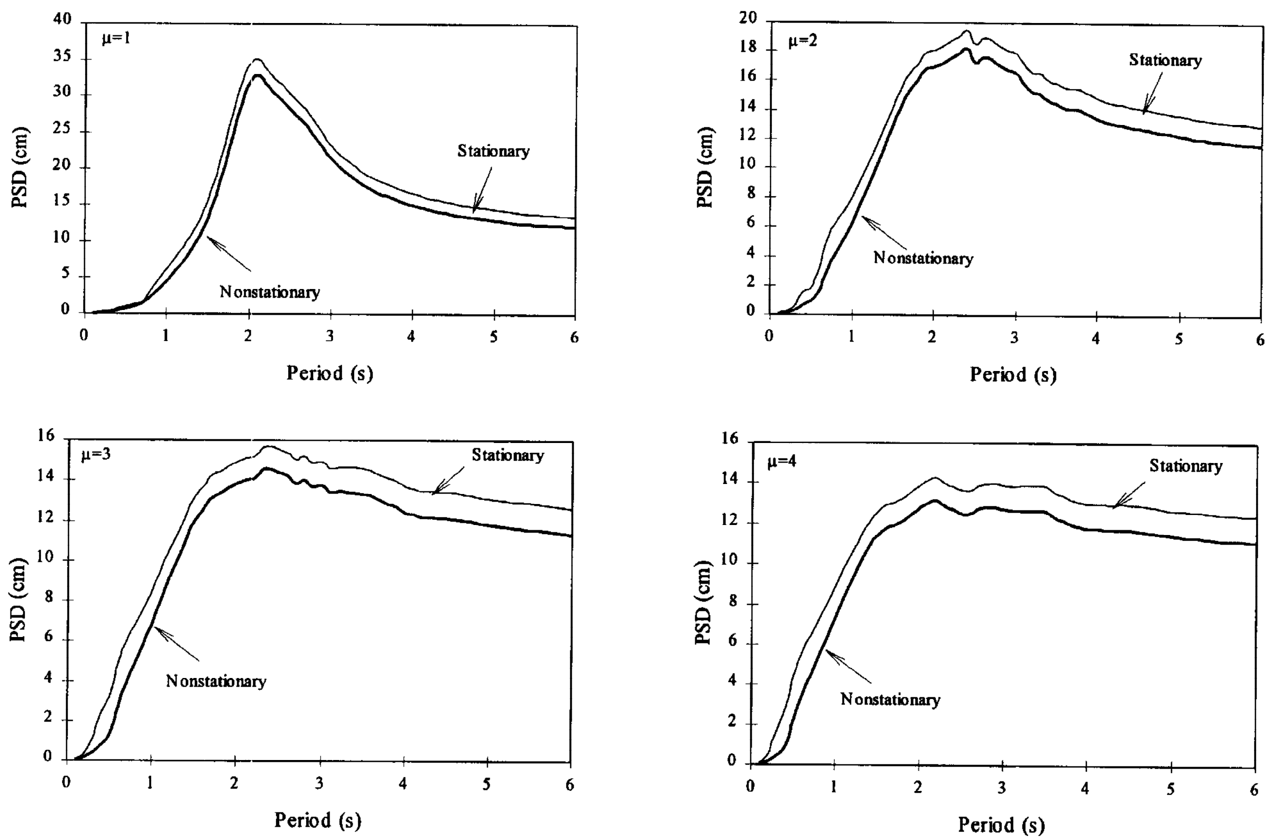

Fig. 3. Influence of the stationarity on the peak standard deviation (*PSD*) of displacement

The ratio $\lambda = PSD_{\text{stationary}} / PSD_{\text{nonstationary}}$ is presented in Fig. 4. The influence of the stationarity is higher for vibration periods smaller than 0.8 s. For periods longer than this, the influence is negligible. From Fig. 4 it is

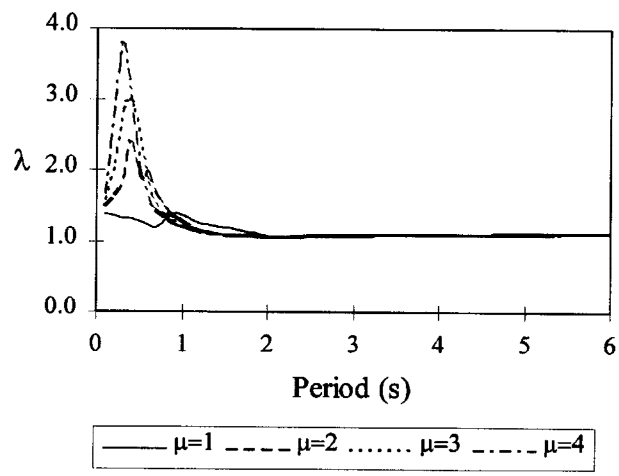

Fig. 4. Ratio $PSD_{\text{stationary}} / PSD_{\text{nonstationary}}$

also seen that the higher the ductility demand value, the higher the differences between nonstationary and stationary results.

Figures 5, 6 and 7 show the *PSD* for three different types of analysis and different μ values. These correspond to stationary, nonstationary and Monte Carlo simulation analysis. The three of them have very similar shapes. The peak factor *r* was obtained by dividing the results of Fig. 7 by those of Fig. 5. Results of the peak factors are shown in Fig. 8, for different ductilities. The calculated peak factors range mainly between 1.5 and 2.5.

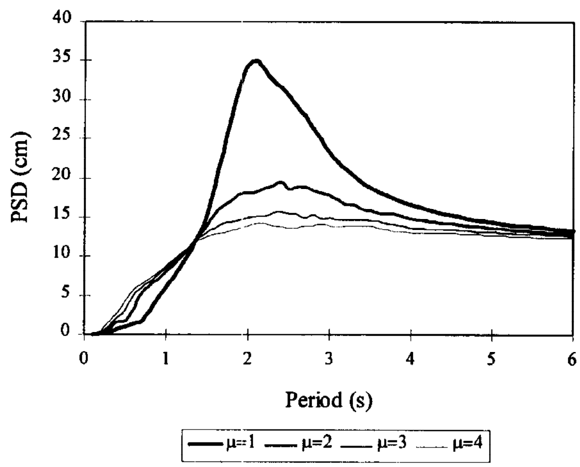

Fig. 5. Stationary analysis

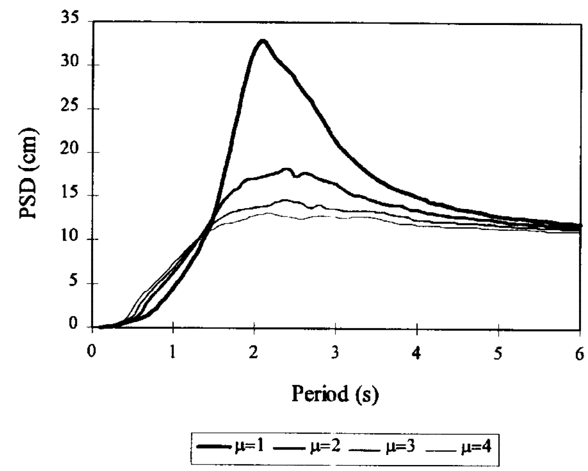

Fig. 6. Nonstationary analysis

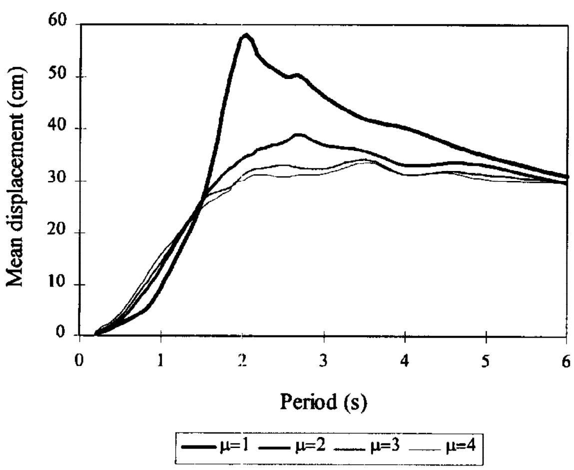

Fig. 7. Monte Carlo simulation

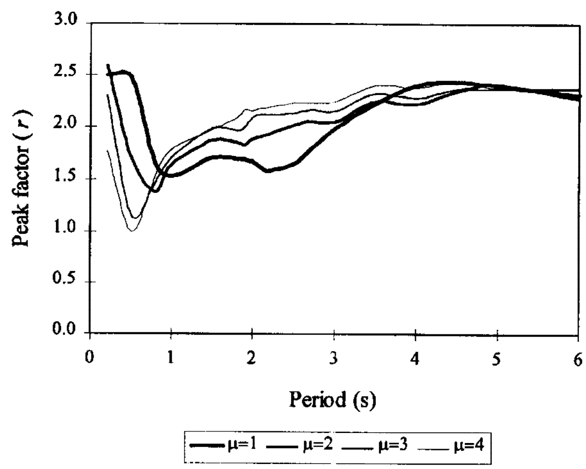

Fig. 8. Peak Factor (*r*)

Histories of the standard deviation of the displacement σ_x were obtained by using two different types of analysis: nonstationary and Monte Carlo simulation. Two systems with the following properties were analyzed: 1) $T=2.1$ *s* and $\mu=1$ (Fig. 9), and 2) $T=3.5$ *s* and $\mu=4$ (Fig. 10). These systems correspond to the maximum displacement obtained for each ductility demand (see Fig. 7). The histories show that the responses are approximately similar for both types of analysis, except in the interval between 25 and 50 s. The reason for this discrepancy could be an unfortunate selection of the modulating function used by Grigoriu *et al*, 1988, or the small number of simulated motions used in this study. The former explanation seems more likely.

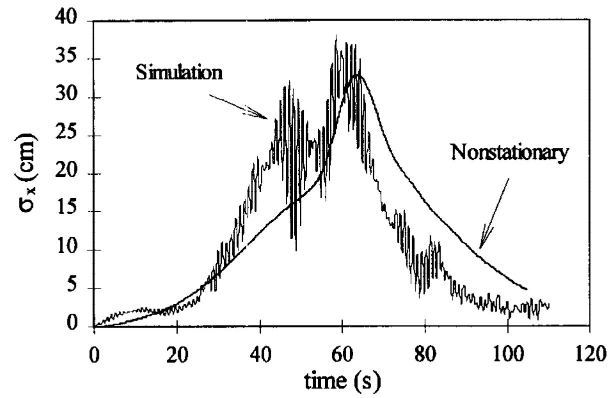

Fig. 9. System with $T=2.1s$ and $\mu=1$

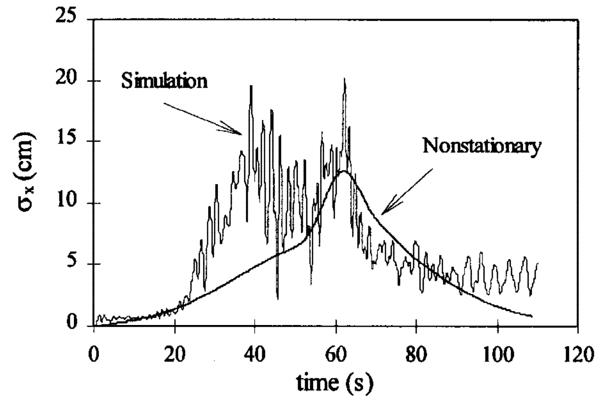

Fig. 10. System with $T=3.5s$ and $\mu=4$

Operative Aspects

The *ELA* is an attractive tool. It can be programmed very easily in a P.C., and it needs smaller computer processing time than Monte Carlo simulation. The following computer times would be consumed in a PC/486/66 Mhz if the total duration of the accelerograms (160 s) were used and if 60 points were necessary to define the spectra shown in Figs. 5, 6 and 7.

Table 1. Computing time

Analysis	Stationary	Nonstationary	Monte Carlo
Computing time	34 minutes	60 hours	118 hours

In this paper, the following considerations were made:

Stationary analysis. The iterations necessary to calculate Σ_Y with the Bartels and Stewart (1972) algorithm were controlled by means of the following expressions (Casciati and Faravelli,1985)

$$C^k = ({C_A}^k - C^{k-1}) / \beta + C^{k-1}$$
$$H^k = ({H_A}^k - H^{k-1}) / \beta + H^{k-1}$$

where k is the iteration number and A is the actual value of C and H corresponding to the k-th iteration. Casciati and Faravelli (1985) use $\beta=10$. In this paper β is taken as 3. The tolerance for the algorithm was 1×10^{-15} and the absolute error allowed in matrixes ${\Sigma_Y}^k$ and ${\Sigma_Y}^{k+1}$ was 1×10^{-10}. The number of iterations was about 100 and 150. In some cases 500 iterations were necessary to obtain convergence.

Nonstationary analysis. In the step-by-step solution an initial tolerance of 1×10^{-10} and an iteration time step of 1×10^{-5} was used. In order to decrease the computing time shown in Table 1, the duration of the excitation was assumed as 80 s instead of 160 s. Thus the real computing time consumed in this study was 30 hours instead of 60 hours mentioned in Table 1.

Monte Carlo analysis. An initial tolerance of 1×10^{-5} and an iteration time step of 1×10^{-5} were assumed. Only 18 systems were used instead of 60, and the duration of the excitation was assumed as 30s instead of 160 s. That duration (30 s) corresponds to the portion associated to the 85% of the total energy of the motion. The real computing time consumed in this study was 7 hours instead of 118 hours mentioned in Table 1.

For larger values of α_6 some numerical problems arose. These are being studied by the authors.

CONCLUSIONS

The accuracy of the Equivalent Linearization Approach was evaluated on the basis of the spectra of peak standard deviations of displacements of hysteretic single of degree freedom systems with ductility demands of μ=1,2,3 and 4, excited with a narrow band process. The following conclusions were obtained.

When Wen's model is applied to hysteretic systems having low ductility demands, the value of α_6 has to be carefully chosen. His influence on the dynamic response of the systems could be important.

For the cases analyzed, a stationary assumption for analyzing long period systems resulted very convenient. For these systems, the differences between stationary and nonstationary results were negligible.

The history of the standard deviation is very sensitive to the shape of the modulation function. Therefore, it is important to choose a realistic modulation function.

For engineering purposes, the Equivalent Linearization is a very efficient and accurate technique for analyzing *SDOF* hysteretic systems subjected to narrow band process. This method can be useful for estimating expected design spectra.

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